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CS 747 : Weekly Quiz 7

a) $Pr(h) = Pr(1\to 1)\cdot Pr(1\to 2) \cdots Pr(2\to 3)\cdot Pr(3\to T)$
where $Pr(x\to y)$ denotes the probability of transitioning from state x to state y.

$$\Rightarrow Pr(h) = p_{11}^3 \cdot p_{12}^3 \cdot p_{21}^3 \cdot p_{13} \cdot p_{32} \cdot p_{23} \cdot p_{3T}$$

b) We know, using Total probability theorem, that $\sum_{s'\in S} p_{ss'} = 1 \quad \forall s \in S\setminus\{T\}$

$$Pr(h) = (p_{11}^3 \cdot p_{12}^3 \cdot p_{13})(p_{21}^3 \cdot p_{23})(p_{32} \cdot p_{3T})$$

Now to maximize $p_{11}^3 \cdot p_{12}^3 \cdot p_{13}$ given the constraint $p_{11}+p_{12}+p_{13}+p_{1T} = 1$, clearly we need to set $p_{1T} = 0$ (otherwise we will need to decrease some other value which is not optimal).

So, we have $p_{11}+p_{12}+p_{13} = 1$

- By AM-GM inequality,

$$\frac{3\left(\frac{p_{11}}{3}\right) + 3\left(\frac{p_{12}}{3}\right) + 1\left(\frac{p_{13}}{1}\right)}{3+3+1} \geq \left[\left(\frac{p_{11}}{3}\right)^3\left(\frac{p_{12}}{3}\right)^3\left(\frac{p_{13}}{1}\right)\right]^{\frac{1}{7}}$$

LHS $= 1/7 \to$ a constant.

The equality is achieved when $\frac{p_{11}^*}{3} = \frac{p_{12}^*}{3} = p_{13}^*$

$$\Rightarrow p_{11}^* = p_{12}^* = \frac{3}{7} \text{ and } p_{13}^* = \frac{1}{7}$$

Similarly, we can calculate the other values. The final values are :-

p_{11}^*	p_{12}^*	p_{13}^*	p_{1T}^*	p_{21}^*	p_{22}^*	p_{23}^*	p_{2T}^*	p_{31}^*	p_{32}^*	p_{33}^*	p_{3T}^*
$\frac{3}{7}$	$\frac{3}{7}$	$\frac{1}{7}$	0	$\frac{3}{4}$	0	$\frac{1}{4}$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$