1. Let Q,Q' be problems with $Q \leqslant Q'$ and $Q' \leqslant Q$. We say that Q and Q' are polynomial-time equivalent. Prove/Disprove: Any two NP-Complete problems are polynomial-time equivalent.

2. DOUBLE-SAT: Decide whether a Boolean formula has at least two satisfying assignments. Prove that DOUBLE-SAT is NP-Complete.

3. A CNF formula is called not-all-equal satisfiable if for some truth assignment of the variables, each clause has at least one true literal and at least one false literal.

NAESAT: Decide whether a Boolean formula in CNF is not-all-equal satisfiable.

Prove that NAESAT is NP-Complete.

4. Let $\Phi(x_1, x_2, \ldots, x_n)$ be a Boolean formula in the conjunctive normal form (CNF). We say that Φ is all-but-one satisfiable if there is a truth assignment of the variables for which all except exactly one of the clauses of Φ evaluate to true. By AB1SAT, we denote the problem of deciding whether the given CNF formula Φ is all-but-one satisfiable.

5. **[Subgraph isomorphism problem]** Given two graphs G and H, decide whether there exists an injective function $f : \mathrm{V}(H) \to \mathrm{V}(G)$ such that

$$(u,v) \in \mathrm{E}(H) \text{ if and only if } (f(u), f(v)) \in \mathrm{E}(G).$$

Prove that SUBGRAPH-ISOMORPHISM is NP-complete.

6. **[Independent Set Problem]** Let $G = (V, E)$ be an undirected graph. A subset U of V is called an independent set if for all $u,v \in U$, we have $(u,v) \notin E$. The independent set problem takes G and a positive integer k as input, and decides whether G contains an independent set with k vertices. Prove that INDEPENDENT-SET is NP-complete.

7. **[Vertex Cover Problem]** Let G = (V, E) be an undirected graph. A subset U of V is called a vertex cover of G if for all $(u,v) \in E$, we have either $u \in U$ or $v \in U$ (or both). The vertex cover problem takes G and a positive integer k as input, and decides whether G contains a vertex cover with k vertices. Prove that VERTEX-COVER is NP-complete.

8. Prove that if P = NP, then every non-trivial problem in this class is NP-complete.

9. Let P and Q be two problems in NP such that the same polynomial-time reduction f can be used for proving both $P \leqslant Q$ and $Q \leqslant P$. Prove/Disprove: f must be a bijection.

10. Every instance of a problem in NP can be encoded in binary. Without loss of generality, we can therefore assume that the space of input instances of all problems in NP is $\{0,1\}^*$. Invalid encodings can be assumed to belong to the REJECT set.

The intersection $P \wedge Q$ of two problems P and Q in NP is the problem having

$$\text{Accept}(P \wedge Q) = \text{Accept}(P) \cap \text{Accept}(Q).$$

(a) Prove that the class NP is closed under intersection.

(b) Prove that the class of NP-complete problems is not closed under intersection.

11. Suppose that we want to factor a positive integer n (may be assumed to be composite). This is not a decision problem. Formulate a decision problem that can be solved in polynomial time if and only if the factoring problem can be solved in polynomial time.