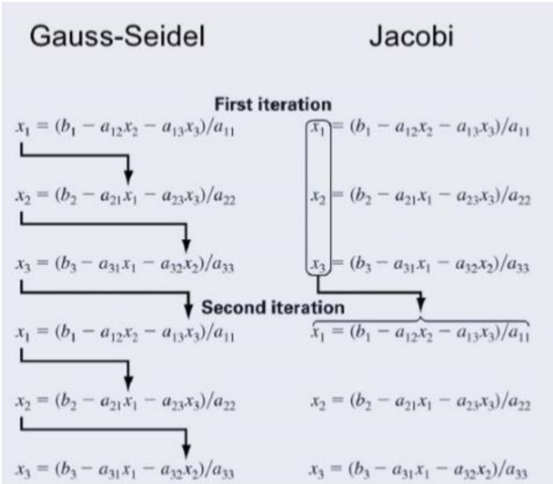
Ejercicio 3*

El método de Gauss-Seidel es una mejora del método de Jacobi, se ilustra en el siguiente cuadro:



La ecuación genérica es como sigue:

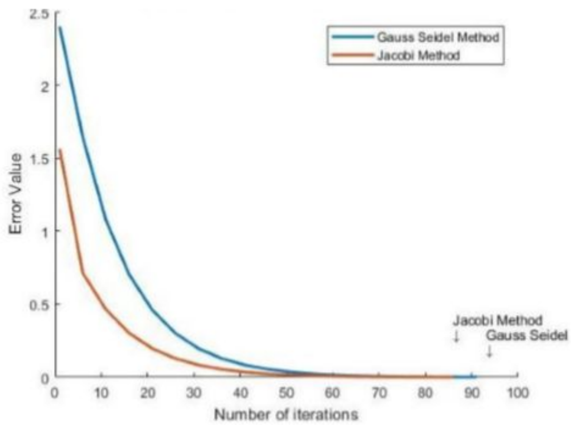
$$x_i^{k-1} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{k-1} - \sum_{j=i-1}^{n} a_{ij}x_j^{k}\right]$$

a. Implemente el algoritmo en Octave.
b. Aplíquelo a la resolución del sistema:

$$A = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 6 & 1 \\ -1 & 1 & 1 \end{bmatrix};\ b = \begin{bmatrix} 3 \\ 9 \\ -6 \end{bmatrix}$$

partiendo de $x^0 = [0,0,0]^t$

c. Resuelva el mismo sistema por Jacobi.
d. Realice una gráfica que muestre la velocidad de convergencia de ambos métodos, similar a la siguiente:



b.

```
Command Window
>> n=3;
>> A=[4 -1 -1; -2 6 1; -1 1 1];
>> b=[3 9 -6];
>> m=500;
>> E=0.00001;
>> [vectPasos, vectErr]=P3_E3_D_gseidel(A,b,E,m)
1  0.750000  1.750000 -7.000000  7.000000
2 -0.562500  2.479167 -9.041667  2.041667
3 -0.890625  2.710069 -9.600694  0.559028
4 -0.972656  2.775897 -9.748553  0.147859
5 -0.993164  2.793704 -9.786868  0.038315
6 -0.998291  2.798381 -9.796672  0.009804
7 -0.999573  2.799588 -9.799161  0.002488
8 -0.999893  2.799896 -9.799789  0.000628
9 -0.999973  2.799974 -9.799947  0.000158
10 -0.999993  2.799993 -9.799987  0.000040
11 -0.999998  2.799998 -9.799997  0.000010
vectPasos =
```

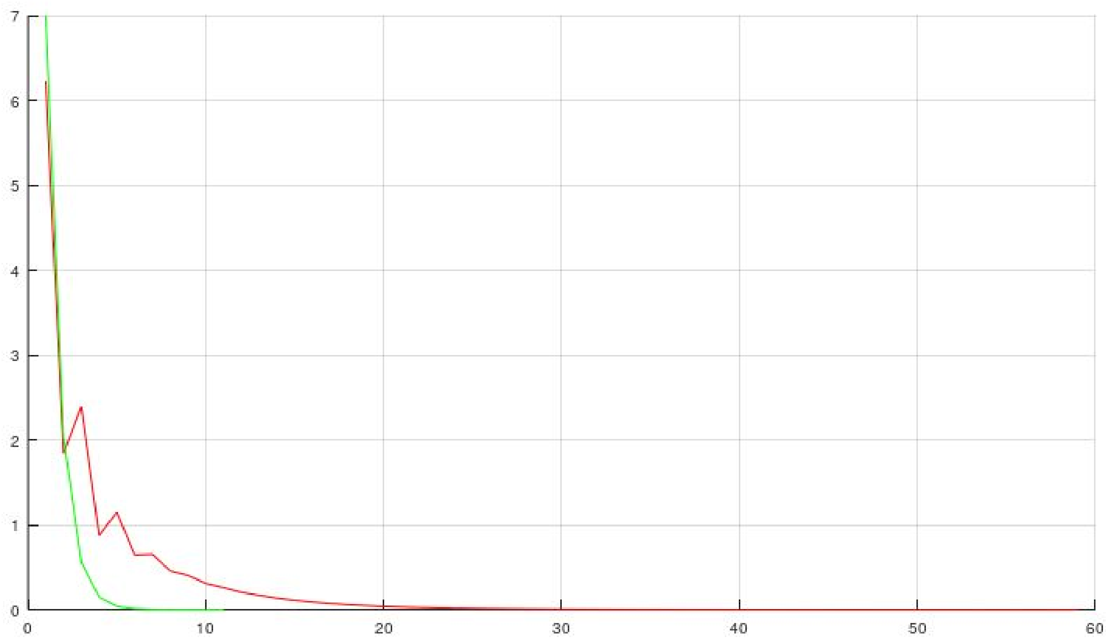
c.

```
Command Window
>> n=3;
>> A=[4 -1 -1; -2 6 1; -1 1 1];
>> b=[3 9 -6];
>> m=500;
>> E=0.00001;
>> [vectPasos, vectErr]=P3_E3_D_jacobi(n,A,b,m,E)
1  0.750000  1.500000 -6.000000  6.229968
2 -0.375000  2.750000 -6.750000  1.841365
3 -0.250000  2.500000 -9.125000  2.391391
4 -0.906250  2.937500 -8.750000  0.873324
5 -0.703125  2.656250 -9.843750  1.147454
6 -1.046875  2.906250 -9.359375  0.644425
7 -0.863281  2.710938 -9.953125  0.651454
8 -1.060547  2.871094 -9.574219  0.456217
9 -0.925781  2.742188 -9.931641  0.403149
10 -1.047363  2.846680 -9.667969  0.308583
11 -0.955322  2.762207 -9.894043  0.258296
12 -1.032959  2.830566 -9.717529  0.204591
13 -0.971741  2.775269 -9.863525  0.167691
14 -1.022064  2.820007 -9.747009  0.134573
15 -0.981750  2.783813 -9.842072  0.109417
16 -1.014565  2.813095 -9.765564  0.088247
17 -0.988117  2.789406 -9.827660  0.071530
18 -1.009563  2.808571 -9.777523  0.057801
19 -0.992238  2.793066 -9.818134  0.046796
20 -1.006267  2.805610 -9.785304  0.037842
21 -0.994924  2.795462 -9.811877  0.030623
22 -1.004104  2.803672 -9.790385  0.024770
23 -0.996678  2.797030 -9.807775  0.020042
24 -1.002686  2.802403 -9.793708  0.016213
25 -0.997826  2.798056 -9.805090  0.013117
26 -1.001758  2.801573 -9.795882  0.010612
27 -0.998577  2.798728 -9.803331  0.008585
28 -1.001151  2.801029 -9.797305  0.006945
29 -0.999069  2.799167 -9.802180  0.005619
```

Command Window

```
28 -1.001151  2.801029 -9.797305  0.006945
29 -0.999069  2.799167 -9.802180  0.005619
30 -1.000753  2.800674 -9.798236  0.004546
31 -0.999391  2.799455 -9.801427  0.003678
32 -1.000493  2.800441 -9.798845  0.002975
33 -0.999601  2.799643 -9.800934  0.002407
34 -1.000323  2.800289 -9.799244  0.001947
35 -0.999739  2.799766 -9.800611  0.001575
36 -1.000211  2.800189 -9.799505  0.001275
37 -0.999829  2.799847 -9.800400  0.001031
38 -1.000138  2.800124 -9.799676  0.000834
39 -0.999888  2.799900 -9.800262  0.000675
40 -1.000090  2.800081 -9.799788  0.000546
41 -0.999927  2.799935 -9.800171  0.000442
42 -1.000059  2.800053 -9.799861  0.000357
43 -0.999952  2.799957 -9.800112  0.000289
44 -1.000039  2.800035 -9.799909  0.000234
45 -0.999969  2.799972 -9.800073  0.000189
46 -1.000025  2.800023 -9.799941  0.000153
47 -0.999979  2.799982 -9.800048  0.000124
48 -1.000017  2.800015 -9.799961  0.000100
49 -0.999987  2.799988 -9.800031  0.000081
50 -1.000011  2.800010 -9.799975  0.000066
51 -0.999991  2.799992 -9.800021  0.000053
52 -1.000007  2.800006 -9.799983  0.000043
53 -0.999994  2.799995 -9.800013  0.000035
54 -1.000005  2.800004 -9.799989  0.000028
55 -0.999996  2.799997 -9.800009  0.000023
56 -1.000003  2.800003 -9.799993  0.000018
57 -0.999998  2.799998 -9.800006  0.000015
58 -1.000002  2.800002 -9.799995  0.000012
59 -0.999998  2.799999 -9.800004  0.000010
```

d.

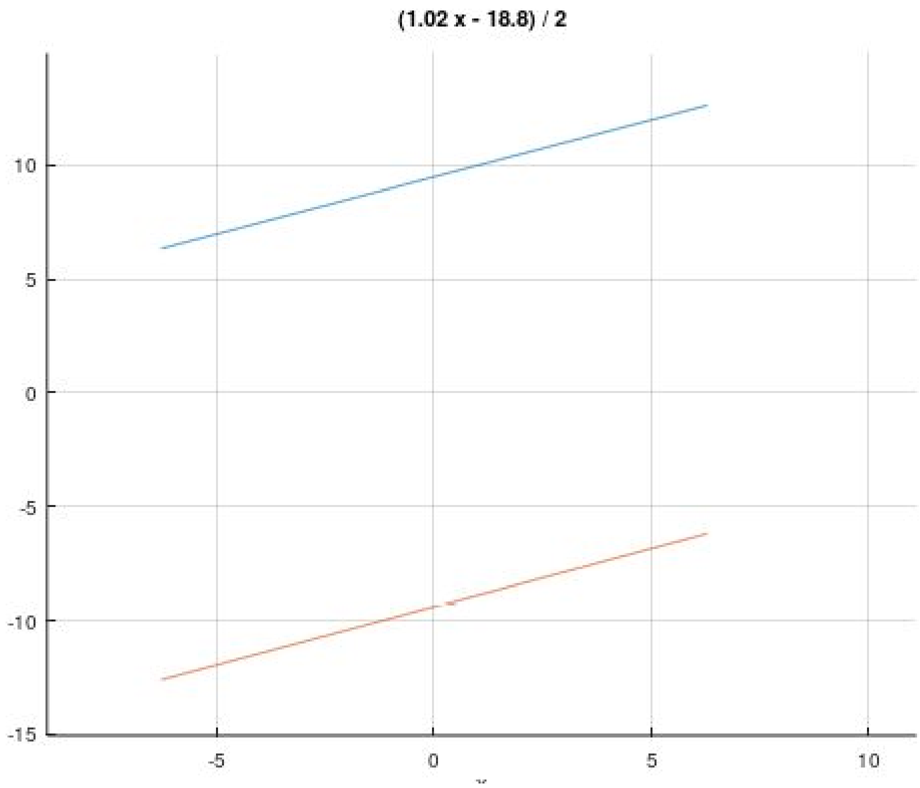


Ejercicio 4*

Dadas las ecuaciones: $\begin{cases} 0.5\,x - y = -9.5 \\ 1.02\,x - 2y = 18.8 \end{cases}$

a. Resolverla gráficamente.
b. Calcule el determinante, usando la función $det()$.
c. Teniendo en cuenta los puntos a. y b. ¿qué esperaría respecto al condicionamiento del sistema?
d. Resuelva el sistema aplicando el método de Gauss.
e. Resuélvalo nuevamente pero modificando el valor a_{11} levemente a 0.52. Interprete el resultado.
f. Calcule el radio espectral $\boldsymbol{\rho}$ e interprételo teniendo en cuenta las conclusiones anteriores.

a.



b.

```
Determinante
ans =  0.020000
Cond
ans =  314.52
>>
```

c.
Es un sistema mal condicionado, una pequeña variación en uno de los coeficientes provoca una gran variación en la solución. El número condición de la matriz es grande.

d.

```
Command Window
>> A=[0.5 -1; 1.02 -2];
>> b=[-9.5;18.8];
>> n=2;
>> [A, b]=P3_E4_D_gauss(A,b,n)
x =

   1890.00    954.50

A =

   0.50000  -1.00000
   0.00000   0.04000

b =

   -9.5000
   38.1800

>>
```

e.

```
Command Window
>> A=[0.52 -1; 1.02 -2];
>> b=[-9.5;18.8];
>> n=2;
>> [A, b]=P3_E4_E_gauss(A,b,n)
x =

  -1890.00   -973.30

A =

   0.52000  -1.00000
   0.00000  -0.03846

b =

   -9.5000
   37.4346

>>
```

Los valores resultantes cambiaron mucho con un pequeño cambio en los coeficientes. El sistema está mal condicionado.

f.

```
Command Window
>> P3_E4_F

Radio espectral
ans =   1.4934
>>
```

Si la matriz es diagonalmente dominante converge, si no es diagonalmente dominante no se sabe si converge.
Con el radio espectral p, converge si 0<p<1, si no cumple esa cond. diverge

Ejercicio 7*

Dado los siguientes datos:

x	0.6	0.7	0.8	1.0
$y = e^x$	1.82212	2.01375	2.22554	2.718228

x	1.3	1.4
$y = e^x$	3.66930	4.05520

a. Encuentre el polinomio de interpolación usando el método de Newton.
b. Calcule $p(0.75)$ y $p(1.1)$.
c. Calcule una cota para el error.

a.
Polinomio resultado

P5(x) = 1.82212 + 1.91630(x - x0) + 1.00800(x-x0)(x-x1) + 0.35950(x-x0)(x-x1)(x-x2)
+0.1097619 (x-x0)(x-x1)(x-x2)(x-x3) + 0.0011905(x-x0)(x-x1)(x-x2)(x-x3)(x-x4)

P5(x) = 1.82212 + 1.91630*(x - 0.6) + 1.00800*(x-0.6)*(x-0.7)
+ 0.35950*(x-0.6)*(x-0.7)*(x-0.8) + 0.1097619*(x-0.6)*(x-0.7)*(x-0.8)*(x-1)
+ 0.0011905*(x-0.6)*(x-0.7)*(x-0.8)*(x-1)*(x-1.3)

b.
En pto x=0.75

P5(0.75) = 1.82212 + 1.91630*(x - 0.6) + 1.00800*(x-0.6)*(x-0.7)
+   0.35950*(x-0.6)*(x-0.7)*(x-0.8) + 0.1097619*(x-0.6)*(x-0.7)*(x-0.8)*(x-1)
+ 0.0011905*(x-0.6)*(x-0.7)*(x-0.8)*(x-1)*(x-1.3) = 2.1170

En pto x=1.1

P5(0.75) = 1.82212 + 1.91630*(x - 0.6) + 1.00800*(x-0.6)*(x-0.7)
+0.35950*(x-0.6)*(x-0.7)*(x-0.8) + 0.1097619*(x-0.6)*(x-0.7)*(x-0.8)*(x-1)
+ 0.0011905*(x-0.6)*(x-0.7)*(x-0.8)*(x-1)*(x-1.3) = 3.0041

c.
Resto de newton puntos no equiespaciados
f(x)-Pn(x)=f'(n+1)(c)/(n+1)!(x-x0)(x-x1)...(x - xn-1)

Mayor valor de la funcion derivada: x=1.4
f''''''(1.4)=e^1.4= 4.0552
4.0552/6! *(x-0.6)*(x-0.7)*(x-0.8)*(x-1)*(x-1.3)*(x-1.4)

Ejercicio 9*

Usando los datos del Ejercicio 7:

a. Calcule el polinomio de interpolación aplicando el método de Lagrange
b. Grafique el polinomio interpolante vs $y = e^x$
c. Determine el valor de $e^{0.9}$ y calcule el error relativo real.

a.
pp0= -30484375*pi*(x - 7/5)*(x - 13/10)*(x - 1)*(x - 4/5)*(x - 7/10)/235466

pp1= 22375*(x - 7/5)*(x - 13/10)*(x - 1)*(x - 4/5)*(x - 3/5)/14

pp2= -189500*pi*(x - 7/5)*(x - 13/10)*(x - 1)*(x - 7/10)*(x - 3/5)/321

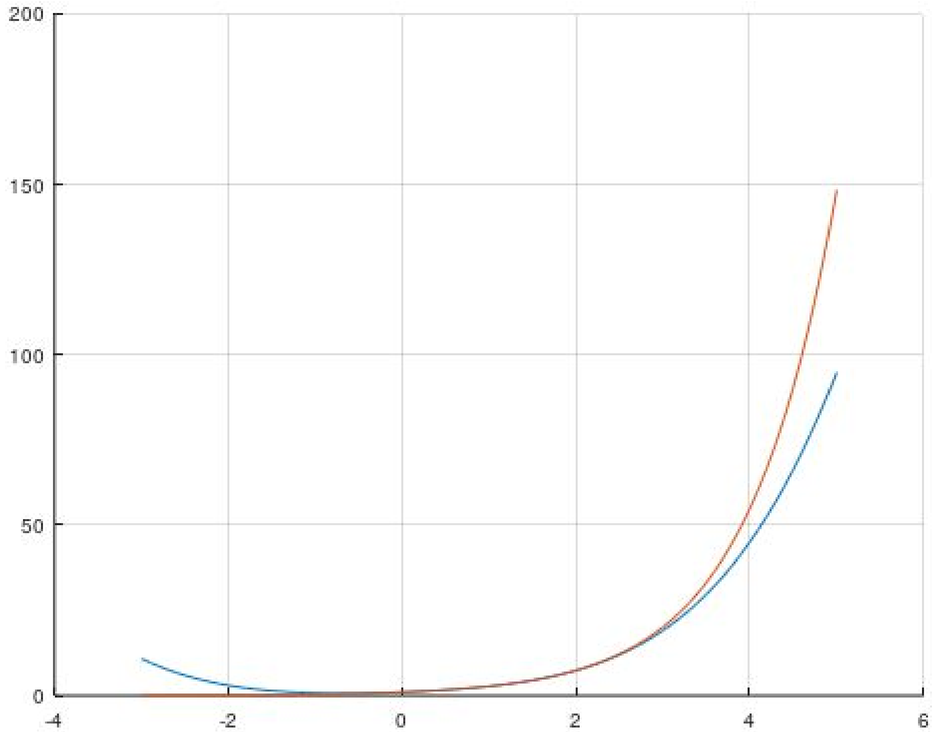
pp3= 11696875*(x - 7/5)*(x - 13/10)*(x - 4/5)*(x - 7/10)*(x - 3/5)/12393

pp4= -16255000*(x - 7/5)*(x - 1)*(x - 4/5)*(x - 7/10)*(x - 3/5)/27909

pp5= 2984375*(x - 13/10)*(x - 1)*(x - 4/5)*(x - 7/10)*(x - 3/5)/9891

ppf= -30484375*pi*(x - 7/5)*(x - 13/10)*(x - 1)*(x - 4/5)*(x - 7/10)/235466 + 22375*(x - 7/5)*(x - 13/10)*(x - 1)*(x - 4/5)*(x - 3/5)/14 -189500*pi*(x - 7/5)*(x - 13/10)*(x - 1)*(x - 7/10)*(x - 3/5)/321 + 11696875*(x - 7/5)*(x - 13/10)*(x - 4/5)*(x - 7/10)*(x - 3/5)/12393 -16255000*(x - 7/5)*(x - 1)*(x - 4/5)*(x - 7/10)*(x - 3/5)/27909 + 2984375*(x - 13/10)*(x - 1)*(x - 4/5)*(x - 7/10)*(x - 3/5)/9891

b.



Command Window

```
>> x0=[0.6 0.7 0.8 1 1.3 1.4];
>> y0=[1.82212 2.01375 2.22554 2.718228 3.6693 4.0552];
>> n=6;
>> x=0.75
x =  0.75000
>> [lagrng] = P3_E9_lagrange(x, x0, y0, n)
lagrng =  2.1170
>>
```

Command Window

```
>> x0=[0.6 0.7 0.8 1 1.3 1.4];
>> y0=[1.82212 2.01375 2.22554 2.718228 3.6693 4.0552];
>> n=6;
>> x=1.1
x =  1.1000
>> [sum] = P3_E9_lagrange(x, x0, y0, n)
sum =  3.0041
>>
```

Command Window

```
>> x0=[0.6 0.7 0.8 1 1.3 1.4];
>> y0=[1.82212 2.01375 2.22554 2.718228 3.6693 4.0552];
>> n=6;
>> x=0.9
x =  0.90000
>> [sum] = P3_E9_lagrange(x, x0, y0, n)
sum =  2.4596
>>
```

c.

```
>> P3_E9_C

x =  0.90000
aprox =  2.4596
error =  0.0000012649
>>
```

Ejercicio 10*

El cuadro siguiente representa la relación entre el tiempo (t) y la velocidad (v):

Tiempo (s)	Velocidad (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97

a. Calcule el valor $v(16)$, aplicando el método de splines lineales.
b. Grafique.

a.

s0= 2838*x/125 para x [0, 10]
s1= 6787*x/250 -1111/25 para x [10, 15]
s2= 15457*x/500 -10093/100 para x [15, 20]
s3= -10347*x/50+3*sqrt(40397)*(2*x/5 -8) + 93123/20 para x [20, 22.5]

s2= 15457*x/500 -10093/100 para x [15, 20] en 16 ans = 393.69

Command Window

```
>> x=16;
>> n=5;
>> xx=[0 10 15 20 22.5];
>> ff=[0 227.04 362.78 517.35 602.97];
>> sol=P3_E10_splines_linear_point(x, xx, ff, n)
n =  4
sol =

   363.26   389.93   393.69   380.36

sol =

   363.26   389.93   393.69   380.36

>>
```

b.