Assignment - 3

1. (**5 points**) Consider the initial value problem:

$$y' = \frac{y}{x} - \left(\frac{y}{x}\right)^2,\ x \in [1,2],\ y(1) = 1.$$

Use the Runge-Kutta method of order 4 to obtain an approximation upto $y(2)$ using stepsize $h = 0.01$. Plot the solution in $x \in [1,2]$.

2. (**5 points**) Write a matlab code to solve the folowing boundary value problem

$$\begin{aligned} -u'' + p(x)u' + q(x)u &= f(x),\ 0 \le x \le 1, \\ u(0) &= g_0, \\ u(1) &= g_1, \end{aligned}$$

where p, q, and f are known functions. Chose the known functions to generate a result. (Do not use examples from tutorial in order to ignore similarity).

3. (**10 points**) Write a program to evaluate $I = \int_{-4}^{4} \frac{dx}{1+x^2}$ using the
(a) Trapezoidal rule
(b) Simpson's rule
(c) composite trapezoidal and composite simpson rule for $n = 10$
(d) Use two point Gauss-Legendre quadrature to evaluate the given integral. Compare with the results obtained in (a), (b) and (c).

4. (i) (**2.5 points**) Solve the following linear system by Gauss-Seidel method, with tolerance $= 10^{-4}$ in the l_∞ norm. Choose initial guess as $x_1 = 1/2 = x_2$

$$\begin{aligned} 10x_1 + x_2 &= 11 \\ x_1 + 10x_2 &= 11. \end{aligned}$$

(ii)(**2.5 points**) Solve the following linear system by Gauss Jacobi method and choose initial guess as $x_1 = x_2 = x_3 = 0$

$$\begin{aligned} 4x_1 + x_2 - x_3 &= 3 \\ 2x_1 + 7x_2 + x_3 &= 19 \\ x_1 - 3x_2 + 12x_3 &= 31. \end{aligned}$$

Instructions:

- Any descriptive answer should be written at the top of the code. Use '%' to comment inside the code.
- Make Matlab script for each of the above problems and submit only the '.m' file in gradescope.
- The final code should run without any error. **Sample inputs required for the code should be specified by yourself.**
- Code will be checked manually. Checker will only hit run, and he/she will not provide any input during checking. Everything should be specified in each code.

- Any Cheating or unfair means will leads to you directly failing the course (i.e.'F' grade/ audit fail) and further disciplinary action by the institute which may include suspension for a semester. Give your exam honestly and do not help others during the exam. There will be no mercy in the case of any unfair means.
- For any clarification feel free to comments on team under this assignment posted.