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CS747 : Weekly Quiz 1

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a) $$P(S_1 = S_2 = \cdots = S_n) = \sum_{k=0}^{m} P(S_1 = S_2 = \cdots = S_n \mid S_1 = k) \cdot P(S_1 = k)$$

$$= \sum_{k=0}^{m} p_1^k (1-p_1)^{m-k} \cdots p_n^k (1-p_n)^{m-k} \cdot ({}^mC_k)^n$$

$$= \sum_{k=0}^{m} ({}^mC_k)^n (p_1 \cdots p_n)^k [(1-p_1) \cdots (1-p_n)]^{m-k}$$

b) Let $X_1, X_2, \ldots, X_{nm}$ denote the random variables corresponding to each pull where $X_i = 1$ if i-th pull gave 1-reward, otherwise $X_i = 0$.

We have,

$$S = X_1 + X_2 + \cdots + X_{nm}$$

$$\Rightarrow E[S] = \sum_{i=1}^{nm} E[X_i]$$

$$= (p_1 + p_2 + \cdots + p_n) + \cdots \text{ m times}$$

$$= m(p_1 + p_2 + \cdots + p_n)$$

Since X_i and X_j are independent for $i \neq j$, we have:-

$$Var(S) = \sum_{i=1}^{nm} Var(X_i)$$

$$= [p_1(1-p_1) + \cdots + p_n(1-p_n)] + \cdots \text{ m times}$$

$$= m[p_1(1-p_1) + \cdots + p_n(1-p_n)]$$