## Assignment - 1

mean of X & Y.

To show y & x lie on the line we need
to prove y = w|x + b where w, & b are
parometers obtained from regression.

So, Let yi represent value of y fax ith somple &
Some follows for n'

Let the linear agression model be  $\tilde{\gamma} = w_1 x + b$ 

Now y = y (i) + Ep (True value + Exxox).

y" = w, x" + E; +b

 $y^{(1)} = w_1 x^1 + b + e_1$  $y^{(2)} = w_1 x^{(2)} + b + e_2$ 

y(n) = w, x(n) + b + En.

Summing above  $w_1 \leq x^{(i)} + bn + \leq \epsilon_i$   $\lim_{i \leq 1} |x_i|^2 = |x_i|^2 = |x_i|^2 + |x_i|^2 + |x_i|^2 = |$ 

dividing by n

1 \( \Sigma \) \( \text{y'} \) = \( \text{w}\_1 \) \( \text{S}\_2 \) \( \text{y'} \) \( \text{y'}

So, SIM to Show that EE; =0, Sum of mean square etwos = \(\frac{1}{2}(y; -\tilde{y};)^2\) MSE = \( \( \forall \) - \( \mu\_1 \) \( \tau\_2 \) For this to be min  $\frac{\partial MSE}{\partial b} = 0$ =) \( \frac{7}{2} \left( \frac{1}{9} \cdot - \partial \cdot \text{1}} \cdot - \partial \cdot \text{2} \left( \frac{1}{9} \cdot \text{2} \left( a) NOW E; = y; -w, n; -b so -25 e, = 0 =) \( \sum\_{15} \) \( \sum\_{15} \) \( \sum\_{15} \) 50,  $y = w, x + b + E_B$ Hence (4, x) lie on the least square fit line

Px,y denote worelation coefficient between x & y Nov, given Py. Z > 0 Dorways Px, y need not be greater than So, using partial correlation,  $\int_{1-\gamma^2 x_2}^{2x_1-2} \sqrt{1-\rho^2 x_4}$ =)  $\int_{XY} = \int_{XZ} \cdot \int_{YZ} + \int_{X7.2} \int_{1-P_{XZ}}^{2} \int_{1-P_{YZ}}^{2}$ NOW by condition of pastial well-18/27.2 (). Pxy = Px2 Py2 + BQ J1-P2 J1-P2 so the upper & lower bound of Pxy are 8x7 942 + J1-92 J1-92 & Sx2 92 - J1-92 J1-92

respectively.

NOW assuming fr = 0.75 & fyz = 0.8 which indicates strong cosselotion by them Bxy E (10.75)10.8) - JI-0.75 JI-0.82, 10.75)10.8) + JI-0.75 JI-0.87 Pxy 6 [ 0.6 - Jo.43\$5 Jo.36, 0.6 + Jo.4375 Jo.36] Px,4 E [ 0.6 - 0.39 , 0.6 + 0.39] Jx, y & [0.21, 0.99], So gry lan be between 0-21 to 0.3 as well, which indicates a work cooxelotion bin x, y, Hence a strong cost clarin is not guaranteed always

DIE Given: X1, X2, X3 - Xn a sequence I iid handom variables  $E(x_1) = E(x_2) = -- E(x_1) = y$ To Prove! - Fox n > 0 1+ x1+x2+- xy -> 4 10 let sn = x1+x2+- xm  $\frac{1+1}{n-1}P\left(\left(\frac{s_n-y}{n}-\frac{y}{n}\right) \neq e\right) \longrightarrow 0 \quad \text{for any } e>0$  $\frac{Pxool}{Sn} = X_1 + X_2 + \cdots + X_n$  $S_n = \frac{S_n}{n}$  $S_n = X_1 + X_2 + \dots + X_n$ 5 = 4n 5n = 4



Now, By Lhebysheu's inequality,

$$P\left[|S_n-u|/J_{\ell}\epsilon\right] \leq \frac{Vox(S_n)}{\epsilon^2}$$

Now

$$Vox(S_n) = Vox(X_1+X_2+-+X_n)$$

= 12 VOS (X1+X2+-. +Xn)

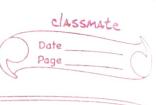
= 
$$\left[ \sqrt{2} \left( \sqrt{2} \left( x_1 \right) + - \sqrt{2} \left( x_2 \right) \right] \right] \left( \frac{2}{110} \right)$$

 $= \frac{\eta 6^2}{\eta^2} \qquad (as \ Vos(x_1) = Vos(x_2) = Vos(x_n) = 6^2 \text{ as they}$ 

 $\frac{\sqrt{\sqrt{\sqrt{2}}}}{\sqrt{\sqrt{2}}} = \frac{6^2}{\sqrt{\sqrt{2}}}$ 

 $\left(\begin{array}{c} axtr \\ n>0 \end{array}\right)$ 

Henre Proved.



To Derive: Maximum A Posseriosi (MAP) Son for 20 linear eigension

NOW, y= WTX + E

6 ~ Nosmai (0,52)

Hen  $\alpha$  y/x Normal  $(w^Tx, \sigma^2)$   $\Rightarrow P(y/x) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-(\eta_i^Tw - y_i^T)^2}$ 

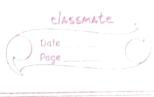
N= argman Plw/y, , x, y, x2 -- yn, xn)

W = argman Ply, x,, - yn, x, 1w) Plw) | Boys (Rule) w Ply, x, yn, xn)

= azgnaz Ply, x, = yn, xn [w) P(w)

Now we assume w to be w ~ N(0, T27) So, PIW) = 1//1 1 VZ + 1/2 VZ TZ Z

 $\frac{\rho(w)}{\sqrt{2\pi}} = \frac{1}{2} \frac{-\omega^2}{2\tau^2}$ 



Now Ply1, x, -- yn, xn | w) = IT Ply1, x1 | w) ] Now Play (2) = P(y|x,z) P(x|z) (as y'depends (x,y) = P(y|x,z) P(x|z) on x' & are  $SO, P(y, n, -y_n, n_{D}|w) = \prod_{i=1}^{N} \left[ P(y; |x_i, w) P(x_i|w) \right]$  $w = \underset{\sim}{\text{argmox}} \prod_{j=1}^{N} P[(j; | x; , \omega i)] P(x; | \omega) - P(\omega)$ = argmar [T P[y; |xi, w] P(xi). P(w). = argmor [ [log P(y; | N; , W) + log P(w)] - (as P(N;)) is constant)  $= \arg \max_{i=1}^{\infty} \left[ \log \left( \frac{1}{\sqrt{2\pi\sigma^2}} \right) + \log \left( e^{-\frac{(n_i)^T W - y_i)^2}{2\sigma^2}} \right) + \log \left( \frac{1}{2\Gamma^2} \right) \right]$   $= \log \left( \frac{1}{\sqrt{2\pi\Gamma^2}} \right)$ = dogmin 1 · E (x, Tw-y;)2 + 21 w Tw 7 constant so ignored.  $w = \underset{N}{\operatorname{orgmin}} \frac{1}{1} \neq (n; Tw - y;)^{2} + \lambda (iwTw) / \underset{n \neq 2}{\operatorname{where}}$