5. Chemical Potential and Gibbs Distribution

Kuan-Hsuan Yeh

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- The **chemical potential** governs the flow of particles between the systems, just as the temperature governs the flow of energy.
- Definition of chemical potential:

  $$\mu(\tau, V, N) \equiv \left(\frac{\partial F}{\partial N}\right)_{t,V}$$

  then

  $$\mu_1 = \mu_2$$

  expresses the condition of diffusive equilibrium.
- Two systems are in combined thermal and diffusive equilibrium when their temperatures and chemical potentials are equal: $\tau_1 = \tau_2; \mu_1 = \mu_2$.
- The chemical potential is equivalent to a true potential energy: the difference in chemical potential between two systems is equal to the potential barrier that will bring the two systems into diffusive equilibrium.
- **Total chemical potential**:

  $$\mu = \mu_{ext} + \mu_{int}$$

- Alternative relation:

  $$\frac{\mu(U, V, N)}{\tau} = -\left(\frac{\partial \sigma}{\partial N}\right)_{U,V}$$

- **Gibbs factor**:

  $$exp[(N\mu - \epsilon)/\tau]$$

- **Gibbs sum** (grand partition function):

  $$\mathcal{Z}(\mu, \tau) = \sum_{ASN} exp[(N\mu - \epsilon_{s(N)})/\tau]$$

  where ASN stands for all states and numbers of particles

- The absolute probability that the system will be found in a state N_1, ϵ_1 is given by:

$$P(N_1, \epsilon_1) = \frac{exp[(N_1\mu - \epsilon_1)/\tau]}{\mathcal{Z}}$$

- **Absolute activity**:

$$\lambda \equiv exp(\mu/\tau)$$