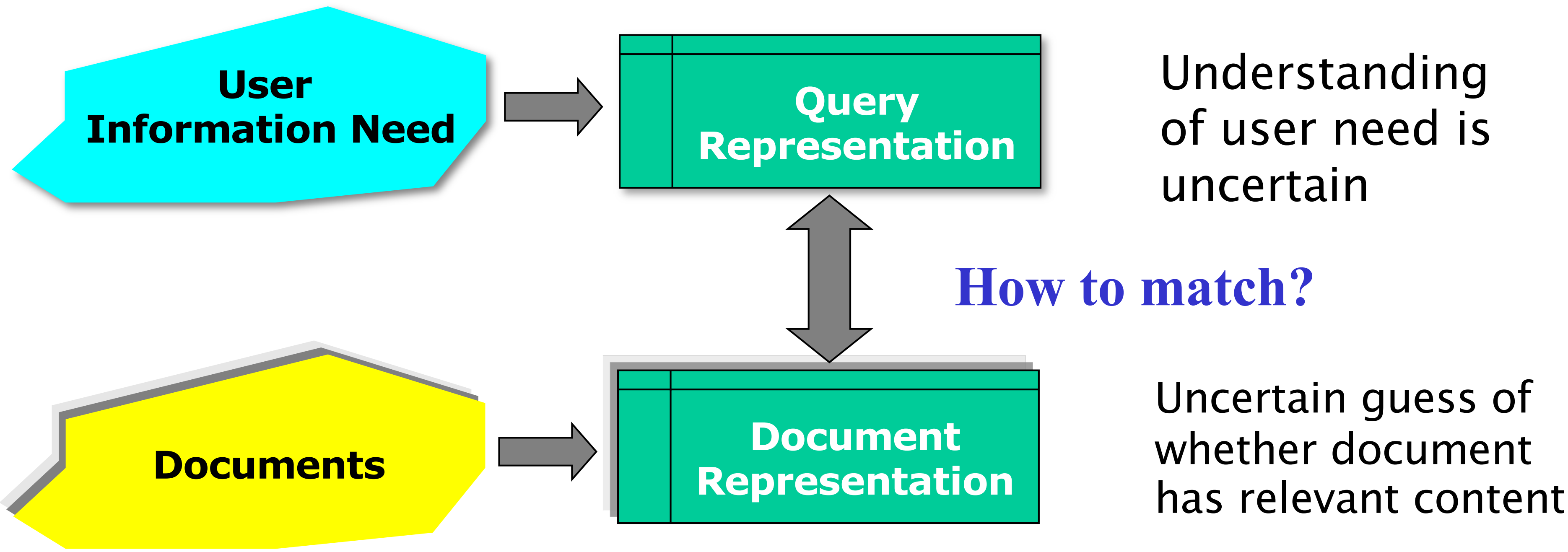
Introduction to

Information Retrieval

Probabilistic Information Retrieval

Language Models for IR

Why probabilities in IR?

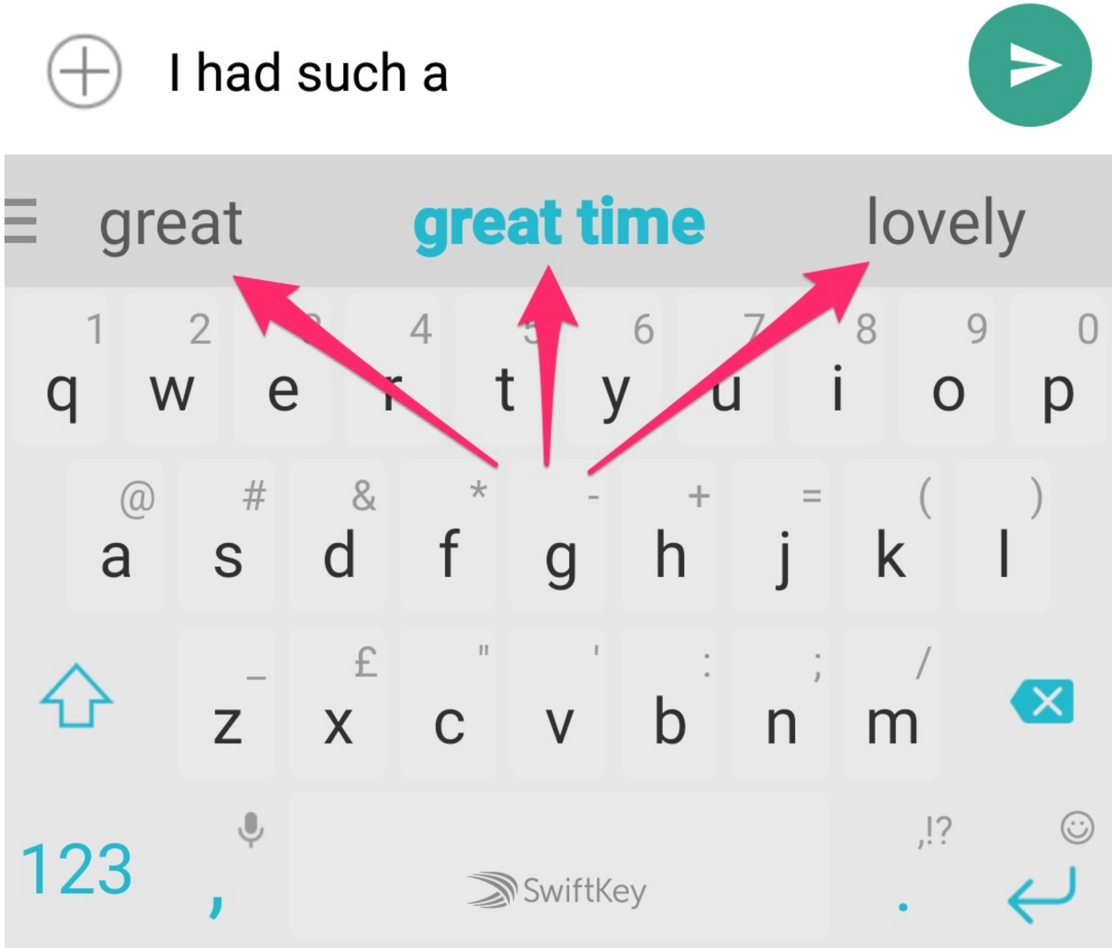


Probabilities provide a principled foundation for uncertain reasoning.
Can we use probabilities to quantify our uncertainties?

Probabilistic IR topics

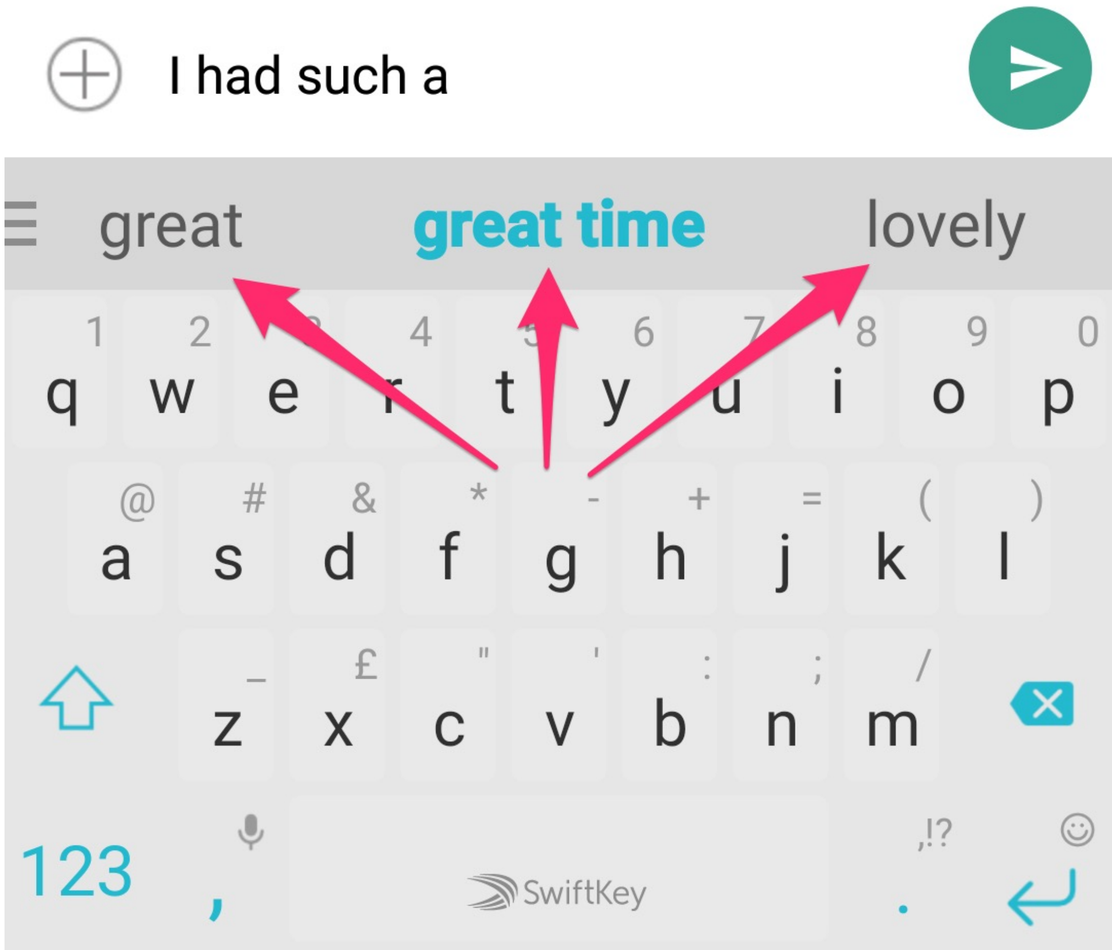
- Language model approach to IR
- Classical probabilistic retrieval model

What is a Language Model (LM)?



Predictive text shown by Google, Whatsapp, …

What is a Language Model (LM)?



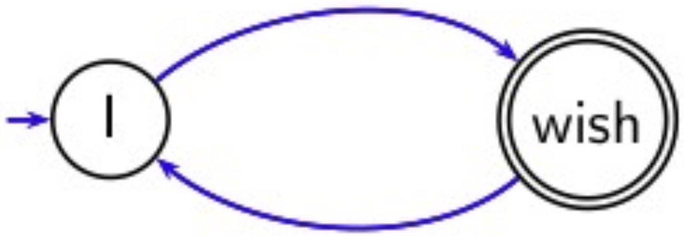
Predictive text shown by Google, Whatsapp, …

LM: a statistical or probabilistic technique to determine the probability of a given sequence of terms

LM gives the probability of a word sequence being "valid" or "expected"

What is a Language Model (LM)?

We can view a finite state automaton as a deterministic language model.

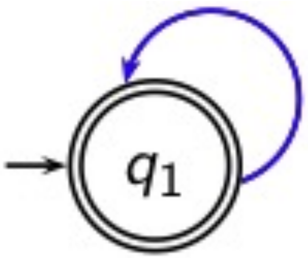


I wish I wish I wish I wish . . .

Cannot generate: “wish I wish” or “I wish I”.

Our basic model: each document was generated by a different automaton like this except that these automata are probabilistic.

A probabilistic language model



w	$P(w\|q_1)$	w	$P(w\|q_1)$
STOP	0.2	toad	0.01
the	0.2	said	0.03
a	0.1	likes	0.02
frog	0.01	that	0.04
		...	...

This is a one-state probabilistic finite-state automaton – a unigram language model – and the state emission distribution for its one state q_1. STOP is not a word, but a special symbol indicating that the automaton stops.

frog said that toad likes frog STOP

$P(\text{string}) = 0.01 \cdot 0.03 \cdot 0.04 \cdot 0.01 \cdot 0.02 \cdot 0.01 \cdot 0.02$

$= 0.0000000000048$

A question

Consider two magazines, one is a sports magazine, the other is a socio-political magazine

A news headline: “UK has a new prime minister”

From which magazine is this headline more likely to come from?

Is this an application of a probabilistic language model?

Using language models (LMs) for IR

- Task: given a query, output a ranked list of documents in decreasing order of "relevance" to the query
  - Same task, but a new way to look at "relevance"
  - We view the document as a generative model that generates the query.
- *What we need to do:*
  - Define the precise generative model we want to use
  - Apply to query and find the document(s) that are most likely to have generated the query
  - Present most likely document(s) to user

A different language model for each document

language model of d_1

w	$P(w\|.)$	w	$P(w\|.)$
STOP	.2	toad	.01
the	.2	said	.03
a	.1	likes	.02
frog	.01	that	.04
		...	...

language model of d_2

w	$P(w\|.)$	w	$P(w\|.)$
STOP	.2	toad	.02
the	.15	said	.03
a	.08	likes	.02
frog	.01	that	.05
		...	...

frog said that toad likes frog STOP

$P(\text{string}|M_{d1}) = 0.01 \cdot 0.03 \cdot 0.04 \cdot 0.01 \cdot 0.02 \cdot 0.01 \cdot 0.02 = 0.0000000000048 = 4.8 \cdot 10^{-12}$

$P(\text{string}|M_{d2}) = 0.01 \cdot 0.03 \cdot 0.05 \cdot 0.02 \cdot 0.02 \cdot 0.01 \cdot 0.02 = 0.0000000000120 = 12 \cdot 10^{-12}$ $P(\text{string}|M_{d1}) < P(\text{string}|M_{d2})$

Thus, document d_2 is "more relevant" to the string "frog said that toad likes frog STOP" than d_1 is.

Recall a few probability basics

- For events *A* and *B:*
- Bayes' Rule

$$p(A,B) = p(A \cap B) = p(A \mid B)p(B) = p(B \mid A)p(A)$$

$$\underbrace{p(A \mid B)}_{\text{Posterior}} = \frac{p(B \mid A)p(A)}{p(B)} = \frac{p(B \mid A)\overbrace{\boxed{p(A)}}^{\text{Prior}}}{\sum_{X=A,\overline{A}} p(B \mid X)p(X)}$$

Using language models in IR

- Each document is treated as (the basis for) a language model.
- Given a query q
- Rank documents based on $P(d|q)$

$$P(d|q) = \frac{P(q|d)P(d)}{P(q)}$$

- $P(q)$ is the same for all documents, so ignore
- $P(d)$ is the prior – often treated as the same for all d
  - But we can give a prior to "high-quality" documents, e.g., those with high PageRank in Web search.
- $P(q|d)$ is the probability of q given d.
- *So to rank documents according to relevance to q, ranking according to $P(q|d)$ and $P(d|q)$ is equivalent*

Where we are

- In the LM approach to IR, we attempt to model the query generation process.
- Then we rank documents by the probability that a query would be observed as a random sample from the respective document model.
- That is, we rank according to $P(q|d)$.
- Next: how do we compute $P(q|d)$?

- Notation: M_d : the document model

How to compute $P(q|d)$

- We will make the same conditional independence assumption as for Naive Bayes.

$$P(q|M_d) = P(\langle t_1, \ldots, t_{|q|} \rangle | M_d) = \prod_{1 \leq k \leq |q|} P(t_k|M_d)$$

($|q|$: length of q; t_k : the token occurring at position k in q)

- This is equivalent to:

$$P(q|M_d) = \prod_{\text{distinct term } t \text{ in } q} P(t|M_d)^{\text{tf}_{t,q}}$$

- $\text{tf}_{t,q}$: term frequency (# occurrences) of t in q
- Multinomial model (omitting constant factor)

Parameter estimation

- Missing piece: Where do the parameters $P(t|M_d)$ come from?
- Start with maximum likelihood

$$\hat{P}(t|M_d) = \frac{\mathrm{tf}_{t,d}}{|d|}$$

($|d|$: length of d; $\mathrm{tf}_{t,d}$: # occurrences of t in d)

- *We have a problem with zeros*
  - A single t with $P(t|M_d) = 0$ will make $P(q|M_d) = \prod P(t|M_d)$
  - We would give a single term "veto power".
  - E.g., for query "Rahman top hits", a document about "Rahman top songs" (but not using the word "hits") would have $P(t|M_d) = 0$; That's bad.
- We need to smooth the estimates to avoid zeros.

Smoothing

- Key intuition: A non-occurring term is possible (even though it didn't occur in the particular document), . . .
- . . . but no more likely than would be expected by chance in the collection.
- Notation: M_c: the collection model; cf_t: the number of occurrences of t in the collection; $T = \sum_t \mathrm{cf}_t$: the total number of tokens in the collection.

$$\hat{P}(t|M_d) = \frac{\mathrm{tf}_{t,d}}{|d|}$$

- We will use $\hat{P}(t|M_c) = \mathrm{cf}_t / T$

Mixture model

- We will use $\hat{P}(t|M_c)$ to "smooth" $P(t|d)$ away from zero.
- $P(t|d) = \lambda P(t|M_d) + (1 - \lambda) P(t|M_c)$
- Mixes the probability from the document with the general collection frequency of the word.

- High value of λ: "conjunctive-like" search – tends to retrieve documents containing all query words.
- Low value of λ: more disjunctive, suitable for long queries
- Correctly setting λ is very important for good performance

Mixture model: Summary

$$P(q|d) \propto \prod_{1 \leq k \leq |q|} (\lambda P(t_k|M_d) + (1-\lambda)P(t_k|M_c))$$

- What we model: The user has a document in mind and generates the query from this document.
- The equation represents the probability that the document that the user had in mind was in fact this one.

Example 1

- Collection of two docs: d_1 and d_2
- d_1 : Jackson was one of the most talented entertainers of all time
- d_2: Michael Jackson anointed himself King of Pop
- Query q: Michael Jackson
- Use mixture model with $\lambda = 1/2$
- $P(q|d_1) = [(0/11 + 1/18)/2] \cdot [(1/11 + 2/18)/2] \approx 0.003$
- $P(q|d_2) = [(1/7 + 1/18)/2] \cdot [(1/7 + 2/18)/2] \approx 0.013$
- Ranking: $d_2 > d_1$

Example 2

- Collection: d_1 and d_2
- d_1 : Xerox reports a profit but revenue is down
- d_2: Lucene narrows quarter loss but decreases further
- Query q: revenue down
- Use mixture model with $\lambda = 1/2$
- $P(q|d_1) = [(1/8 + 2/16)/2] \cdot [(1/8 + 1/16)/2] = 1/8 \cdot 3/32 = 3/256$
- $P(q|d_2) = [(1/8 + 2/16)/2] \cdot [(0/8 + 1/16)/2] = 1/8 \cdot 1/32 = 1/256$
- Ranking: $d_1 > d_2$

Language Models vs. Vector Space

LMs vs. vector space model

- LMs have some things in common with vector space models.
    - Role of Term frequency is similar
    - But TF is not scaled in LMs
- Probabilities are inherently “length-normalized”.
    - Cosine normalization does something similar for vector space
- Mixing document and collection frequencies has an effect similar to IDF
    - Terms rare in the general collection, but common in some documents will have a greater influence on the ranking.

LMs vs. vector space model: Assumptions

- Simplifying assumption: Queries and documents are objects of same type.
    - The vector space model makes the same assumption
    - May not be true!
- Simplifying assumption: Terms are conditionally independent.
    - Again, vector space model (and Naive Bayes) makes the same assumption.
    - Not true in most cases
- Cleaner statement of assumptions than vector space
- Thus, better theoretical foundation than vector space
    - … but “pure” LMs perform much worse than “tuned” LMs.

LMs vs. vector space model: differences

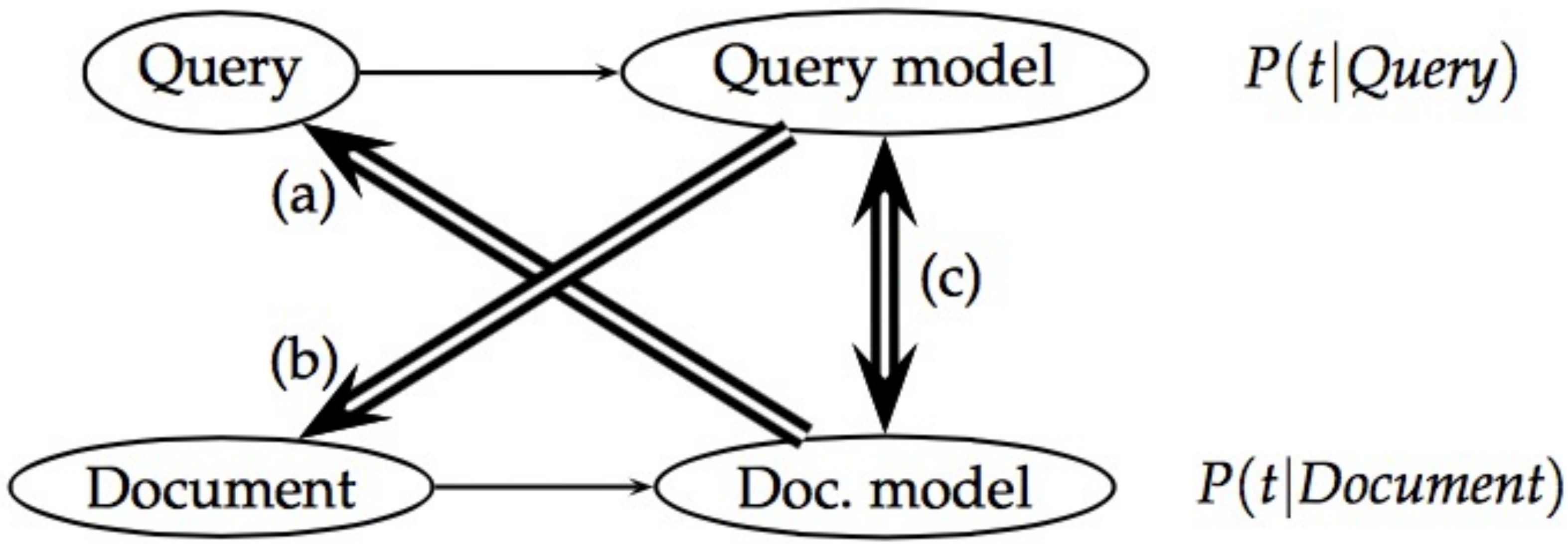
- LMs vs. vector space model: differences
  - LMs: based on probability theory
  - Vector space: based on similarity, a geometric/ linear algebra notion
  - Collection frequency vs. document frequency
  - Details of term frequency, length normalization, etc.

Alternative Language Modeling approaches: Brief discussion

Alternative LM approaches

- Rather than looking at the probability of a document LM generating the query, can look at the probability of a query LM generating the document
  - Challenge: much less text to estimate a LM based on query
  - Advantage: easier to incorporate relevance feedback
- Can make two LMs from the document (M_d) and the query (M_q), and then ask how different these two LMs are
  - Develop a risk minimization approach for retrieval: compute risk of retrieving a document d as relevant to query q
  - Risk can be estimated as the KL divergence of M_d from M_q

Three ways of developing language modeling approach



Kullback-Leibler Divergence

$$R(d;q) = KL(M_d \| M_q) = \sum_{t \in V} P(t|M_q) \log \frac{P(t|M_q)}{P(t|M_d)}$$