4. Thermal Radiation and Planck Distribution

Kuan-Hsuan Yeh

October 23, 2017

- The Planck distribution describes the spectrum of the electromagnetic radiation in thermal equilibrium within a cavity.
- Thermal electromagnetic radiation is often called black body radiation.
- **Mode** characterizes a particular oscillation amplitude pattern in the cavity.
- Frequency of the radiation: $\omega = 2\pi f$
- Energy ϵ_s of the state with s quanta in the mode:

$$\epsilon_s = s\hbar\omega$$

- Zero point energy: $\frac{1}{2}\hbar\omega$
- These energies are the same as the energies of a quantum harmonic oscillator, but there is a difference between the concepts. s for the oscillator is called the quantum number, and s for the quantized electromagnetic mode is called the number of photons in the mode.
- **Planck distribution function** for the thermal average number of photons in a single mode of frequency ω:

$$< s >= \frac{1}{exp(\hbar\omega/\tau) - 1}$$

- Thermal average energy in the mode is:

$$< \epsilon >=< s > \hbar\omega = \frac{\hbar\omega}{exp(\hbar\omega/\tau - 1)}$$

- A transverse wave is a moving wave that consists of oscillations occurring perpendicular to the direction of the energy transfer (or the propagation of the wave).
- Frequency ω of the mode in terms of n: $\omega_n = n\pi c/L$

- Energy per unit volume (**Stefan-Boltzmann law of radiation**):

$$\frac{U}{V} = \frac{\pi^2}{15\hbar^3 c^3}\tau^4$$

- Spectral density (**Planck radiation law** - the frequency distribution of thermal radiation):

$$u_\omega = \frac{\hbar}{\pi^2 c^3}\frac{\omega^3}{exp(\hbar\omega/\tau) - 1}$$

- **Stefan-Boltzmann constant** - a body that radiates at this rate is said to radiate as a black body:

$$\sigma_B = \pi^2 k_B^4/60\hbar^3 c^2 = 5.670 \times 10^{-8} W m^2 K^{-4}$$

- The flux density of radiation energy is $J_U = \sigma_B T^4$