Algorithms-II
Autumn 2022
Tutorial- 1

1. Consider a stack with a fixed size K with PUSH, POP operations (assume PUSH operations fail in O(1) time if stack is full and POP operations fail in O(1) time if stack is empty). Which of the following are NOT valid potential function for amortized analysis of a sequence of n PUSH and POP operations (choose all that apply)?

   a) The number of elements in the stack
   b) The number of free spaces in the stack
   c) The number of free spaces - the number of elements in the stack
   d) The number of free spaces + the number of elements in the stack

2. A sequence of stack operations is performed on stack whose size never exceeds k. After every k operations, a copy of the entire stack is made for backup purposes. Show that the cost of n stack operations, including copying the stack, is O(n) using the accounting method.

3. Consider the implementation of a queue using two stacks A and B (I am sure you all know this implementation). Find the amortized cost of a sequence of n enqueue and dequeue operation using each of aggregate, accounting, and potential method.

4. Suppose a graph, in addition to capacities on edges, has a capacity on every vertex such that the total inflow (and therefore the total outflow from the node) cannot exceed the capacity of the vertex. The other constraints remain the same as in the standard maximum-flow problem. How will you find the maximum flow in this graph?

5. Suppose all capacities in a graph are distinct. Will the max-flow be unique?

6. Consider a p × q matrix X. Each element of X is a non-negative real number. However, the sum of the elements in any row and in any column is an integer (may be different for different rows and columns). Construct a p × q matrix Y such that each element of Y is a non-negative integer, and the sum of the elements in any row and in any column in Y is the same as the sum in the corresponding row and column respectively in X. You must use maximum flow concepts to construct Y.

7. A company running a factory has to arrange for extra maintenance staff to work over vacation periods.
   - There are K vacation periods in the year, numbered from 1 to K, with the i-th vacation period having d_i days.
   - There are N maintenance staff available numbered from 1 to N, with the j-th maintenance being available for a total of c_j days over all vacation periods. In addition, for each vacation period i, each staff can only work on a subset of the d_i days (for example, if a vacation period spans over Friday, Saturday and Sunday, staff 1 may be available for only Friday and Saturday but not Sunday, staff 2 may be available for only Friday and Sunday but not Saturday, and staff 3 may be available for all days).

   What is the maximum no. of vacation days that can be covered by the company with its available staff?

8. A college has N students X_1, X_2,…, X_N, M departments D_1, D_2,…, D_M, and P societies S_1, S_2,…, S_P. Each student is enrolled in exactly one department, and is a member of at least one society. The college has a student association (like your gymkhana) with one member from each society (You can assume that every society has at least one member). However, the society members have to be chosen such that the student association has at most Q_k members from any department D_k. Design an algorithm to form the association. Will you be able to form such an association always? You must model the problem as a maximum flow problem.