User Cancellation Avoidance

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1 Introduction to the Problem

We have implemented a finite element method using FEniCS to solve a time-dependent Navier-Stokes equations

$$\frac{\partial}{\partial t}\mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} - \nu \Delta \mathbf{u} + \nabla p = \mathbf{f}, \text{ in } \Omega \times (0, T], \tag{1.1}$$

$$\nabla \cdot \mathbf{u} = 0, \text{ in } \Omega \times (0, T], \tag{1.2}$$

$$\mathbf{u} = \mathbf{u}_D, \text{ on } \Gamma_D \times (0, T], \tag{1.3}$$

$$p = p_N, \text{ on } \Gamma_N \times (0, T], \tag{1.4}$$

which are a system of equations for the velocity $\mathbf{u}$ and pressure p in an incompressible fluid. $\mathbf{f}$ is a given force per unit volume and ν measures the viscosity of the fluid. The boundary of the spatial domain Ω has been split into Γ_D and Γ_N where different boundary conditions are imposed. Next we walk through the procedures of applying the finite element methods, implement them in FEniCS to a specific problem, and finally discuss the resulting numerics.

2 Mixture Model

Generative process

$$\mathbb{P}(\mathbf{y}, \mathbf{c}, \boldsymbol{\theta}) = \prod_{k=1}^{K} G_0(\theta_k) \prod_{n=1}^{N} F(y_n|\theta_{c_n}) P(c_n) \tag{2.1}$$

- $\mathbf{y} = \{y_1, y_2, \cdots, y_N\}$, observations
- $\mathbf{c} = \{c_1, c_2, \cdots, c_N\}$, cluster assignments
- $\boldsymbol{\theta} = \{\theta_1, \theta_2, \cdots, \theta_K\}$, cluster parameters
- $G_0(\theta)$, a prior over the cluster parameters
- $P(c)$, a prior over the mixing distribution
- $F(y_n|\theta_{c_n})$, a hypothetical distribution over the observations
- Assumptions: each observation is conditionally independent given its latent cluster assignments and the cluster parameters

Posterior probability of assignments

$$\mathbb{P}(\mathbf{c}|\mathbf{y}) = \frac{\mathbb{P}(\mathbf{y}|\mathbf{c})P(\mathbf{c})}{\sum_{\mathbf{c}} \mathbb{P}(\mathbf{y}|\mathbf{c})P(\mathbf{c})}, \tag{2.2}$$

where

$$\mathbb{P}(\mathbf{y}|\mathbf{c}) = \int_{\theta} \left[\prod_{n=1}^{N} F(y_n|\theta_{c_n}) \prod_{k=1}^{K} G_0(\theta_k) \right] \mathrm{d}\theta \tag{2.3}$$

BNP clustering assume that there is an infinite number of latent clusters (namely $K \to +\infty$), but that a finite number of them is used to generate the observed data. There are an infinite number of clusters, though a finite data set only exhibits a finite number of active clusters.

3 Conclusion

We derived the general weak form solution for a time-dependent Navier-Stokes equation, and then implemented in FEniCS to seek for the numerical solution for a specific problem. Moreover, we considered a diffusion-reaction process that acts as external force term on the flow, by coupling the governing PDEs into one compound system. The numerical results for flow velocity, pressure and chemicals concentration are displayed in color map. We discussed the evolution of these profiles over time, and also observed how the reaction impacting factor and fluid viscosity influence the velocity profile. Either larger m or smaller ν results in a more irregular dynamics for the velocity in the long term.