1. Suppose that in Graham scan, pairs of points (but not three or more) may have the same x-coordinates. How can you modify the algorithm to handle this degeneracy.

2. [Incremental Convex Hull construction]

(a) Let C be a convex polygon, and P a point. Propose an algorithm to determine whether P is inside or outside C.

(b) Let $P_1, P_2, \ldots, P_n$ be a set of n points in the general position. We want to compute CH($P_1, P_2, \ldots, P_n$). Use Part (a) to convert CH($P_1, P_2, \ldots, P_i$) to CH($P_1, P_2, \ldots, P_{i+1}$). Total running time?

(c) Propose an O($n \log n$)-time algorithm using this idea.

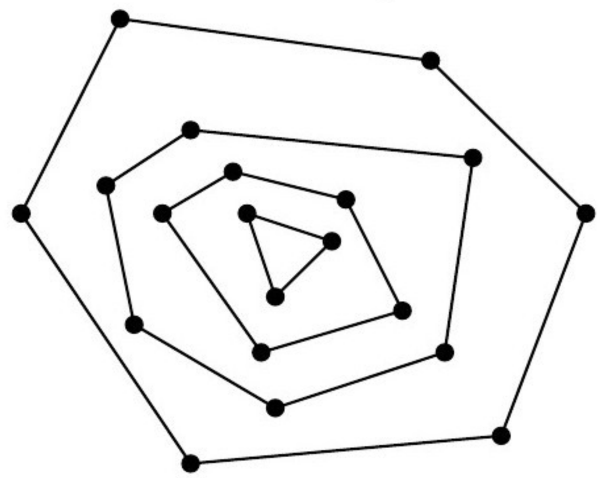
3. Let S and T be two disjoint sets of points in the Euclidean plane. S and T need not be horizontally separated. You have computed the two convex hulls CH(S) and CH(T).

(a) Propose an O(n)-time algorithm to merge these two hulls to CH($S \cup T$), where $n = | S \cup T |$.

(b) Prove that the problem of Part (a) cannot be solved in o(n) time in the worst case.

4. [Onion layers]

Let S be a set of n points in the plane. Let L_1 denote the set of vertices of CH(S). Remove the points of L_1 from S, and compute the convex hull of S again. Let L_2 denote the set of vertices of this convex hull. Repeat.



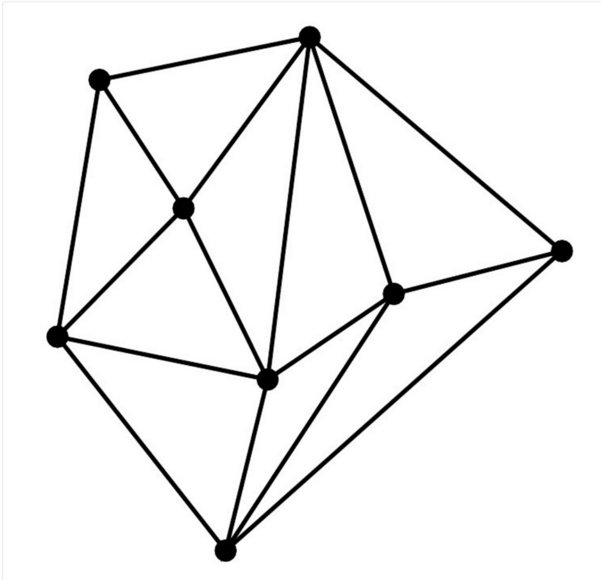
What is the worst-case running time for computing all onion layers if you use

(a) Jarvis march
(b) Graham scan.

Give tight bounds.

5. You are given n points in the plane in general position. Arrange the points to a list $P_1, P_2, \ldots, P_n$ such that for different i, j, the segments P_iP_{i+1} and P_jP_{j+1} do not intersect except perhaps at some endpoint, and each $P_iP_{i+1}P_{i+2}$ is a right turn.

6. (a) Prove that the number e of edges in any triangulation of CH(S) with $|S| = n$ satisfies $2n - 3 \leq e \leq 3n - 6$.



(b) How can you use the incremental convex-hull construction to triangulate CH(S)? Running time?

7. [Farthest Pair]

Let S be a set of n points in general position in the plane. We want to find P, $Q \in S$ such that $d(P, Q)$ is the maximum. This distance is called the diameter of S.

(a) Prove that P and Q are vertices of CH(S).

(b) Let P be a point on CH(S). Demonstrate that the distances of the points on CH(S) from P need not be unimodal.

(c) Let Q, Q' be consecutive vertices on CH(S). Let L be the line QQ'. The perpendicular distances of the vertices of CH(S) from L are unimodal. Let P be the farthest point from L. Build a collection C of pairs (P, Q) and (P, Q'). Prove that the farthest pair (P, Q) can be found in C.

(d) If the points in S are in general position, what is the maximum size of C?

(e) Propose an $O(n \log n)$-time algorithm for computing the farthest pair in S.

8. [A point sweep algorithm in one dimension]

(a) You are given n intervals $[a_i, b_i]$ standing for activities (like classes, seminars, and so on), all of which must be scheduled. Propose an efficient algorithm to find out the minimum number of classrooms needed. Assume that the interval endpoints are in general position.

(b) Lift the general-position restriction from the previous exercise. How can you make your algorithm work (in the same running time)?

(c) Assume that each activity has a preparation time c_i and a closing time d_i. How can you make your algorithm work in this setting?

9. Suppose that in the line-sweep algorithm for the line-segment intersection problem, some lines are allowed to be vertical, but different vertical lines have different x-coordinates. Explain how you can handle a "vertical segment" event. Do not use rotation. Instead modify the line-sweep algorithm covered in the class.