7. Fermi and Bose Gases

Kuan-Hsuan Yeh

December 4, 2017

- Whenever $n \geq n_Q \equiv (M\tau/2\pi\hbar^2)^{3/2}$ the gas is said to be in the quantum regime and is called a **qunatum gas**.
- The quantum regime obtains when the temperature is below

  $$\tau_0 \equiv (2\pi\hbar^2/M)n^{2/3}$$

  defined by the condition $n = n_Q$
- A gas in the quantum regime with $\tau \ll \tau_0$ is often said to be a **degenerate gas**.
- Fermions: at absolute zero all orbials with $0 < \epsilon < \epsilon_F$ will be occupied with $f = 1$, where ϵ_F (**Fermi energy**) is the energy below which there are just enough orbitals to hold the number of particles assigned to the system, or the energy of the highest filled orbital at absolute zero.
- Bosons: at absolute zero the ground orbital is occupied by all the particles in the system.
- $n_F = (3N/\pi)^{1/3}$, radius of the sphere at fermi energy.
- $\epsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} \equiv \tau_F$ (fermi temeperature)
- Total ground state kinetic energy:

  $$U_0 = \frac{3}{5}N\epsilon_F$$

- At constant N the energy increases as the volume decreases, so that the Fermi energy gives a repulsive contribution to the binding of any material.
- $\mathcal{D}(\epsilon)$: **density of states -** the number of orbitals of energy between ϵ and $\epsilon + d\epsilon$
- $$\mathcal{D}(\epsilon) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\epsilon^{1/2}$$
- **Einstein condensation**: a substantial fraction of the total number of noninteracting boson particles in the system occupy the ground orbital