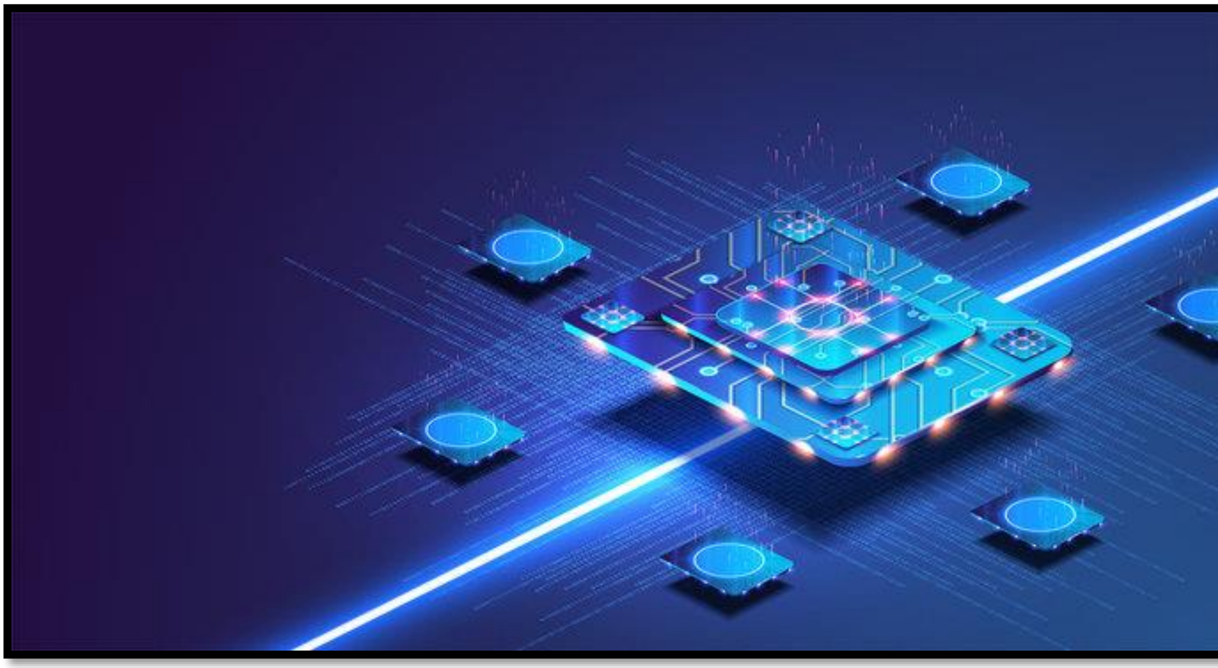
QUANTUM ENTANGLEMENT & QUANTUM COMPUTING ASSIGNMENT - ISA05



NAME: VIBHAV V K SAMAGA

SRN: PES1UG20EC221

SEC: D

SEM: 5

PROBLEM STATEMENT: *TO FIND OUT THE AMPLITUDE OF THE STATE |00> FROM A GIVEN QUANTUM CIRCUIT WHOSE FUNCTION IS BALANCED*

INTRODUCTION:

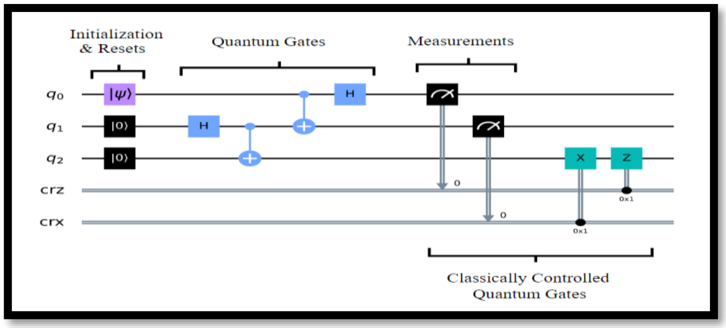
This quantum assignment mainly deals with the implementation of the Deutsch-Jozsa Algorithm for building specific quantum circuits, and involves us to figure out the amplitude of a given state provided that the function involved in the quantum circuit is balanced. This obviously raises many questions like : "What is a Quantum Circuit" , "What is the Deutsch-Jozsa algorithm and why is it important" and "What are balanced functions". All these questions will be answered by the end of this report. So to begin with let us first understand what a Quantum Circuit is.

WHAT IS A QUANTUM CIRCUIT?

Now a quantum circuit is a computational routine consisting of coherent quantum operations on quantum data, that are essentially qubits, and measurements.

As studied before in the previous unit, quantum operations can be performed on quantum data by using quantum gates according to your specific output needs, that is different quantum gates yield different outputs.As mentioned before, the basic skeletal structure of a quantum circuit involves : i) Initialization and Resets ii) Quantum Gates iii) Measurements (refer Fig 1.1)

Now, it is imperative to understand that sometimes adding a lot of quantum gates can result in a lot of confusion and can get quite convoluted. So to avoid that we resort to using functions but at times we need to construct our functions on the basis of out desired outputs. In such cases, we need to find out certain characteristics of our quantum machine or function. To do so, we construct something known as an Oracle or, in simple terms, a Black Box.



Example of a Quantum Circuit (Fig 1.1)

WHAT IS AN ORACLE?

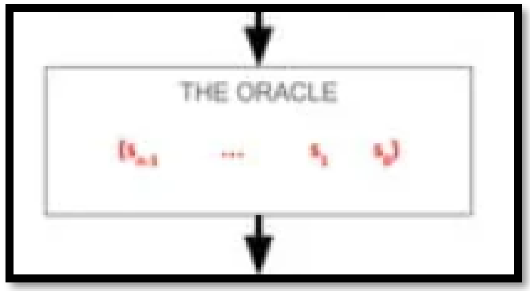
An Oracle is an abstract machine used to study decision problems. This is the more general definition of an Oracle, but the quantum definition is that an Oracle is an unexposed operation that is used as input to another algorithm.(refer Fig 1.2)

Often, such operations are defined using a classical function $f:\{0,1\}^n\rightarrow\{0,1\}^m$, which takes an n-bit binary input and produces an m-bit binary output.

There are many types of Oracles but one of which is very common is the Phase Oracle which involves encoding a function , 'f', into an Oracle, 'O', by applying a phase based on the input to O.

For example, you might define O such that : $O|x\rangle=(-1)pow(f(x))|x\rangle$.

Choosing the best way to implement an oracle depends heavily on how this oracle is to be used within a given algorithm. One such algorithm which uses this concept is the Deutsch – Josza Algorithm, which leads us to the next question as to what is the Deutsch-Josza Algorithm?



Oracle (Fig 1.2)

THE DEUTSCH-JOSZA ALGORITHM:

The Deutsch-Jozsa Algorithm was the first example of a quantum algorithm that performs better than the best classical algorithm. It showed that there can be advantages to using a quantum computer as a computational tool for a specific problem.

THE PROBLEM:

We are given a hidden Boolean function f, which takes as input a string of bits, and returns either 0 or 1, that is:

f({x0,x1,x2,...})→0 or 1 , where xn is 0 or 1

The property of the given Boolean function is that it is guaranteed to either be balanced or constant. Now what is a constant and a balanced function? A constant function returns all 0's or all 1's for any input given, while a balanced function returns 0's for exactly half of all inputs and 1's for the other half.

Our task is to determine whether the given function is balanced or constant. It is also important to note that that the Deutsch-Josza problem is an n bit extension of the single bit Deutsch-Josza problem.

Now there are two solutions to this problem : The Classical Solution and The Quantum Solution.

1)THE CLASSICAL SOLUTION:

Classically, in the best case, two queries to the oracle can determine if the hidden Boolean function, f(x), is balanced: e.g. if we get both f(0,0,0,...)→0 and f(1,0,0,...)→1, then we know the function is balanced as we have obtained the two different outputs.

This is the easiest way to find out if a function is balanced or constant. Furthermore, if n bits are present and if 2^(n-1) +1[more than 50%] states yield 0 or 1, then it implies it is a Constant Function. Failing the above equation would thus mean it's a Balanced Function.

If we get the same result continually in succession, we can express the probability that the function is constant as a function of k inputs as:

$$P_{\text{constant}}(k) = 1 - \frac{1}{2^{k-1}} \qquad \text{for } 1 < k \leq 2^{n-1}$$

2)THE QUANTUM SOLUTION:

Using a quantum computer, we can solve this problem with 100% confidence after only one call to the function f(x), provided we have the function f implemented as a quantum oracle, which maps the state |x⟩|y⟩ to |x⟩|y⊕f(x)⟩, where ⊕ is addition modulo 2.(refer fig 1.3)

Now there are a series of steps that need to be followed to arrive to this solution by constructing a suitable quantum circuit.

Step 1- **We must have two quantum registers, the first is an n-qubit register initialized to state 0 and the second is a one qubit register initialized to state 1.**

$$|\psi_0\rangle = |0\rangle^{\otimes n}|1\rangle$$

<u>Step 2-</u> Apply the H Gate to both the qubits

$$|\psi_1\rangle = \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x\rangle \left(|0\rangle - |1\rangle\right)$$

<u>Step 3-</u> Apply the Quantum Oracle, which over here maps :
|x⟩|y⟩ to |x⟩|y⊕f(x)⟩

$$\begin{aligned} |\psi_2\rangle &= \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x\rangle(|f(x)\rangle - |1 \oplus f(x)\rangle) \\ &= \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} (-1)^{f(x)} |x\rangle(|0\rangle - |1\rangle) \end{aligned}$$

<u>Step 4-</u> Next apply the H Gate to only the first register

$$\begin{aligned} |\psi_3\rangle &= \frac{1}{2^n} \sum_{x=0}^{2^n-1} (-1)^{f(x)} \left[\sum_{y=0}^{2^n-1} (-1)^{x \cdot y} |y\rangle\right] \\ &= \frac{1}{2^n} \sum_{y=0}^{2^n-1} \left[\sum_{x=0}^{2^n-1} (-1)^{f(x)} (-1)^{x \cdot y}\right] |y\rangle \end{aligned}$$

<u>Step 5-</u> Measure the first register

Now If the measured value is a 0 state then it implies that f(0)=f(1), which further implies that the Oracle used is a “CONSTANT” one, else it implies that f(0) not equals f(1) and the Oracle used is a “BALANCED” one.(refer fig 1.4)

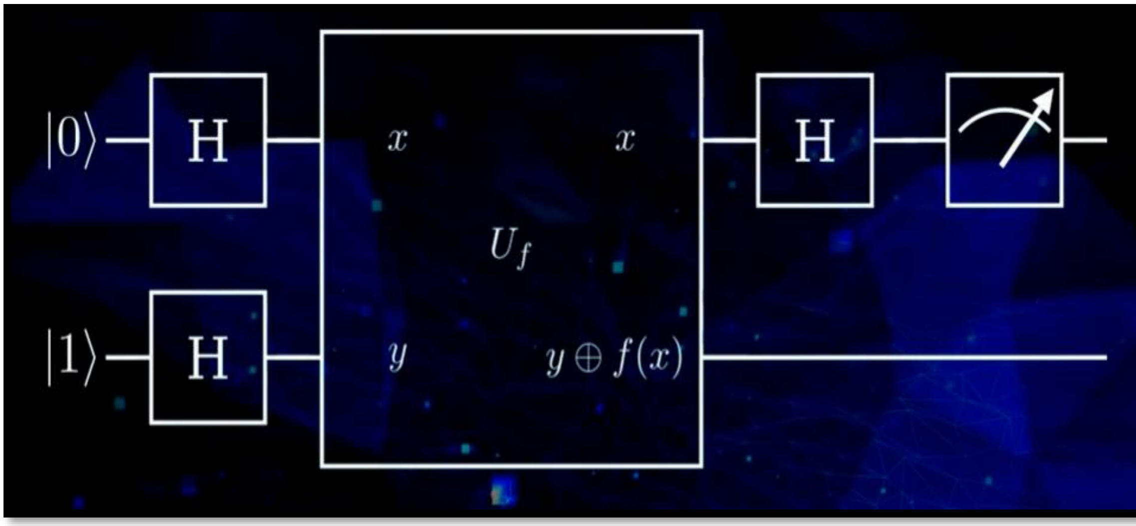
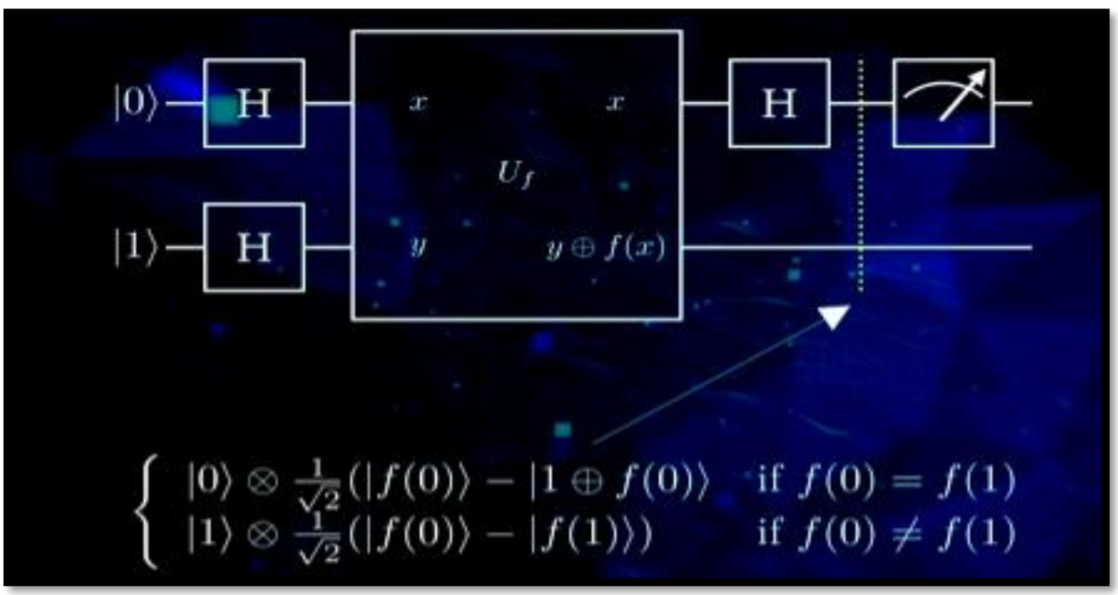


Diagram of the Quantum Solution to the Deutsch-Josza Problem(Fig 1.3)



Final Solution before measurement(Fig 1.4)

IMPLEMENTATION:

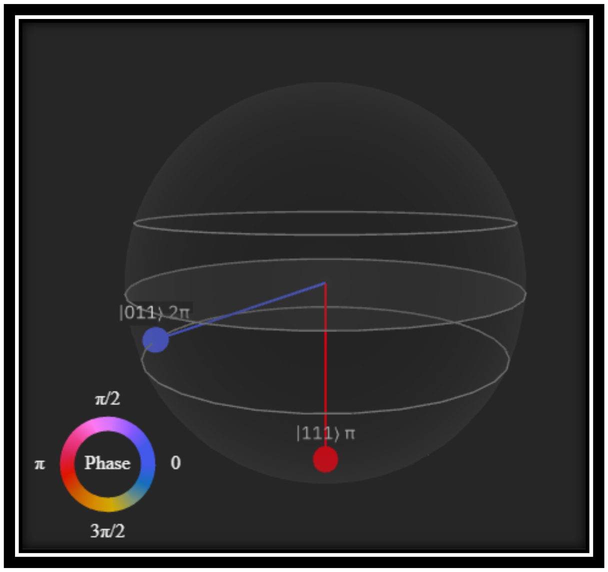
This was implemented on IBM Quantum Composer and uses QASM 2.0 PL. Here it was given that the Oracle is predefined to be a balanced one and we are to find the amplitude of 00 State.

CODE:

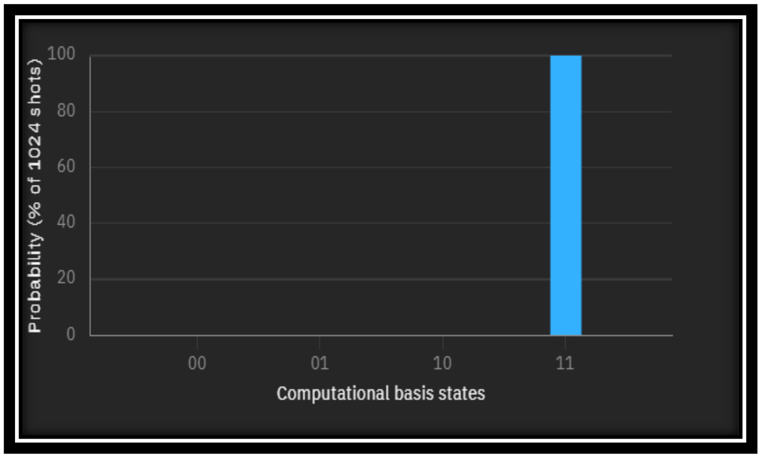
```
OPENQASM 2.0;
include "qelib1.inc";

gate oracle_balanced q0, q1, q2 {
  cx q0, q2;
  cx q1, q2;
}
 qreg q[3];
 creg c[2];
reset q[0];
reset q[1];
reset q[2];
x q[2];
h q[0];
h q[1];
h q[2];
oracle_balanced q[0], q[1], q[2];
h q[0];
h q[1];
measure q[0] -> c[0];
measure q[1] -> c[1];
```

CIRCUIT:

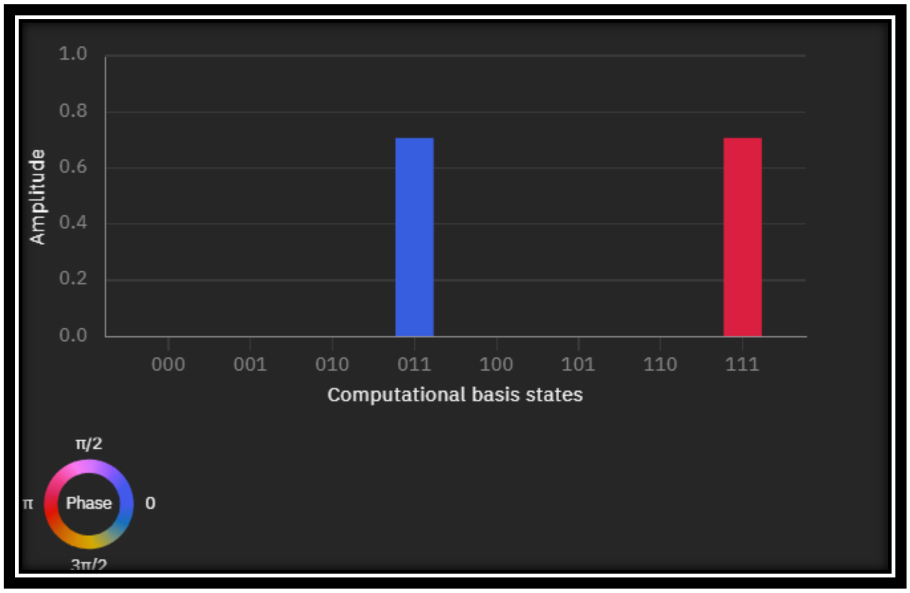
BLOCKSPHERE GRAPH:



PROBABILITIES GRAPH:



STATEVECTOR GRAPH:



CONCLUSION AND RESULT:

We can see that since the Oracle is a BALANCED one, the output state vector shows a 100% probability of being 11 which is true to the output as it shouldn't yield 00. So the Deutsch Josza Algorithm is a very versatile and powerful algorithm capable of outperforming regular classical functions!

Hence, to finalise the result, we can conclude that the Amplitude of 00 State will be 0 and states 111 and 011 will have amplitude 0.707.

REFERENCES:

1)https://www.sciencedirect.com/topics/engineering/hadamard-gate

2)https://www.sciencedirect.com/topics/mathematics/hadamard-gate

3) https://qiskit.org/textbook/ch-states/single-qubit-gates.html#xgate

4) https://en.wikipedia.org/wiki/Controlled_NOT_gate

5)https://en.wikipedia.org/wiki/Deutsch%E2%80%93Jozsa_algorithm

6) https://en.wikipedia.org/wiki/Oracle_machine

7) https://en.wikipedia.org/wiki/Quantum_decoherence

8)https://www.youtube.com/watch?v=_BHvE_pwF6E&ab_channel=sentdex

9) https://qiskit.org/textbook/ch-algorithms/defining-quantum-circuits.html

10) https://learn.microsoft.com/en-us/azure/quantum/concepts-oracles

11)https://qiskit.org/documentation/stubs/qiskit.circuit.library.PhaseOracle.html

12)https://www.youtube.com/watch?v=47FxvVZA4GM&ab_channel=PSPACE

13) https://qiskit.org/textbook/ch-algorithms/deutsch-jozsa.html

14) https://quantum-computing.ibm.com/composer/docs/iqx/guide/deutsch-jozsa-algorithm

15) https://quantum-computing.ibm.com/composer/docs/iqx/guide/grovers-algorithm

16) https://quantum-computing.ibm.com/composer/docs/iqx/build-circuits#build-your-circuit-with-openqasm-code