

Se HW-3 Vibhor Agarwal 2020349

Crivent DER, A is triangular.
To Prove. 9t's eigenvalues are its diagonal element

Non A being triangulax matrix will always be mon singular:

We know jor triangular matrices det (A) = product of diagonal entries.

ie [a11 0 0], for all matrices of cuch kind |

a21 a22 0 & any dimensions - nxn,

a31 a32 a33 new have always have

(determinant) = P T g; deisn.

Now, A Let à se on eigenvalue.

digonal mobile so non diagonal entries genoin

un touched

So, |A-XI|=0 for eigenvalue.

ie $\pi |\alpha_{ii} - \lambda| = 0$

Due

But this means $\lambda = 9 \%$ for atteast is [1, n]

So this means that eigenvalues in case of triangulars
mutin is its diagonal entries.

03

Given: u, v e R, uTg=1.
A = ugT

To Find: Eigen values of A.

Now we know Rank(AB) = min(RankA), Rank(B))

Since U, of are now & column vectors

Rank(u) = Rank(UT) = 1

ie Rank (A) = Rank(vot) = 1

Mence A is not full hank.
But this means we will have Roots: O
as eigenvalues of A. Here A is always diagonalisable it

Uv 70.

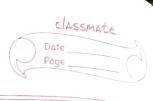
Now from previous Qz ROSUIT.

RONK (UUT) = No. of non zero eigen values.

⇒ No, of non zero eigenvalues = 1.

Hence zero eigenvalues = n-1. [AS A most howen eigenvalues]

Co moltiplicity of \$0 = n-1.



Now for other nonzero eigenvalue, we propose

= volument de les ligenvalues We now verify this, by taking u as eigenvector

 $Au = uv^Tu$ WA = NOTU

So, Au=u=u=uoTu.
This indeed verifies oTu is eigenvalue of A=uoT

So the eigenvalues of A are a O, UTu.

Now for power iterations.

Ux = x, K (av + c) (Az) K + -- + Co (An) On

Enhere (1,0,) are (eigenvalue, eigenvector) pair.

8 up denotes estimated eigenvector in kthitetration.

Gi, Cz - cn E R.

NOW 1,=1 & Az, Az - - In=0, so,

4x=1×[40+--0]

Yz = CIU, which is independent of K.
So it will converge in just a stugle instial
literation & won't converge any further.

Je chow: Eigenvalues of symmetric the definite are
real & 70.

So, jet's accome $Ax = Ax & x \neq 0$. Hien, $A = A^{T}$.

 $\lambda \pi^{T} x = \pi^{T}(\lambda \pi)$ $= \pi^{T}(A\pi)$ $= \pi^{T}($

 $= (A\pi)^{T} \times$ $= (A\pi)^{T} \times$

 $\frac{2}{2} \left[\frac{\lambda \lambda}{\lambda} \right] = 0$

Nov since xx +0 Bs x +0 so , x= x.

But this means & is also Heal.

Now we need to show >70

Let A be a symmetric mean the definite making ie, xTAX70 + x eR". [dej'nik] Now let x be one a corresponding eigenvector $A x = \lambda z$ $\Rightarrow x^{T} A x = x^{T} \lambda x$ $\frac{1}{2} = \frac{1}{2} \frac{1}{12} \frac{1}{12}$ NOW XTAX 708 11711270 ζο, λ 70 Hence A70 and is heal which completes

Grivan: A $\in \mathbb{R}^{n \times n}$ is symetric & +ve definite.

i.e. eigenvalues are sean & +ve. $e^{A} = \underbrace{\mathbb{E}}_{K=0} \underbrace{\mathbb{E}}_{K} \underbrace{\mathbb{E}}_{K}$

Since A is symmetric so A is also diagonalisable.

So $A = Q D Q^{-1}$ where Q is a orthogonal modine with n is independent vectors. C $D = \begin{bmatrix} \Omega, & 0 & 0 & -1 \\ 0 & \lambda & 2 & 0 & -2 \end{bmatrix}$

0 220-c

Nov

 $e^{\gamma} = \begin{bmatrix} \frac{2}{2} \frac{\lambda_1}{\lambda_1} & 0 & 0 & -\frac{1}{2} \\ \frac{2}{2} \frac{\lambda_1}{\kappa_1} & 0 & 0 & -\frac{1}{2} \\ \frac{2}{2} \frac{\lambda_2}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} \\ 0 & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} \\ 0 & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} \\ 0 & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} \\ 0 & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} \\ 0 & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} \\ 0 & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} & \frac{2}{2} \frac{\lambda_1}{\kappa_1} \\ 0 & \frac{2}{2} \frac{\lambda_1}{\kappa_1} &$

 $e^{2} = \begin{bmatrix} a^{1} & 0 & 0 & - & - & \\ 0 & e^{2} & 0 & - & \\ & & & \\ & & & \\ &$

$$A = QDQ^{-1}$$

$$\Rightarrow Q^{-1}AQ = D$$

$$e^{D} = \sum_{k=0}^{\infty} (Q^{-1}AQ)^{k}$$

$$A^{2} = QDQ^{-1}QDQ^{-1}$$

$$A^{3} = QD^{2}Q^{-1}QDQ^{-1}$$

$$A^{3} = QD^{2}Q^{-1}QDQ^{-1}$$

$$A^{3} = QD^{2}Q^{-1}QDQ^{-1}$$

$$A^{4} = QD^{4}Q^{-1}$$

$$A^{5} = QD^{4}Q^{-1}QDQ^{-1}$$

$$A^{6} = QD^{6}Q^{-1}$$

$$A^{7} = QD^{7}Q^{-1}$$

$$A^{7} =$$

Vector to di eigenvalue of A

de liver! - A & R " A is non defective .

To lave: Rook (A) = No. of non zero eigenvalues of

Sinc A is diagonalisable. [Ais non define iff diagnolis

A = Q \(\int \alpha \) where \(\int \int \alpha \) is
Inversible as A is non defective so all eigenvectors who linearly independent NOW ROOK (OA) = ROOK (A) when Q is invertible

So, QAX=0 iff AX=0 ie QA&A has the

NOW AERUAM & DERMAN.

So, Mank (QA) = Hank (A)

A By Rank So Gank (QA) = n- Ken (QA) Nollity]

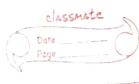
Mank (Q) = n- Ken (A) = n- Ken (QA)

20 PRANK (A) ~

MANK(A^T) mank (AR) = Mank (AR)) = Mank(QTAT) as QT is invertible & honk(QA) = honk
(A) < nan K(AT)

= Mank(A).

So eank (OA) - Hank (AO) = Man & CA),



mark (OA) - stank (A) MONK (AO) = Mank (A) [Multiplying by Pank (0 AQ) = Mank (A) a nuchible as grank (A) = honk (D). Q is invertible But we know tank of a diagonal makix is no . of non zero entries or eigenvalues So, Mank(A) = Noi of non zero eigenvolves of A.

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SC Assignment 3

2020349

Problem 6

The results that we obtain in this problem are as follows

- We see that the actual eigenvalues found by numpy function is [11 -2 -3]
- The Normalised Power Iteration as expected can find the largest magnitude eigenvalue 11.
- The corresponding eigen vector found is [0.5 1 0.75] which is very close to the actual eigenvector found by numpy ie. [0.371 0.742 0.557]
- Inverse Power Iteration can also find the eigenvalue of the smallest magnitude ie 2 However its not able to find the correct sign because we are using vector infinite norm for getting the magnitude which is always positive but the found magnitude is correct.
- The eigenvector found by inverse iteration corresponding to eigenvalue -2 is [-0.2 -0.4 1] which is close to actual eigenvector of [0.182 0.36 -0.91]. Note that we can always multiply eigenvector by a scalar and so on multiplying it by -1 gives us [0.2 0.4 -1] which is fairly accurate as compared to eigenvector found by Numpy.

Problem 7

```
Shifted Inverse Iteration:
The found eigenvalue is 2.133074475348525
The corresponding eigenvector is [-0.60692002 1. 0.34691451]
Numpy eigenvalues and eigenvectors:
The eigenvalues are [0.57893339 2.13307448 7.28799214]
The corresponding eigenvector columns are
[[-0.0431682 -0.49742503 -0.86643225]
[-0.35073145 0.8195891 -0.45305757]
[ 0.9354806 0.28432735 -0.20984279]]
```

- The actual eigenvalues found by numpy function is [7.28 2.13 0.57]
- We see that shifted inverse iteration can find the eigenvalue nearest to 2 ie. 2.133
- The eigenvector found by Shifted Inverse Iteration is [-0.606 1 0.3469] and that found by numpy corresponding to 2.133 is [-0.49 0.81 0.28] which is fairly close to that by shifted inverse iteration.

Problem 8

The results that we obtain are as follows

```
Rayleigh Quotient Iteration:
The found eigenvalue is: 11.000000000000018
The corresponding eigenvector is: [0.5 1. 0.75]
The convergence rate is: 0.2340098684432489
Numpy eigenvalues and eigenvectors:
The eigenvalues are [11. -2. -3.]
The corresponding eigenvector columns are
[[3.71390676e-01 1.82574186e-01 -5.26283806e-16]
[7.42781353e-01 3.65148372e-01 -5.54700196e-01]
[5.57086015e-01 -9.12870929e-01 8.32050294e-01]]
```

- The eigenvalue that we obtain using Rayleigh Quotient Iteration for matrix in Q6 is 11. We take a random matrix here. We observe from the result of Numpy Eigenvalue that 11 is indeed an eigenvalue of the matrix.
- The corresponding eigenvector found by Rayleigh Quotient Iteration is [0.5 1 0.75] which is close to the corresponding eigenvector of 11 found by Numpy as [0.37 0.74 0.557]
- The largest magnitude eigenvalue is found by Numpy is 11 and we calculate the convergence rate using 11 and the approximate convergence rate comes out to be 0.23. For subsequent iterations this convergence rate comes close to 2 and 6.26.

Problem 9

The results that we obtain are as follows

```
For matrix in Q6:
Eigen values using QR Iteration: [11. -3. -2.]
Eigen values using numpy: [11. -2. -3.]
For matrix in Q7:
Eigen values using QR Iteration: [7.28799214 2.13307448 0.57893339]
Eigen values using numpy: [7.28799214 2.13307448 0.57893339]
```

As we observe for Matrix is problem 6 the eigenvalues are [11,-2,-3] and that obtained by our QR Iteration are also [11,-3,-2] which is exactly the same.

As we observe for Matrix is problem 7 the eigenvalues are [7.28,2.13,0.57] and that obtained by our QR Iteration are also [7.28,2.133,0.578] which is the same and hence verifies our algorithm.