# CS747 : Weekly Quiz 4

VIBHAV AGGARWAL
190050128

Sol. Let $P(s_t = s)$ denote the probability that the agent is in state $s$ at time $t$ for $t \geq 0$.

Let $t \geq 1$, then by the total probability theorem, we have

$$P(s_t = s) = \sum_{s' \in S} P(s_t = s \mid s_{t-1} = s') \cdot P(s_{t-1} = s')$$

$$= \sum_{s' \in S} T(s', \pi(s'), s) \cdot P(s_{t-1} = s')$$

[By using the def of T]

In the question, $P(s_t = s)$ is represented by $X[t][s]$. So, we get a recursive relation as:

$$X[t][s] = \sum_{s' \in S} T(s', \pi(s'), s) \cdot X[t-1][s'] \quad \text{for } t \geq 1$$

This can be computed using dynamic programming as :-

Input : $S, T, s_{start}, s_{finish}, t, \pi$
Output : $X[t][s_{finish}]$
Pseudocode:

```
For s ∈ S : X[0][s] = 0
  X[0][s_start] = 1
  For t' in {1..t} :
      For s ∈ S:
          X[t'][s] = 0
          For s' ∈ S : X[t'][s] = X[t'][s] + T(s', π(s'), s)·X[t'-1][s']
Return X[t][s_finish]
```

The running time complexity of this algorigthm is $O(t|S|^2)$ because there are $t|S|$ DP states and evaluation of each state takes $O(|S|)$ time.