Project 1

As discussed in class, two groups of asteroid accompanying the planet Jupiter in its orbit around the Sun—the Trojan asteroids. The two groups sit at points of stable equilibrium (the Lagrange points) preceding and trailing Jupiter at an angular distance of π/3 radians. The combination of the gravitational attraction of the Sun and Jupiter gives a resultant force on a Trojan asteroid towards the center of mass of the two bodies such that the centripetal acceleration causes the asteroid to orbit with the same period as the planet.

Note that units suitable for this system are normalized for the Solar System: the mass of the Sun (M⊙) is taken as unit mass, unit distance is the astronomical unit (AU), and unit time is one year. For such a system, the gravitational constant G is 4π2. The mass of Jupiter can be taken to be 0.001 M⊙ and the average distance of Jupiter from the Sun is 5.2 AU. The asteroids' masses are negligible in this system.

```
import matplotlib    # Library used for plotting
import numpy as np   # Numerical library
import matplotlib.pyplot as plt # Plot commands

import scipy         # Another numerical library
from scipy import integrate

# Define some colors using the RGB format

CF_red = (204/255, 121/255, 167/255)
CF_vermillion = (213/255, 94/255, 0)
CF_orange = (230/255, 159/255, 0)
CF_yellow = (240/255, 228/255, 66/255)
CF_green = (0, 158/255, 115/255)
CF_sky = (86/255, 180/255, 233/255)
CF_blue = (0, 114/255, 178/255)
CF_black = (0, 0, 0)
```

Part 1 - Setup

Write a program to solve the equations of motion for this system numerically, and demonstrate that the asteroids will stay at the Lagrange points over many orbits of Jupiter around the Sun. This should show that your program is operating correctly with no numerical instabilities being propagated during the computation.

```
# Parameters
days_J = 4331    # Days per Jupiter year
M_S = 1          # Mass of the Sun
M_J = 0.001      # Mass of Jupiter
radius_J = 5.2   # Radius of Jupiter's orbit
```

```
t_min = 0
t_max = 100 * days_J
epsilon = 10

velocity_J = 2 * np.pi * radius_J / days_J # Average velocity of
Jupiter per day
radius_S = radius_J * M_J / (M_S + M_J) # Radius of the Sun's orbit
velocity_S = 2 * np.pi * radius_S / days_J # Average velocity of the
Sun per day

position_x_L4 = radius_J * np.cos(np.pi / 3)
position_y_L4 = radius_J * np.sin(np.pi / 3)
position_x_L5 = radius_J * np.cos(np.pi / 3)
position_y_L5 = -radius_J * np.sin(np.pi / 3)

velocity_x_L4 = velocity_J * -0.866
velocity_y_L4 = velocity_J * 0.5
velocity_x_L5 = velocity_J * 0.866
velocity_y_L5 = velocity_J * 0.5

init = np.array([-radius_S, 0, 0, -velocity_S, # Sun
                radius_J, 0, 0, velocity_J, # Jupiter
                position_x_L4, position_y_L4, velocity_x_L4,
velocity_y_L4, # Asteroid L4
                position_x_L5, position_y_L5, velocity_x_L5,
velocity_y_L5]) # Asteroid L4
```

Here we define the parameters used in the model of the three-body problem. I define the number of Earth days in a Jupiter year, the masses of the Sun and Jupiter, and the radius of Jupiter's orbit. I also define the time ranges and the maximum step size for the numerical solution of the equations of motion. Using these parameters, I calculate some important parameters for the system, such as the velocity of Jupiter's orbit, the radius of the Sun's orbit and the velocity of the Sun's orbit.

Now, I can calculate the positions and velocities of the asteroids in the Lagrange points using some trigonometry. Then, I create an initial state containing the initial positions and velocities of the Sun, Jupiter, an asteroid in the L4 point, an asteroid in the L5 point.

```
# Functions
d = -2.96 * 10 ** (-4) # G * M / AU^3 * 86400^2

def general_derivative(t, state, mass_J):
    x_S, y_S, v_x_S, v_y_S, \
        x_J, y_J, v_x_J, v_y_J, \
        x_A4, y_A4, v_x_A4, v_y_A4, \
        x_A5, y_A5, v_x_A5, v_y_A5 = state

    # Distances
    r_JS = np.sqrt((x_J - x_S) ** 2 + (y_J - y_S) ** 2)  # Distance
from Jupiter to Sun
```

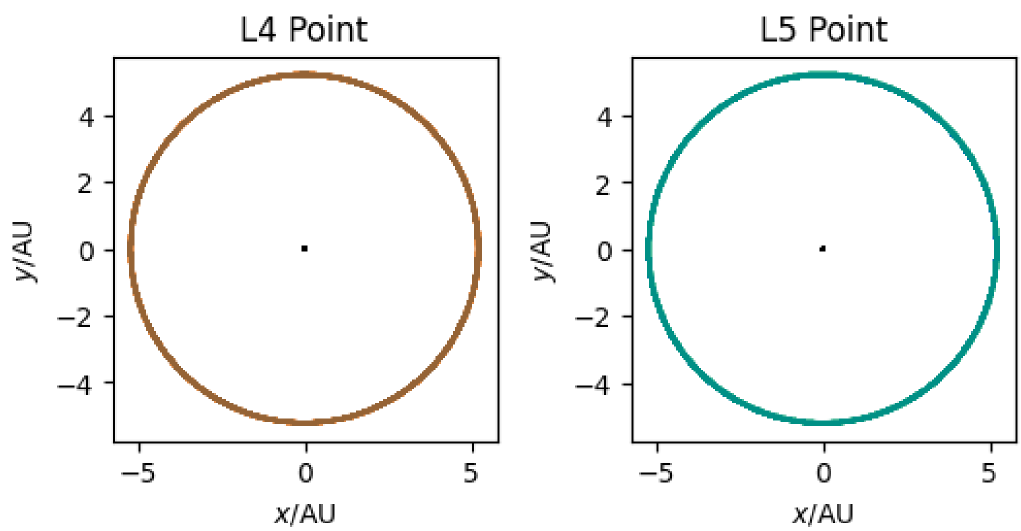
```
    r_A4S = np.sqrt((x_A4 - x_S) ** 2 + (y_A4 - y_S) ** 2)  # Distance from L4 Asteroid to Sun
    r_A4J = np.sqrt((x_A4 - x_J) ** 2 + (y_A4 - y_J) ** 2)  # Distance from L4 Asteroid to Jupiter

    r_A5S = np.sqrt((x_A5 - x_S) ** 2 + (y_A5 - y_S) ** 2)  # Distance from L5 Asteroid to Sun
    r_A5J = np.sqrt((x_A5 - x_J) ** 2 + (y_A5 - y_J) ** 2)  # Distance from L5 Asteroid to Jupiter

    # Accelerations
    a_x_SJ =  d * mass_J * (x_S - x_J) / (r_JS ** 3) # Sun's acceleration due to Jupiter
    a_y_SJ =  d * mass_J * (y_S - y_J) / (r_JS ** 3)

    a_x_JS = d * M_S * (x_J - x_S) / (r_JS ** 3)  # Jupiter's acceleration due to the Sun
    a_y_JS = d * M_S * (y_J - y_S) / (r_JS ** 3)

    a_x_A4S = d * M_S * (x_A4 - x_S) / (r_A4S ** 3)  # L4 Asteroid's acceleration due to the Sun
    a_y_A4S = d * M_S * (y_A4 - y_S) / (r_A4S ** 3)
    a_x_A4J = d * mass_J * (x_A4 - x_J) / (r_A4J ** 3)  # L4 Asteroid's acceleration due to Jupiter
    a_y_A4J = d * mass_J * (y_A4 - y_J) / (r_A4J ** 3)
    a_x_A4 = a_x_A4S + a_x_A4J  # Total x acceleration of the L4 Asteroid
    a_y_A4 = a_y_A4S + a_y_A4J  # Total y acceleration of the L4 Asteroid

    a_x_A5S = d * M_S * (x_A5 - x_S) / (r_A5S ** 3)  # L5 Asteroid's acceleration due to the Sun
    a_y_A5S = d * M_S * (y_A5 - y_S) / (r_A5S ** 3)
    a_x_A5J = d * mass_J * (x_A5 - x_J) / (r_A5J ** 3)  # L5 Asteroid's acceleration due to Jupiter
    a_y_A5J = d * mass_J * (y_A5 - y_J) / (r_A5J ** 3)
    a_x_A5 = a_x_A5S + a_x_A5J  # Total x acceleration of the L5 Asteroid
    a_y_A5 = a_y_A5S + a_y_A5J  # Total y acceleration of the L5 Asteroid

    return np.array([v_x_S, v_y_S, a_x_SJ, a_y_SJ, \
                     v_x_J, v_y_J, a_x_JS, a_y_JS, \
                     v_x_A4, v_y_A4, a_x_A4, a_y_A4, \
                     v_x_A5, v_y_A5, a_x_A5, a_y_A5])
```

Before writing the general derivative function, I first define the constant 'd' calculated the same way as we did in class using $G \times M / AU^3 \times 86400^2$. Then, I define the general derivative function that takes in the current state of the system and the mass of Jupiter and returns the next state.

This is done by first desconstructing the state that is passed in and calculating the accelerations due to the different bodies. To calculate the accelerations, I calculate the distances between the bodies and then calculate the accelerations as we did in class. We need to account for Sun's acceleration due to Jupiter, Jupiter's acceleration due to the Sun, and the asteroids' acceleration due to the Sun and Jupiter.

Finally, I return the derivative of the state, where the next positions are the previous velocities and the next velocities are the accelerations.

```
# Solution
def derivative(t, state):
    return general_derivative(t, state, M_J)

solution = integrate.solve_ivp(derivative, [t_min, t_max], init,
max_step = epsilon)

x_S = solution.y[0]
y_S = solution.y[1]

x_J = solution.y[4]
y_J = solution.y[5]

x_A4 = solution.y[8]
y_A4 = solution.y[9]

x_A5 = solution.y[12]
y_A5 = solution.y[13]

# Plotting
fig, (ax_L4, ax_L5) = plt.subplots(1, 2)
fig.set_size_inches(6, 3)

ax_L4.plot(x_S, y_S, color = CF_black)
ax_L4.plot(x_J, y_J, color = CF_blue)
ax_L4.plot(x_A4, y_A4, color = CF_vermillion, alpha = 0.7)
ax_L4.set(title = "L4 Point", xlabel = "$x$/AU", ylabel = "$y$/AU")
ax_L4.set_aspect("equal")

ax_L5.plot(x_S, y_S, color = CF_black)
ax_L5.plot(x_J, y_J, color = CF_blue)
ax_L5.plot(x_A5, y_A5, color = CF_green, alpha = 0.7)
ax_L5.set(title = "L5 Point", xlabel = "$x$/AU", ylabel = "$y$/AU")
ax_L5.set_aspect("equal")

plt.subplots_adjust(wspace = 0.35)
```



To check that everything is working as expected, I produced a plot. I define the specific derivative with the correct value for the mass of Jupiter. Then using the `solve_ivp` function, I solve the equations of motion for the system using the initial state and the derivative function for the time range and maximum step. I then plot the positions of the Sun, Jupiter, and the asteroids over 100 years. The plot looks as expected and the asteroids are staying at the Lagrange points over many orbits of Jupiter around the Sun.

Part 2 - Position Stability

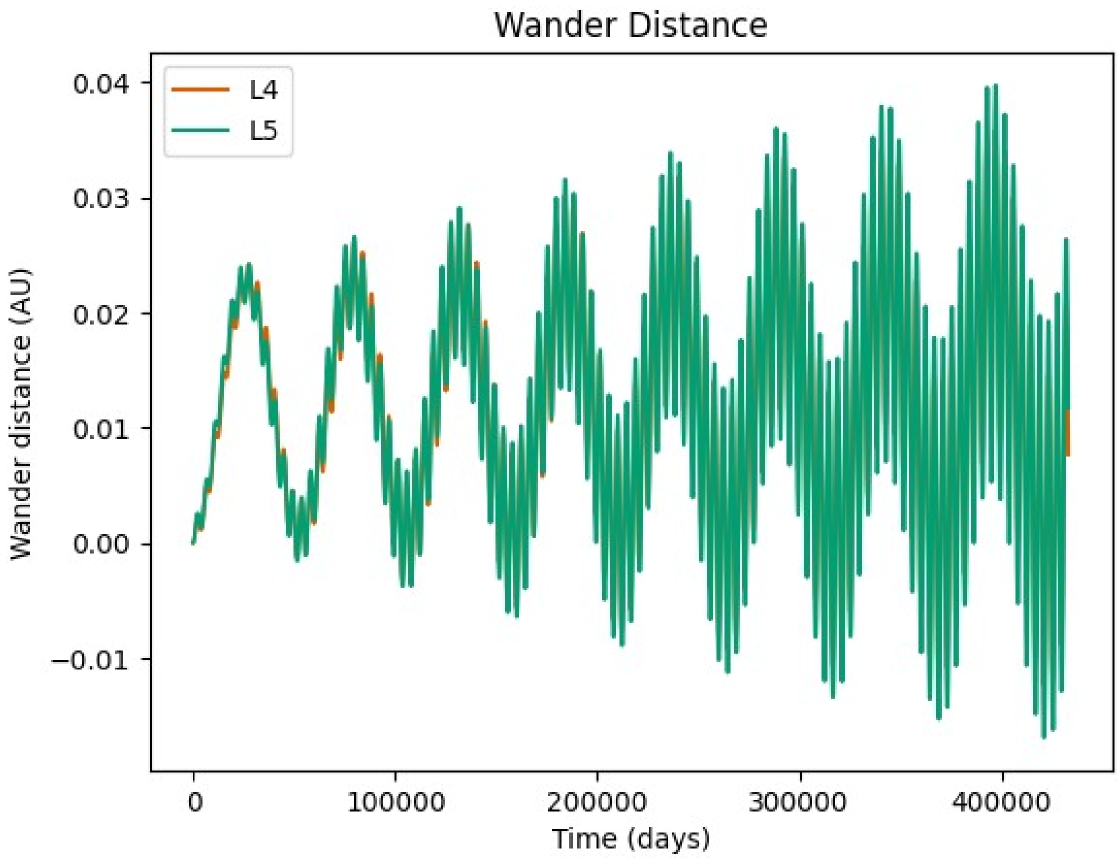
Vary the initial positions of the asteroids and plot their positions through a few hundred orbits. Your results should show that they oscillate about the Lagrange points, but do not escape from the stable position. Considering both the leading and trailing groups, determine a way to quantitatively describe how far the stable orbits wander from the Lagrange points. Plot their wander as a function of their initial position perturbation.

```
def wander(solution):
    X_L4 = solution.y[8]
    Y_L4 = solution.y[9]

    X_L5 = solution.y[12]
    Y_L5 = solution.y[13]

    wander_distance_L4 = np.sqrt(X_L4 ** 2 + Y_L4 ** 2) - 5.2
    wander_distance_L5 = np.sqrt(X_L5 ** 2 + Y_L5 ** 2) - 5.2

    return wander_distance_L4, wander_distance_L5
```

I define a helpful `wander` function that takes in the solution and finds the distance of the asteroids from the origin (subtracting 5.2 to account for the radius of Jupiter's orbit). This function is then used to calculate the wander of the asteroids due to various different initial conditions.

```python
radius_delta = 0    # Delta for the radius of Jupiter's orbit
angle_delta = 0     # Delta for the angle of the L4 and L5 points

position_x_L4 = (radius_J + radius_delta) * np.cos((np.pi / 3) +
angle_delta)
position_y_L4 = (radius_J + radius_delta) * np.sin((np.pi / 3) +
angle_delta)
position_x_L5 = (radius_J + radius_delta) * np.cos((np.pi / 3) +
angle_delta)
position_y_L5 = -(radius_J + radius_delta) * np.sin((np.pi / 3 +
angle_delta))

init_p2 = np.array([-radius_S, 0, 0, -velocity_S, # Sun
                radius_J, 0, 0, velocity_J, # Jupiter
                position_x_L4, position_y_L4, velocity_x_L4,
velocity_y_L4, # Asteroid L4
                position_x_L5, position_y_L5, velocity_x_L5,
velocity_y_L5]) # Asteroid L4

solution_p2 = integrate.solve_ivp(derivative, [t_min, t_max], init_p2,
max_step = epsilon)

ts = solution_p2.t
wander_distance_L4, wander_distance_L5 = wander(solution_p2)

fig, ax = plt.subplots(1, 1)
ax.plot(ts, wander_distance_L4, color = CF_vermillion, label = "L4")
ax.plot(ts, wander_distance_L5, color = CF_green, label = "L5")
ax.set(title = "Wander Distance", xlabel = "Time (days)", ylabel =
"Wander distance (AU)")
ax.legend()
```

```
<matplotlib.legend.Legend at 0x72300bedc450>
```



Here, I tried to make the code more dynamic by introducing the `radius_delta` and the `angle_delta`. The radius delta changes the radius of Jupiter's orbit which is used to calculated the positions of the L4 and L5 points. The angle delta changes the angle of the L4 and L5 points which also affects the position of the L4 and L5 points.

The next part is the same as before and I calculate the varied positions of the asteroids and update the initial state to run the simulation. I then plot the wander of the asteroids as a function of the time. When there are no deltas and initial state is the unvaried one, the plot shows that the wander of the asteroids increases slowly as the time increases. Although the values are still within 0.04 AU for 100 orbits, this shows that the simulation is not perfectly accurate and might have some numerical instabilities.

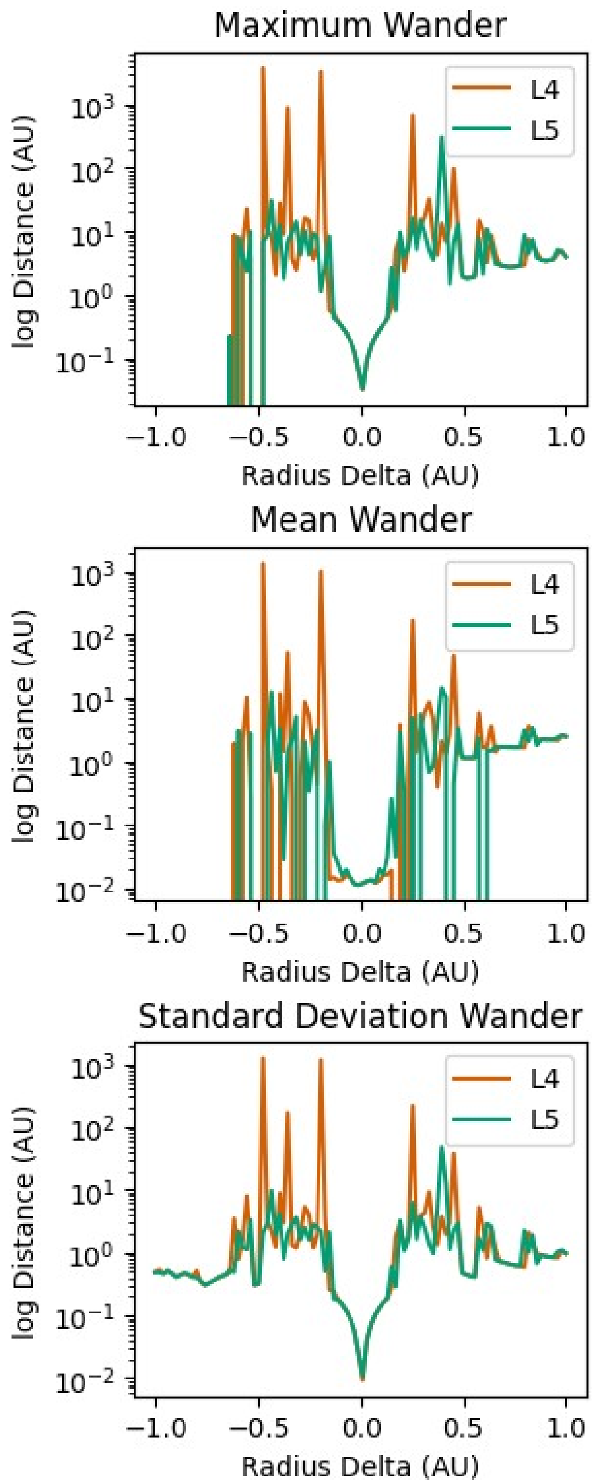
```
# Changing the radius of Jupiter's orbit
radius_delta = np.linspace(-1, 1, 100)
angle_delta = 0

max_wander_L4 = np.zeros_like(radius_delta)
max_wander_L5 = np.zeros_like(radius_delta)

mean_wander_L4 = np.zeros_like(radius_delta)
mean_wander_L5 = np.zeros_like(radius_delta)

std_wander_L4 = np.zeros_like(radius_delta)
```

```
std_wander_L5 = np.zeros_like(radius_delta)

for i, rd in enumerate(radius_delta):
    position_x_L4_p2 = (radius_J + rd) * np.cos((np.pi / 3) +
angle_delta)
    position_y_L4_p2 = (radius_J + rd) * np.sin((np.pi / 3) +
angle_delta)
    position_x_L5_p2 = (radius_J + rd) * np.cos((np.pi / 3) +
angle_delta)
    position_y_L5_p2 = -(radius_J + rd) * np.sin((np.pi / 3 +
angle_delta))

    init_p2 = np.array([-radius_S, 0, 0, -velocity_S, # Sun
                        radius_J, 0, 0, velocity_J, # Jupiter
                        position_x_L4_p2, position_y_L4_p2, velocity_x_L4,
velocity_y_L4, # Asteroid L4
                        position_x_L5_p2, position_y_L5_p2, velocity_x_L5,
velocity_y_L5]) # Asteroid L4

    solution_p2 = integrate.solve_ivp(derivative, [t_min, t_max],
init_p2, max_step = epsilon)
    wander_distance_L4, wander_distance_L5 = wander(solution_p2)

    max_wander_L4[i] = np.max(wander_distance_L4)
    max_wander_L5[i] = np.max(wander_distance_L5)
    mean_wander_L4[i] = np.mean(wander_distance_L4)
    mean_wander_L5[i] = np.mean(wander_distance_L5)
    std_wander_L4[i] = np.std(wander_distance_L4)
    std_wander_L5[i] = np.std(wander_distance_L5)
```

First, I vary the radius of Jupiter's orbit. I chose the range from -1 to 1 with 100 points to see how varying the initial radius of Jupiter's orbit affects the wander of the asteroids. In particular, I store the maximum wander distance, the mean wander distance and the standard deviation of the wander distance. I expect that with values closer to 0, there is little change in the mean and standard deviation of the wander distance. However, as the radius increases, the wander of the asteroids is expected to increase.

```
# Plotting the wander distance statistics across multiple radius
deltas
fig, (ax_max, ax_mean, ax_std) = plt.subplots(3, 1)
fig.set_size_inches(3, 9)

ax_max.plot(radius_delta, max_wander_L4, color = CF_vermillion, label
= "L4")
ax_max.plot(radius_delta, max_wander_L5, color = CF_green, label =
"L5")
ax_max.set(title = "Maximum Wander", xlabel = "Radius Delta (AU)",
ylabel = "log Distance (AU)")
ax_max.set_yscale("log")
```

```
ax_max.legend()

ax_mean.plot(radius_delta, mean_wander_L4, color = CF_vermillion,
label = "L4")
ax_mean.plot(radius_delta, mean_wander_L5, color = CF_green, label =
"L5")
ax_mean.set(title = "Mean Wander", xlabel = "Radius Delta (AU)",
ylabel = "log Distance (AU)")
ax_mean.set_yscale("log")
ax_mean.legend()

ax_std.plot(radius_delta, std_wander_L4, color = CF_vermillion, label
= "L4")
ax_std.plot(radius_delta, std_wander_L5, color = CF_green, label =
"L5")
ax_std.set(title = "Standard Deviation Wander", xlabel = "Radius Delta
(AU)", ylabel = "log Distance (AU)")
ax_std.set_yscale("log")
ax_std.legend()

plt.subplots_adjust(hspace = 0.4)
```

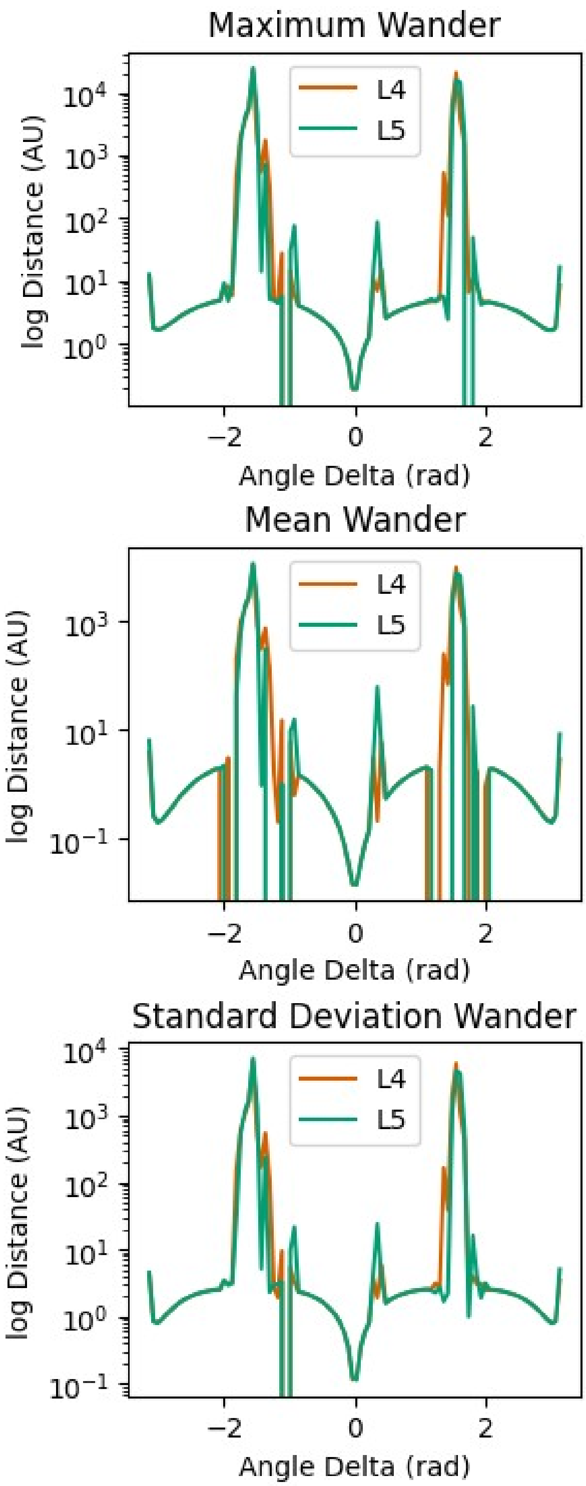


The y-axis of the graphs are logarithmic to better visualize the data in the parts where there is not much change. This is done for all the graphs related to the wander distance and its statistics. As expected, when there is a small change in the radius of Jupiter's orbit, the mean and standard deviation of the wander distance are close to 0. However, as the radius increases, the wander of the asteroids increases significantly. The graphs are also quite symmetric and it is interesting to see that when the radius is changed by around 0.2 to 0.5 AU, the maximum wander distance and other statistics are quite erratic.

```
# Changing the initial angle of the asteroid
radius_delta = 0
angle_delta = np.linspace(-np.pi, np.pi, 100)

max_wander_l4 = np.zeros_like(angle_delta)
max_wander_l5 = np.zeros_like(angle_delta)

mean_wander_l4 = np.zeros_like(angle_delta)
mean_wander_l5 = np.zeros_like(angle_delta)

std_wander_l4 = np.zeros_like(angle_delta)
std_wander_l5 = np.zeros_like(angle_delta)

for i, ad in enumerate(angle_delta):
    position_x_L4_p2 = (radius_J + radius_delta) * np.cos((np.pi / 3)
+ ad)
    position_y_L4_p2 = (radius_J + radius_delta) * np.sin((np.pi / 3)
+ ad)
    position_x_L5_p2 = (radius_J + radius_delta) * np.cos((np.pi / 3)
+ ad)
    position_y_L5_p2 = -(radius_J + radius_delta) * np.sin((np.pi / 3
+ ad))

    init_p2 = np.array([-radius_S, 0, 0, -velocity_S, # Sun
                        radius_J, 0, 0, velocity_J, # Jupiter
                        position_x_L4_p2, position_y_L4_p2, velocity_x_L4,
velocity_y_L4, # Asteroid L4
                        position_x_L5_p2, position_y_L5_p2, velocity_x_L5,
velocity_y_L5]) # Asteroid L4

    solution_p2 = integrate.solve_ivp(derivative, [t_min, t_max],
init_p2, max_step = epsilon)
    wander_distance_L4, wander_distance_L5 = wander(solution_p2)

    max_wander_L4[i] = np.max(wander_distance_L4)
    max_wander_L5[i] = np.max(wander_distance_L5)
    mean_wander_L4[i] = np.mean(wander_distance_L4)
    mean_wander_L5[i] = np.mean(wander_distance_L5)
    std_wander_L4[i] = np.std(wander_distance_L4)
    std_wander_L5[i] = np.std(wander_distance_L5)
```

Next, I vary the angle of the asteroids from Juptier. I chose the range from $-\pi$ to π with 100 points to see how varying the angle affects the wander of the asteroids. Once again, I store the maximum wander distance, the mean wander distance and the standard deviation of the wander distance. Similarly, I expect that with values closer to 0, there is little change in the statistics and as the angle increases, the wander of the asteroids is expected to increase. Moreover, I expect the graph to be more symmetric as we are dealing with angles around a circular orbit here.

```
fig, (ax_max, ax_mean, ax_std) = plt.subplots(3, 1)
fig.set_size_inches(3, 9)

ax_max.plot(angle_delta, max_wander_L4, color = CF_vermillion, label =
"L4")
ax_max.plot(angle_delta, max_wander_L5, color = CF_green, label =
"L5")
ax_max.set(title = "Maximum Wander", xlabel = "Angle Delta (rad)",
ylabel = "log Distance (AU)")
ax_max.set_yscale("log")
ax_max.legend()

ax_mean.plot(angle_delta, mean_wander_L4, color = CF_vermillion, label
= "L4")
ax_mean.plot(angle_delta, mean_wander_L5, color = CF_green, label =
"L5")
ax_mean.set(title = "Mean Wander", xlabel = "Angle Delta (rad)",
ylabel = "log Distance (AU)")
ax_mean.set_yscale("log")
ax_mean.legend()

ax_std.plot(angle_delta, std_wander_L4, color = CF_vermillion, label =
"L4")
ax_std.plot(angle_delta, std_wander_L5, color = CF_green, label =
"L5")
ax_std.set(title = "Standard Deviation Wander", xlabel = "Angle Delta
(rad)", ylabel = "log Distance (AU)")
ax_std.set_yscale("log")
ax_std.legend()

plt.subplots_adjust(hspace = 0.4)
```



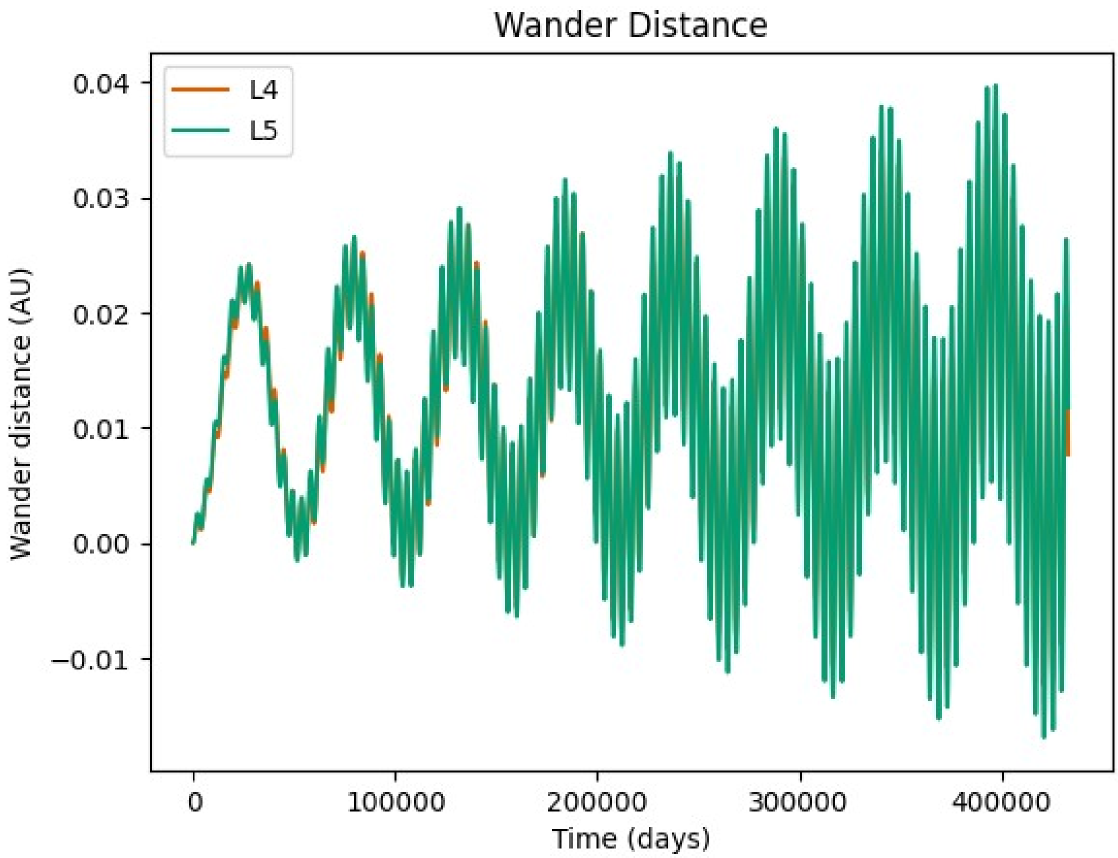
This graph does appear to be more symmetric than the graph where the radius of the orbit is changed. Moreover, the shift in the mean and standard deviation of the wander distance is more gradual than the one for the radius. This is expected as the angle of the asteroids from Jupiter is more continuous and less discrete than the radius of Jupiter's orbit.

Part 3 - Velocity Stability

Give your particles some initial velocity relative to the Lagrange points. Determine the range of initial velocity vectors for which the orbits remain captured by the Lagrange points.

```
velocity_delta = 0

init_p3 = np.array([-radius_S, 0, 0, -velocity_S, # Sun
                radius_J, 0, 0, velocity_J, # Jupiter
                position_x_L4, position_y_L4, velocity_x_L4 -
velocity_delta, velocity_y_L4 + velocity_delta, # Asteroid L4
                position_x_L5, position_y_L5, velocity_x_L5 +
velocity_delta, velocity_y_L5 + velocity_delta]) # Asteroid L4

solution_p3 = integrate.solve_ivp(derivative, [t_min, t_max], init_p3,
max_step = epsilon)

ts = solution_p3.t
wander_distance_L4, wander_distance_L5 = wander(solution_p3)

fig, ax = plt.subplots(1, 1)
ax.plot(ts, wander_distance_L4, color = CF_vermillion, label = "L4")
ax.plot(ts, wander_distance_L5, color = CF_green, label = "L5")
ax.set(title = "Wander Distance", xlabel = "Time (days)", ylabel =
"Wander distance (AU)")
ax.legend()
```

```
<matplotlib.legend.Legend at 0x72300829a6d0>
```



Similar to the start of Part 2, I start by making the code more dynamic by introducing the `velocity_delta`. The velocity delta changes the velocity of the asteroids orbit of the asteroids. Again, I calculate the varied positions of the asteroids and update the initial state to run the simulation. I then plot the wander of the asteroids as a function of the time. When there is no velocity change, the plot is identical to the one in the previous part because nothing has been changed yet.

An interesting to note is that I add the additional velocity in the same direction as the correct initial state. So for the velocity in the x direction of the asteroid in L4, the velocity delta is subtracted instead of added as it is in the opposite direction.

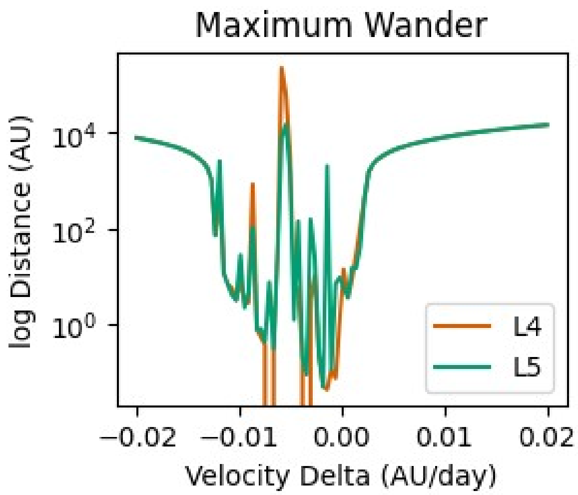
```
velocity_delta = np.linspace(-0.02, 0.02, 100)

max_wander_l4 = np.zeros_like(velocity_delta)
max_wander_l5 = np.zeros_like(velocity_delta)

mean_wander_l4 = np.zeros_like(velocity_delta)
mean_wander_l5 = np.zeros_like(velocity_delta)

std_wander_l4 = np.zeros_like(velocity_delta)
std_wander_l5 = np.zeros_like(velocity_delta)

for i, vd in enumerate(velocity_delta):
```

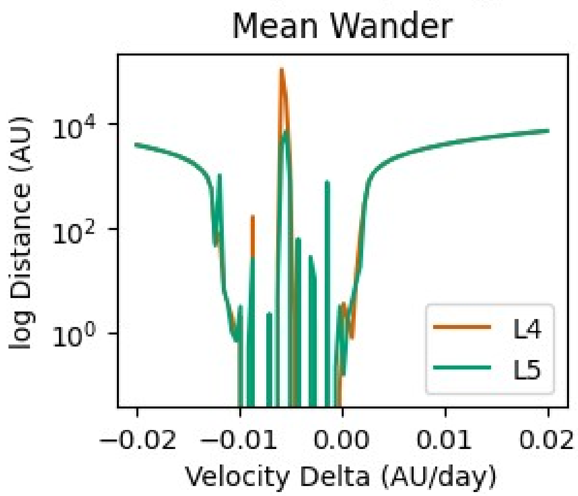
```
    init_p3 = np.array([-radius_S, 0, 0, -velocity_S, # Sun
                        radius_J, 0, 0, velocity_J, # Jupiter
                        position_x_L4, position_y_L4, velocity_x_L4 - vd,
velocity_y_L4 + vd, # Asteroid L4
                        position_x_L5, position_y_L5, velocity_x_L5 + vd,
velocity_y_L5 + vd]) # Asteroid L4

    solution_p3 = integrate.solve_ivp(derivative, [t_min, t_max],
init_p3, max_step = epsilon)
    wander_distance_L4, wander_distance_L5 = wander(solution_p3)

    max_wander_L4[i] = np.max(wander_distance_L4)
    max_wander_L5[i] = np.max(wander_distance_L5)
    mean_wander_L4[i] = np.mean(wander_distance_L4)
    mean_wander_L5[i] = np.mean(wander_distance_L5)
    std_wander_L4[i] = np.std(wander_distance_L4)
    std_wander_L5[i] = np.std(wander_distance_L5)
```

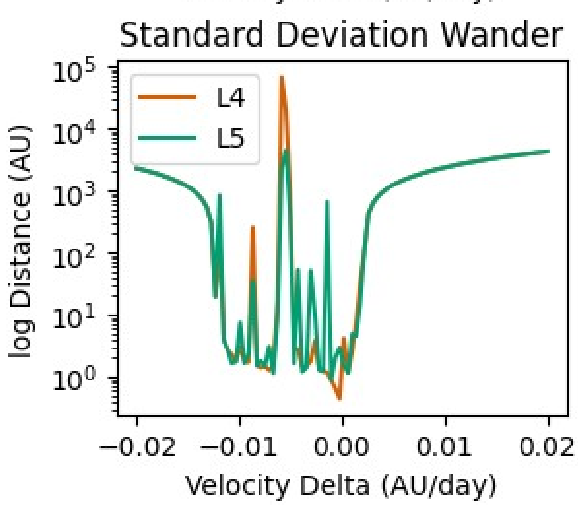
I vary the velocity of the asteroids using 100 points from the range -0.02 to 0.02. Once again, I store the statistics about the wander distance and expect that there is little distubance when the values are closer to 0. However, I expect more wandering when the velocity is increased on the intuitions that the asteroids are moving faster and are more likely to escape the Lagrange points.

```
fig, (ax_max, ax_mean, ax_std) = plt.subplots(3, 1)
fig.set_size_inches(3, 9)

ax_max.plot(velocity_delta, max_wander_L4, color = CF_vermillion,
label = "L4")
ax_max.plot(velocity_delta, max_wander_L5, color = CF_green, label =
"L5")
ax_max.set(title = "Maximum Wander", xlabel = "Velocity Delta
(AU/day)", ylabel = "log Distance (AU)")
ax_max.set_yscale("log")
ax_max.legend()

ax_mean.plot(velocity_delta, mean_wander_L4, color = CF_vermillion,
label = "L4")
ax_mean.plot(velocity_delta, mean_wander_L5, color = CF_green, label =
"L5")
ax_mean.set(title = "Mean Wander", xlabel = "Velocity Delta (AU/day)",
ylabel = "log Distance (AU)")
ax_mean.set_yscale("log")
ax_mean.legend()

ax_std.plot(velocity_delta, std_wander_L4, color = CF_vermillion,
label = "L4")
ax_std.plot(velocity_delta, std_wander_L5, color = CF_green, label =
"L5")
ax_std.set(title = "Standard Deviation Wander", xlabel = "Velocity
```

```
Delta (AU/day)", ylabel = "log Distance (AU)")
ax_std.set_yscale("log")
ax_std.legend()

plt.subplots_adjust(hspace = 0.4)
```







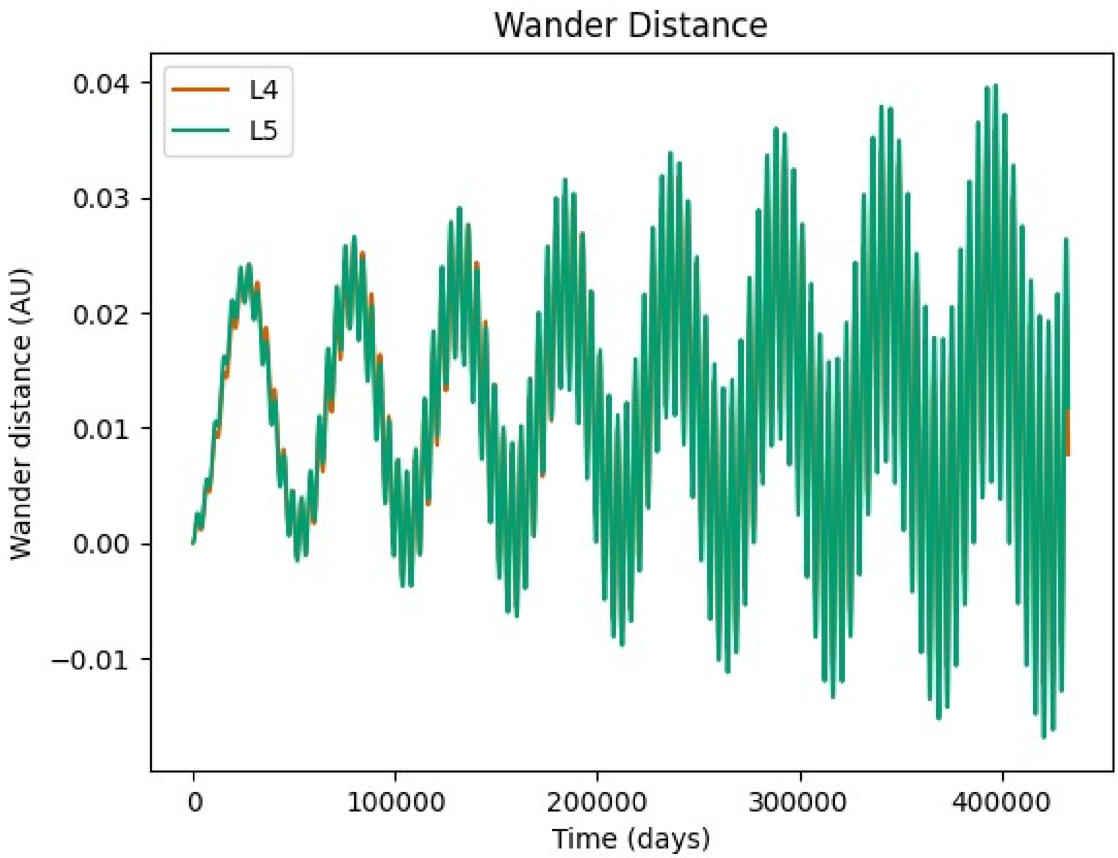
The effects of changing the velocity seem more severe than changing the position. This suggests that the asteroid orbits are more sensitive to changes in velocity than changes in position. The graphs are also more erratic and the wander distance increases more rapidly than when the position is changed. Even at some points close to 0 change, the wander distance is quite high.

Part 4 - Mass Dependence

Similar "Trojan" asteroids are also present in the orbit of Mars. Run your program for a range of planetary masses (within two orders of magnitude of Jupiter's mass should work). Explore the range of distances that the asteroids wander as a function of the planetary mass.

```
M_J = 0.001

derivative_p4 = lambda t, state: general_derivative(t, state, M_J)

solution_p4 = integrate.solve_ivp(derivative_p4, [t_min, t_max], init,
max_step = epsilon)

ts = solution_p4.t
wander_distance_L4, wander_distance_L5 = wander(solution_p4)

fig, ax = plt.subplots(1, 1)
ax.plot(ts, wander_distance_L4, color = CF_vermillion, label = "L4")
ax.plot(ts, wander_distance_L5, color = CF_green, label = "L5")
ax.set(title = "Wander Distance", xlabel = "Time (days)", ylabel =
"Wander distance (AU)")
ax.legend()
```

```
<matplotlib.legend.Legend at 0x72300bfb7650>
```



Once again, I setup the code to check that changing the mass of Jupiter is working as expected. This also highlights the importance of making more paramaterized code as it is much easier to play around with different planetary masses once a general derivative is defined. The graph of the wander distance as a function of time is identical to the one in Part 2 when the mass of Jupiter is not changed.

```
def plot_for_mass(mass, ax):
    derivative_p4 = lambda t, state: general_derivative(t, state, 
mass)
    solution_p4 = integrate.solve_ivp(derivative_p4, [t_min, t_max], 
init, max_step = epsilon)
    wander_distance_L4, wander_distance_L5 = wander(solution_p4)

    ax.plot(solution_p4.t, wander_distance_L4, color = CF_vermillion, 
label = "L4")
    ax.plot(solution_p4.t, wander_distance_L5, color = CF_green, label 
= "L5")
    ax.set(title = f"Wander for Mass {mass}", xlabel = "Time (days)", 
ylabel = "Wander distance (AU)")
    ax.legend()

    print(f"""
Statistics for Mass {mass}
```

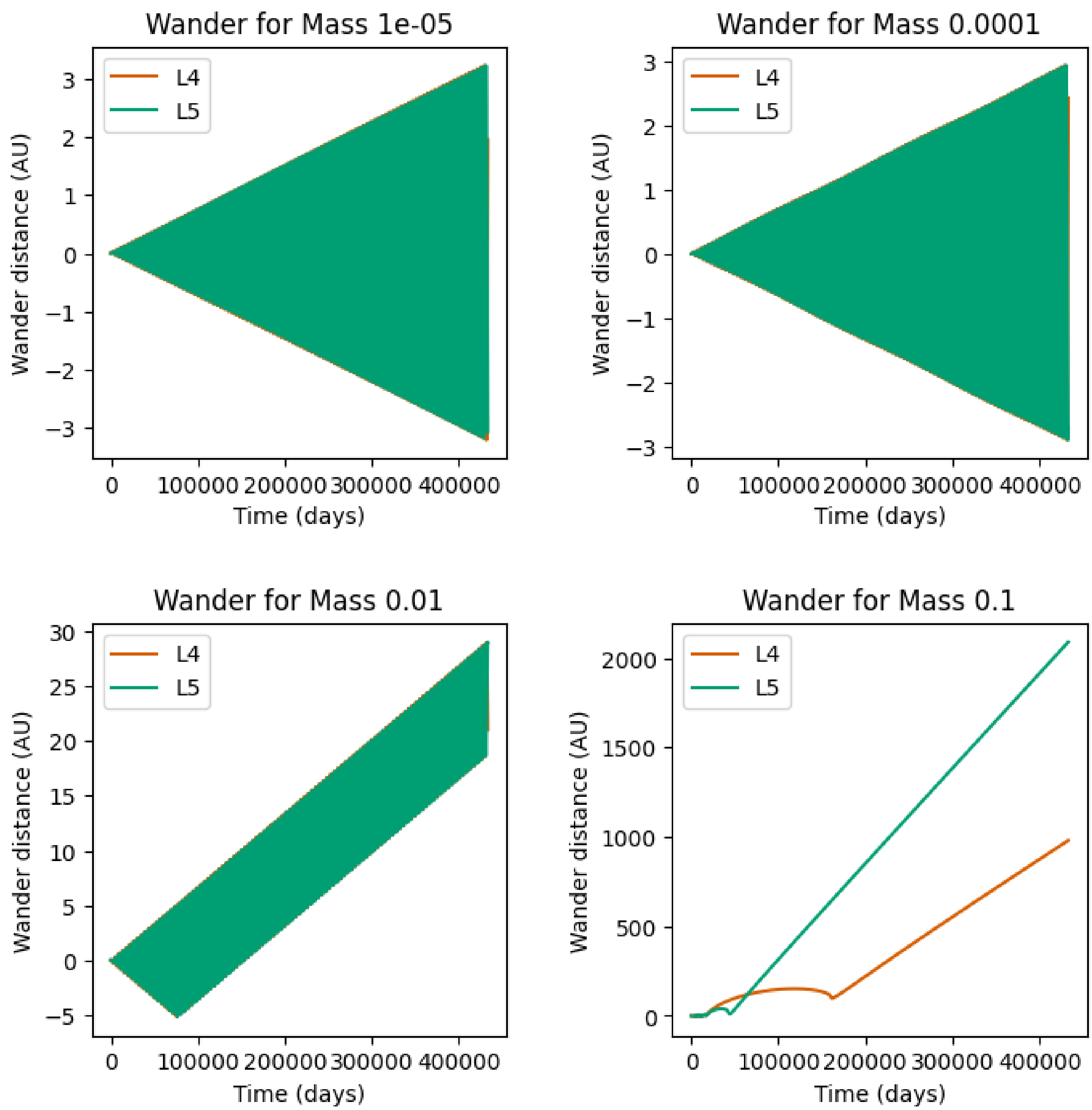
```
L4 Maximum Wander Distance: {np.max(wander_distance_L4)}
L4 Mean Wander Distance: {np.mean(wander_distance_L4)}
L4 Standard Deviation Wander Distance: {np.std(wander_distance_L4)}

L5 Maximum Wander Distance: {np.max(wander_distance_L5)}
L5 Mean Wander Distance: {np.mean(wander_distance_L5)}
L5 Standard Deviation Wander Distance: {np.std(wander_distance_L5)}
""")
```

I define a `plot_for_mass` function that given a platerary mass, creates the updated derivative function and uses it to solve the equations of motion for the system using the initial state for the time range and maximum step. It then plots the wander of the asteroids as a function of the time. I then plot the wander of the asteroids as a function of the time. Additionally, the function also prints out the maximum wander distance, the mean wander distance and the standard deviation of the wander distance for both the L4 and L5 asteroids.

```
fig, ((ax_M1, ax_M2), (ax_M3, ax_M4)) = plt.subplots(2, 2)
fig.set_size_inches(8, 8)

plot_for_mass(1.e-05, ax_M1)
plot_for_mass(1.e-04, ax_M2)
plot_for_mass(1.e-02, ax_M3)
plot_for_mass(1.e-01, ax_M4)
plt.subplots_adjust(wspace = 0.4, hspace = 0.4)
```

```
Statistics for Mass 1e-05

L4 Maximum Wander Distance: 3.2313343922260076
L4 Mean Wander Distance: 0.18796212936882983
L4 Standard Deviation Wander Distance: 1.3003142235596663

L5 Maximum Wander Distance: 3.2256179884892076
L5 Mean Wander Distance: 0.1902183411788807
L5 Standard Deviation Wander Distance: 1.2941570813481236


Statistics for Mass 0.0001

L4 Maximum Wander Distance: 2.944876043393694
L4 Mean Wander Distance: 0.15705485462456695
L4 Standard Deviation Wander Distance: 1.1846414124933742

L5 Maximum Wander Distance: 2.942615026042584
L5 Mean Wander Distance: 0.1557658325978277
L5 Standard Deviation Wander Distance: 1.1808652971489106


Statistics for Mass 0.01
```

```
L4 Maximum Wander Distance: 28.97021780420963
L4 Mean Wander Distance: 10.340662358115262
L4 Standard Deviation Wander Distance: 8.323682321624027

L5 Maximum Wander Distance: 29.01132794162741
L5 Mean Wander Distance: 10.255380167366468
L5 Standard Deviation Wander Distance: 8.301426635331143


Statistics for Mass 0.1

L4 Maximum Wander Distance: 980.110785067687
L4 Mean Wander Distance: 378.4869169451836
L4 Standard Deviation Wander Distance: 293.7429247656761

L5 Maximum Wander Distance: 2088.9896549105597
L5 Mean Wander Distance: 947.8085459754622
L5 Standard Deviation Wander Distance: 649.2752280547942
```



These graphs show that the wander distance of the asteroids increases as the mass of Jupiter changes. This is expected as the effect of the gravitational pull of Jupiter is changed, no longer keeping the balance of forces that sustained the Lagrange points. However, it is interesting to note that when the mass of Jupiter is decreased, the asteroid wanders away as time goes by up to around 3 AU. However, when the mass of Jupiter is increased, the wander distance increases much more rapidly. When the mass is 0.01 solar masses, the wander distance increases to up to 30 AU after 100 orbits. Moreover, when the mass is 0.1 solar masses, the wander distance increases to more than 2000 AU after 100 orbits. This suggests that the wander distance is not linearly dependent on the mass of Jupiter. Personally, I find it counterintuitive that the wander distance increases more rapidly when the mass of Jupiter is increased as I would expect them to be pulled closer into Jupiter.