CS 747 (Autumn 2021): Weekly Quizzes

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Note. Provide justifications/calculations/steps along with each answer to illustrate how you arrived at the answer. You will not receive credit for giving an answer without sufficient explanation.

Submission. Write down your answer by hand, then scan and upload to Moodle. Write clearly and legibly. Be sure to mention your roll number.

Week 1

Question. Consider the family of n-armed bandit instances, $n \geq 2$, in which each arm $a \in \{1, 2, \ldots, n\}$ generates a 1-reward with probability p_a and a 0-reward with probability $1 - p_a$. Thus, each instance of the family is fixed by a vector $(p_1, p_2, \ldots, p_n)$, where $p_a \in [0, 1]$ for $a \in \{1, 2, \ldots, n\}$.

A round-robin algorithm undertakes $m \geq 2$ passes over the set of arms; the sequence of pulls $1, 2, \ldots, n$ is repeated m times. For each arm $a \in \{1, 2, \ldots, n\}$, let s_a denote the number of 1-rewards (interpreted as "successes") from its m pulls, and let f_a denote the number of 0-rewards (interpreted as "failures") from its m pulls (hence $s_a + f_a = m$).

a. For a fixed bandit instance $(p_1, p_2, \ldots, p_n)$, what is the probability that $s_1 = s_2 = \cdots = s_n$? Give your answer in terms of $p_1, p_2, \ldots, p_n$, and m. [2 marks]

b. Denote the total number of successes after the m passes $S = s_1 + s_2 + \cdots + s_n$. What are the mean and variance of S? Again, your answer must be in terms of $p_1, p_2, \ldots, p_n$, and m. [2 marks]

It will help to view the reward given by each pull as a random variable, noting that it is independent of the $(nm - 1)$ others. This view can facilitate an easy computation of the variance of S in part b—in your answer, be sure to explain why.