

# Introduction to **Information Retrieval**

## Lecture 9: Index Compression

# This lecture

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BRUTUS → 

1	2	4	11	31	45	173	174
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CAESAR → 

1	2	4	5	6	16	57	132	...
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CALPURNIA → 

2	31	54	101
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- Collection statistics in more detail
  - How big are the dictionary and postings likely to be, for a given text documents collection?
- Dictionary compression
- Postings compression

# Why compression (in general)?

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- Use less disk space
  - Saves a little money
- **Keep more stuff in memory**
  - Increases speed due to caching of more data
- **Increase speed of data transfer from disk to memory**
  - [read compressed data | decompress] is faster than [read uncompressed data]
  - Premise: Decompression algorithms are fast
    - True of the decompression algorithms we use

# Why compression for inverted indexes?

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- Dictionary
  - Make it small enough to keep in main memory
  - Make it so small that you can keep some postings lists in main memory too
- Postings file(s)
  - Reduce disk space needed
  - Decrease time needed to read postings lists from disk
  - Large search engines keep a significant part of the postings in memory (compression lets you keep more in memory)
- We will devise various IR-specific compression schemes

# Sample text collection: Reuters RCV1

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- **symbol    statistic    value**
- N documents            800,000
- L avg. # tokens per doc 200
- M terms (= word types) ~400,000
- avg. # bytes per token 6  
              (incl. spaces/punct.)
- avg. # bytes per token 4.5  
              (without spaces/punct.)
- avg. # bytes per term 7.5
- non-positional postings  
100,000,000

# Observations

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- Preprocessing greatly affects the size of dictionary and number of postings
  - Stemming, case folding, stop word removal
- Percentage reduction can be different based on properties of the collections
  - E.g., lemmatizer for French reduces dictionary size much more than Porter stemmer for English

# Index parameters vs. what we index (details IIR Table 5.1, p.80)

size of	word types (terms)			non-positional postings			positional postings		
	dictionary			non-positional index			positional index		
	Size (K)	$\Delta\%$	cumul %	Size (K)	$\Delta\%$	cumul %	Size (K)	$\Delta\%$	cumul %
Unfiltered	484			109,971			197,879		
No numbers	474	-2	-2	100,680	-8	-8	179,158	-9	-9
Case folding	392	-17	-19	96,969	-3	-12	179,158	0	-9
30 stopwords	391	-0	-19	83,390	-14	-24	121,858	-31	-38
150 stopwords	391	-0	-19	67,002	-30	-39	94,517	-47	-52
stemming	322	-17	-33	63,812	-4	-42	94,517	0	-52

Exercise: give intuitions for all the '0' entries. Why do some zero entries correspond to big deltas in other columns?

# Lossless vs. lossy compression

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- Lossless compression: All information is preserved.
  - What we mostly do in IR.
- Lossy compression: Discard some information
  - Makes sense when the discarded information is unlikely to be ever used by the IR system
- Several of the preprocessing steps can be viewed as lossy compression: case folding, stop words, stemming, number elimination.



# Vocabulary vs. collection size

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- How big is the term vocabulary?
  - That is, how many distinct words are there?
- Can we assume an upper bound?
- In practice, the vocabulary will keep growing with the collection size

# Vocabulary vs. collection size

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- Heaps' law:  $M = kT^b$
- $M$  is the **size of the vocabulary**,  $T$  is the **number of tokens in the collection**
- Typical values:  $30 \leq k \leq 100$  and  $b \approx 0.5$
- In a log-log plot of vocabulary size  $M$  vs.  $T$ , Heaps' law predicts a line with slope about  $1/2$ 
  - It is the simplest possible relationship between the two in log-log space
  - An empirical finding ("empirical law")

# Heaps' Law

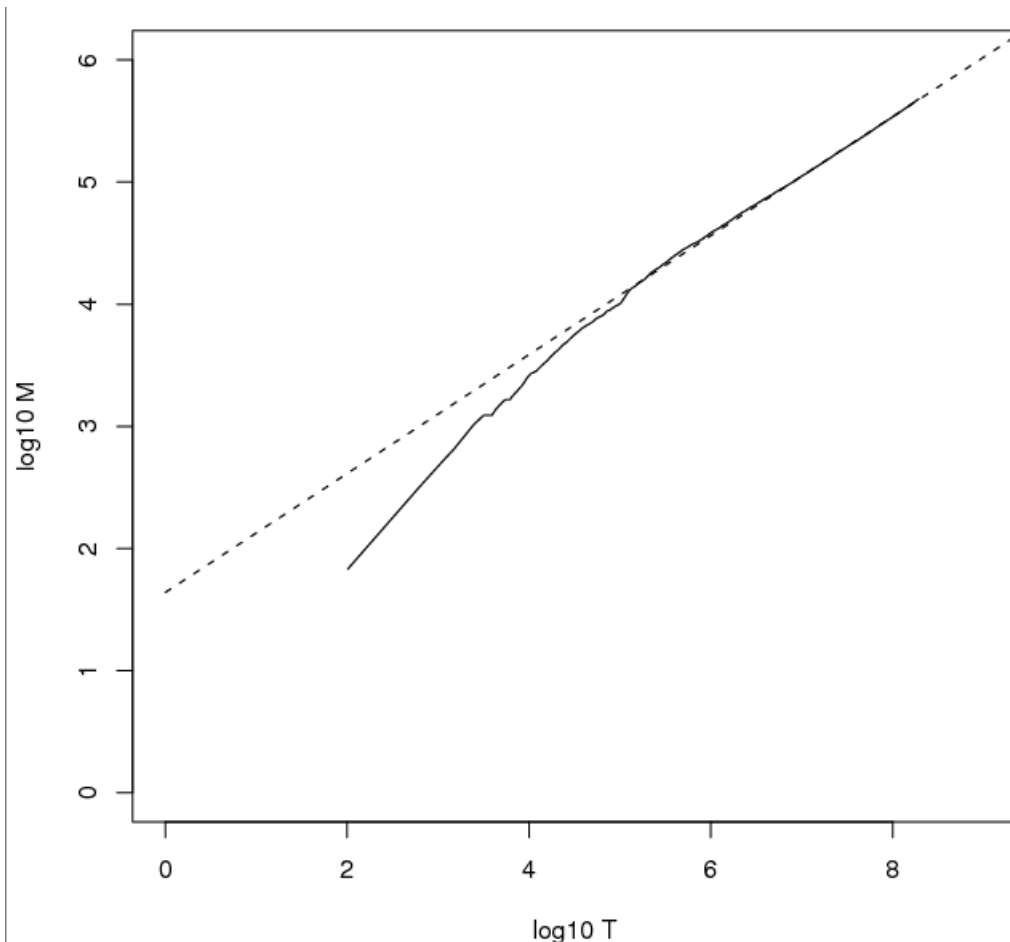
For RCV1, the dashed line  
 $\log_{10} M = 0.49 \log_{10} T + 1.64$   
is the best least squares  
fit.

Thus,  $M = 10^{1.64} T^{0.49}$  so  $k = 10^{1.64} \approx 44$  and  $b = 0.49$ .

Good empirical fit for  
Reuters RCV1 !

For first 1,000,020 tokens,  
law predicts 38,323 terms;  
actually, 38,365 terms

Fig 5.1 p81



# Heap's Law suggests that

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- The size of the dictionary is quite large for large collections
- The dictionary continues to increase with more documents in the collection, rather than a maximum vocabulary size being reached

# Exercises

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- What is the effect of including spelling errors, vs. automatically correcting spelling errors on Heaps' law?
- Compute the vocabulary size  $M$  for this scenario:
  - Looking at a collection of web pages, you find that there are 3000 different terms in the first 10,000 tokens and 30,000 different terms in the first 1,000,000 tokens.
  - Assume a search engine indexes a total of 20,000,000,000 ( $2 \times 10^{10}$ ) pages, containing 200 tokens on average
  - What is the size of the vocabulary of the indexed collection as predicted by Heaps' law?

# Zipf's law

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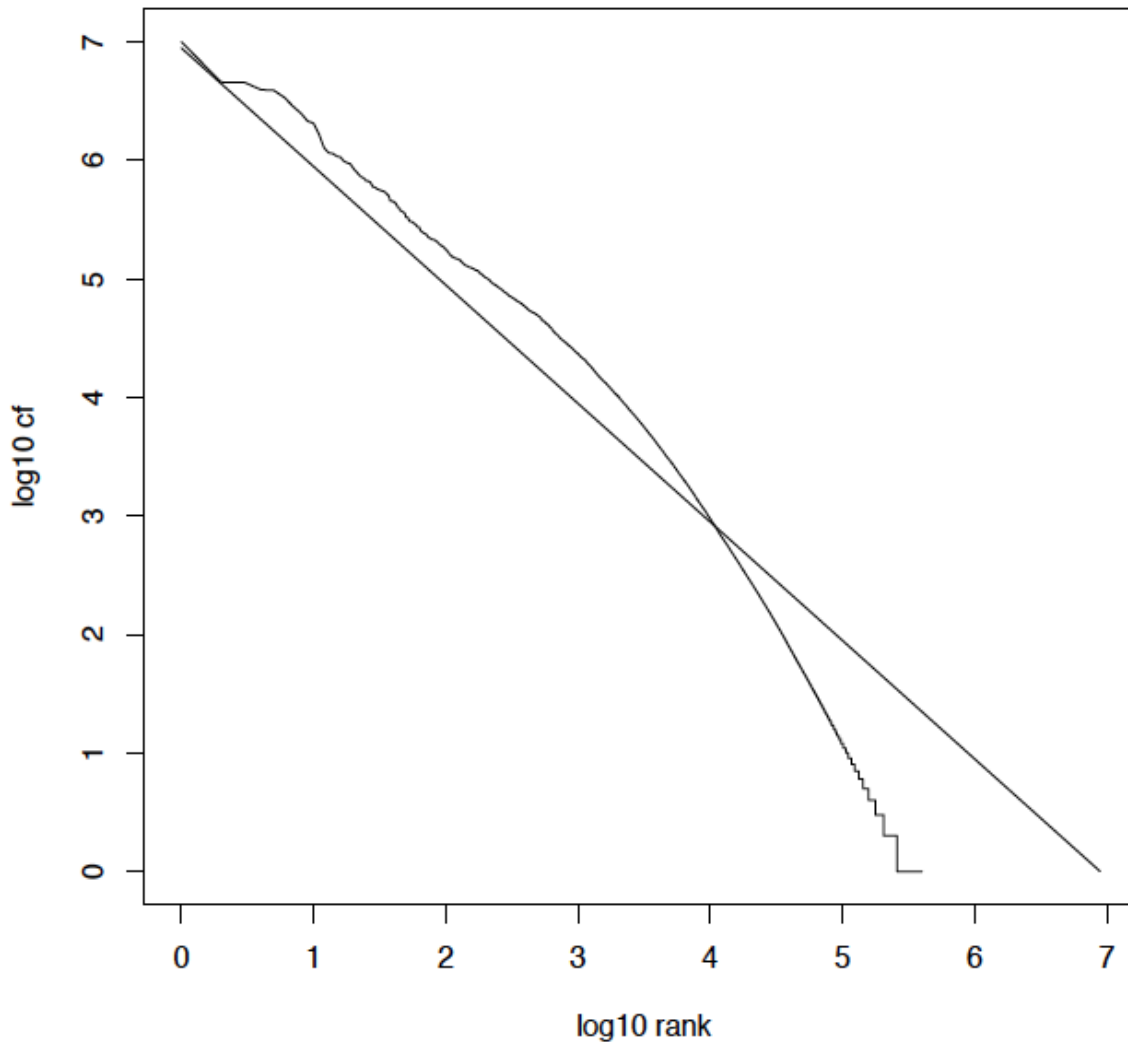
- Heaps' law gives the vocabulary size in collections.
- We also study the relative frequencies of terms.
- *In natural language, there are a few very frequent terms and very many very rare terms.*
- **Zipf's law: The  $i$ -th most frequent term has frequency proportional to  $1/i$ .**
- $cf_i \propto 1/i = K/i$  where  $K$  is a normalizing constant
- $cf_i$  is collection frequency: the number of occurrences of the term  $t_i$  in the collection.

# Zipf's Law consequences

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- If the most frequent term (*the*) occurs  $cf_1$  times
  - then the second most frequent term (*of*) occurs  $cf_1/2$  times
  - the third most frequent term (*and*) occurs  $cf_1/3$  times
  - ...
- Equivalent:  $cf_i = K/i$  where  $K$  is a normalizing factor, so
  - $\log cf_i = \log K - \log i$
  - Linear relationship between  $\log cf_i$  and  $\log i$
- Another power law relationship

# Zipf's law for Reuters RCV1





# Compression

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- Now, we will consider compressing the space for the dictionary and postings
  - Basic Boolean index only
  - No study of positional indexes, etc.
- We will consider compression schemes
  - Dictionary compression
  - Postings list compression

# **DICTIONARY COMPRESSION**

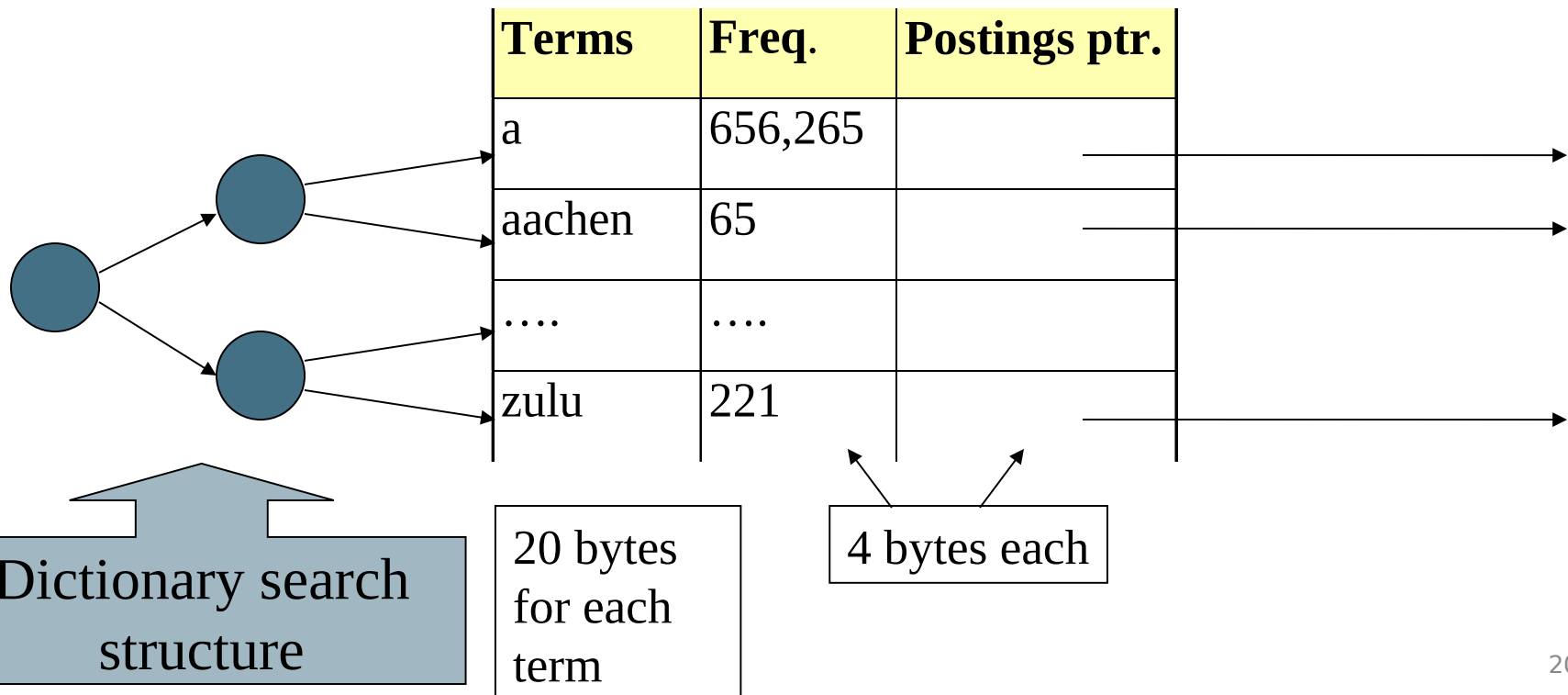
# Why compress the dictionary?

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- Search begins with the dictionary
- We want to keep it in memory
- Memory footprint: competition with other applications
- Embedded/mobile devices may have very little memory
- Even if the dictionary isn't in memory, we want it to be small for a fast search startup time
- So, compressing the dictionary is important

# Dictionary storage - first cut

- Array of fixed-width entries
  - ~400,000 terms; 28 bytes/term = 11.2 MB.



# Fixed-width terms are wasteful

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- Most of the bytes in the **Term** column are wasted – we allot 20 bytes even for 1 letter terms.
  - And we still can't handle *terms with more than 20 chars*
- Written English averages ~4.5 characters/word.
- Ave. dictionary word in English: ~8 characters
  - How do we use ~8 characters per dictionary term?
- Short words dominate token counts but not type average.

# Compressing the term list:

## Approach 1: Dictionary-as-a-String

- Store dictionary as a (long) string of characters:

- Pointer to next word shows end of current word

- Hope to save up to 60% of dictionary space.

Freq.	Postings ptr.	Term ptr.
33		
29		
44		
126		

...sysztylesyzggeticsz4y9%alszysztaihelviteszczecinszomo....

Total string length =  
400K x 8B = 3.2MB

Pointers resolve 3.2M  
positions:  $\log_2 3.2M =$   
22bits = 3bytes

# Space for dictionary as a string

- 4 bytes per term for Freq.
  - 4 bytes per term for pointer to Postings
  - 3 bytes per term pointer
  - Avg. 8 bytes per term in term string
  - 400K terms x 19  $\Rightarrow$  7.6 MB (against 11.2MB for fixed width)
- } Now avg. 11 bytes/term, not 20.

# Approach 2: Blocking

- Store pointers to every  $k$ -th term string.
  - Example below:  $k=4$ .
- Need to store term lengths (1 extra byte)

....**7***systile***9***syzygetic***8***syzygial***6***syzygy***11***szaibelyite***8***szczecin***9***szomo*....

Freq.	Postings ptr.	Term ptr.
33		
29		
44		
126		
7		

} Save 9 bytes  
on 3  
pointers.

← Lose 4 bytes on  
term lengths.



# Blocking

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- Group terms into blocks, each having  $k$  terms
- Store a term pointer only for first term of each block
- Store the length of each term as one additional byte at the beginning of each term
- Search for terms in the compressed dictionary
  - Locate the term's block by **binary search**
  - Then locate term's position within the block by **linear search within the block**
- By increasing block size  $k$ : tradeoff between better compression and speed of term lookup

# Net saving

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- Example for block size  $k = 4$
- Where we used 3 bytes/pointer without blocking
  - $3 \times 4 = 12$  bytes,

now we use  $3 + 4 = 7$  bytes.

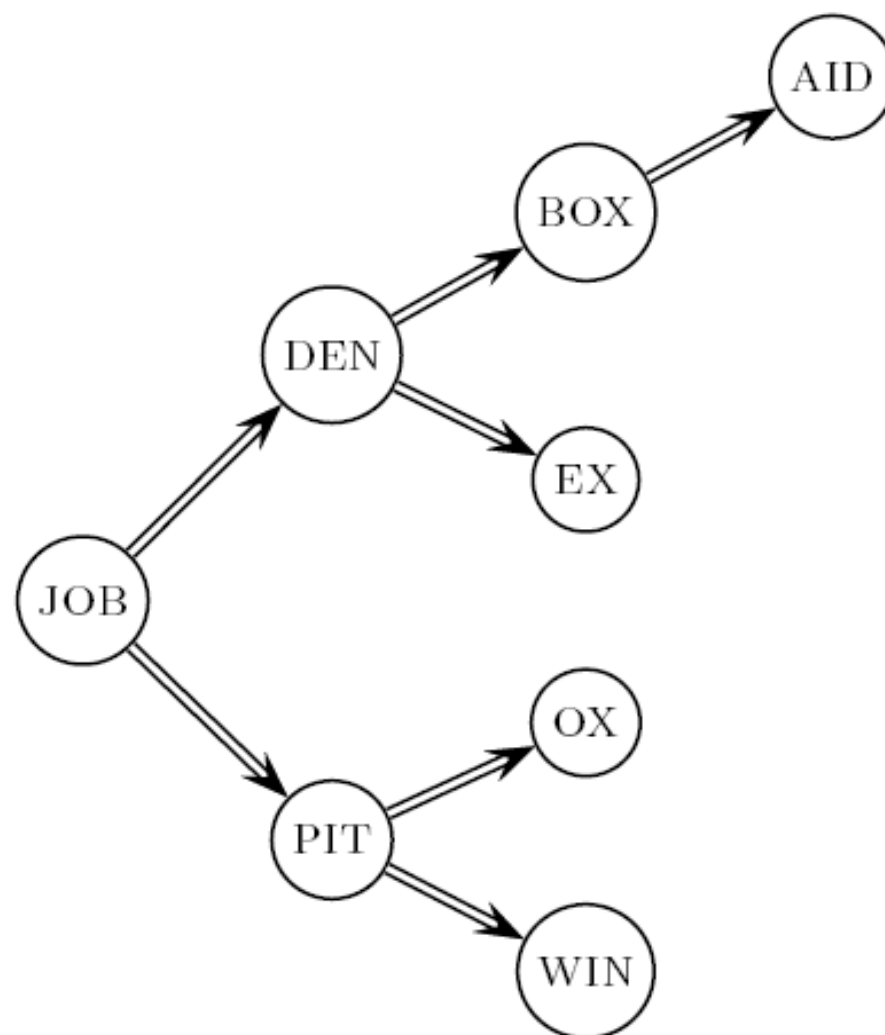
Saved another  $\sim 0.5$  MB. This reduces the size of the dictionary from 7.6 MB to 7.1 MB.  
We can save more with larger  $k$ .

Why not go with larger  $k$ ?

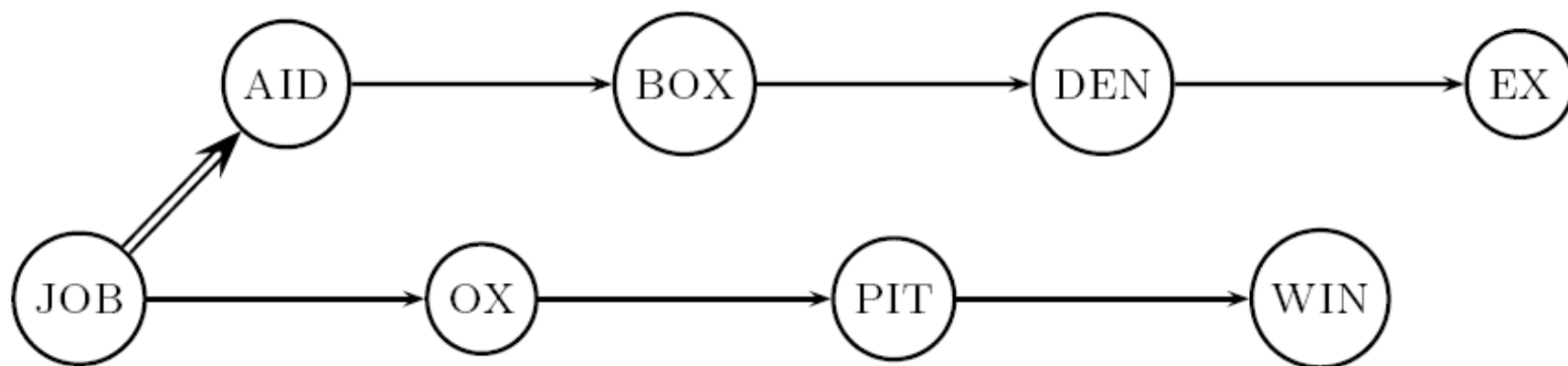
# Dictionary search without blocking

- Assuming each dictionary term equally likely in query (not really so in practice!), average number of comparisons =  
 $(1+2 \cdot 2+4 \cdot 3+4)/8 \sim 2.6$

Exercise: what if the frequencies of query terms were non-uniform but known, how would you structure the dictionary search tree?



# Dictionary search with blocking



- Binary search down to 4-term block;
  - Then linear search through terms in block.
- Blocks of 4 (binary tree), avg. =  
 $(1+2 \cdot 2+2 \cdot 3+2 \cdot 4+5)/8 = 3$  compares

# Exercise

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- Estimate the impact on search performance (and slowdown compared to  $k=1$ ) with blocking, for block sizes of  $k = 4, 8$  and  $16$ .

# Approach 3: Front coding

- Front-coding:
  - Sorted words commonly have long common prefix – store differences only
  - (for last  $k-1$  in a block of  $k$ )

**8automata8automate9automatic10automati  
on**

→ **8automat\*a1♦e2♦ic3♦ion**

Encodes **automat**

Extra length  
beyond **automat.**

Begins to resemble general string compression. 30

# RCV1 dictionary compression summary

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Technique	Size in MB
Fixed width	11.2
Dictionary-as-String with pointers to every term	7.6
Also, blocking $k = 4$	7.1
Also, Blocking + front coding	5.9

# POSTINGS COMPRESSION



# Postings compression

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- The postings file is much larger than the dictionary, factor of at least 10.
- Key desideratum: store each posting compactly.
- A posting for our purposes is a docID.
- For Reuters (800,000 documents), we would use 32 bits per docID when using 4-byte integers.
- Alternatively, we can use  $\log_2 800,000 \approx 20$  bits per docID.
- Our goal: use far fewer than 20 bits per docID.

# Postings: two conflicting forces

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- A term like ***arachnocratic*** occurs in maybe one doc out of a million – we would like to store this posting using  $\log_2 1M \sim 20$  bits.
- A term like ***the*** occurs in virtually every doc, so 20 bits/posting is too expensive.

# Postings file entry

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- We store the list of docs containing a term in increasing order of docID.
  - **computer**: 33,47,154,159,202 ...
- Consequence: it suffices to **store gaps**.
  - 33,14,107,5,43 ...
- Hope: most gaps can be encoded/stored with far fewer than 20 bits.

# Three postings entries

	encoding	postings list					
THE	docIDs	...	283042	283043	283044	283045	...
	gaps		1	1	1		...
COMPUTER	docIDs	...	283047	283154	283159	283202	...
	gaps		107	5	43		...
ARACHNOCENTRIC	docIDs	252000	500100				
	gaps	252000	248100				

# Variable length encoding

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- Aim:
  - For **arachnocratic**, we will use  $\sim 20$  bits/gap entry.
  - For **the**, we will use  $\sim 1$  bit/gap entry.
- If the average gap for a term is  $G$ , we want to use  $\sim \log_2 G$  bits/gap entry.
- Key challenge: encode every integer (gap) with about as few bits as needed for that integer.
- This requires a *variable length encoding*
- Variable length codes achieve this by using short codes for small numbers

# Variable Byte (VB) codes

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- For a gap value  $G$ , we want to use close to the fewest bytes needed to hold  $\log_2 G$  bits
- Begin with one byte to store  $G$  and **dedicate 1 bit in it to be a continuation bit  $c$**
- If  $G \leq 127$ , binary-encode it in the 7 available bits and set  $c = 1$
- Else encode  $G$ 's lower-order 7 bits and then use additional bytes to encode the higher order bits using the same algorithm
- At the end **set the continuation bit of the last byte to 1 ( $c = 1$ )** – and for the other bytes  $c = 0$ .

# Example

docIDs	824	829	215406
gaps		5	214577
VB code	00000110 10111000	10000101	00001101 00001100 10110001

Postings stored as the byte concatenation

000001101011100010000101000011010000110010110001

Key property: VB-encoded postings are uniquely prefix-decodable.

For a small gap (5), VB uses a whole byte.

# Other variable unit codes

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- Instead of bytes, we can also use a different “unit of alignment”: 32 bits (words), 16 bits, 4 bits (nibbles).
- Variable byte alignment wastes space if you have many small gaps – nibbles do better in such cases.
- Variable byte codes:
  - Used by many commercial/research systems
  - Good low-tech blend of variable-length coding and sensitivity to computer memory alignment matches (vs. bit-level codes, which we look at next).



# Variable bit-level codes: Unary code

- Represent  $n$  as  $n$  1s with a final 0.
- Unary code for 3 is 1110.
- Unary code for 40 is

[illegible]

- Unary code for 80 is:

[illegible]

- This doesn't look promising, but....

# Gamma codes

- We can compress better with bit-level codes
  - The Gamma code is the best known of these.
- Represent a gap  $G$  as a pair *length* and *offset*
- *offset* is  $G$  in binary, with the leading bit cut off
  - For example  $13 \rightarrow 1101 \rightarrow 101$
- *length* is the length of offset
  - For 13 (offset 101), this is 3.
- We encode *length* with *unary code*: 1110.
- Gamma code of 13 is the concatenation of *length* and *offset*: 1110101

# Gamma code examples

number	length	offset	$\gamma$ -code
0			none
1	0		0
2	10	0	10,0
3	10	1	10,1
4	110	00	110,00
9	1110	001	1110,001
13	1110	101	1110,101
24	11110	1000	11110,1000
511	111111110	11111111	111111110,11111111
1025	11111111110	0000000001	11111111110,0000000001

# Gamma code properties

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- $G$  is encoded using  $2 \lfloor \log G \rfloor + 1$  bits
  - Length of offset is  $\lfloor \log G \rfloor$  bits
  - Length of length is  $\lfloor \log G \rfloor + 1$  bits
- All gamma codes have an odd number of bits
- Almost within a factor of 2 of best possible,  $\log_2 G$
- Gamma code is uniquely prefix-decodable, like VB
- Gamma code can be used for any distribution
- Gamma code is parameter-free

# RCV1 compression

Data structure	Size in MB
dictionary, fixed-width	11.2
dictionary, term pointers into string	7.6
with blocking, $k = 4$	7.1
with blocking & front coding	5.9
collection (text, xml markup etc)	3,600.0
collection (text)	960.0
Term-doc incidence matrix	40,000.0
postings, uncompressed (32-bit words)	400.0
postings, uncompressed (20 bits)	250.0
postings, variable byte encoded	116.0
postings, $\gamma$ -encoded	101.0

# Index compression summary

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- We can now create an index for highly efficient Boolean retrieval that is very space efficient
- Only 4% of the total size of the collection
- Only 10-15% of the total size of the text in the collection
- However, we've ignored positional information
- Hence, space savings are less for indexes used in practice
  - But techniques substantially the same.