

# False Positives and False Negatives

## Test Says "Yes" ... or does it?

When you have a test that can say "Yes" or "No" (such as a medical test), you have to think:

- It could be **wrong** when it says "Yes".
- It could be **wrong** when it says "No".

## Wrong?



It is like being told you **did** something when you **didn't**!  
Or you didn't do it when you really did.

They each have a special name: **"False Positive"** and **"False Negative"**:

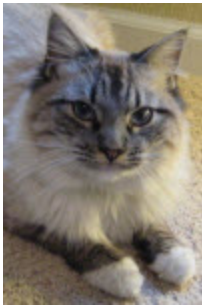
	They say you <b>did</b>	They say you <b>didn't</b>
You really did	<i>They are right!</i>	<b>"False Negative"</b>
You really didn't	<b>"False Positive"</b>	<i>They are right!</i>

Here are some examples of "false positives" and "false negatives":

- **Airport Security:** a "false positive" is when ordinary items such as keys or coins get mistaken for weapons (machine goes "beep")
- **Quality Control:** a "false positive" is when a good quality item gets rejected, and a "false negative" is when a poor quality item gets accepted. (A "positive" result means there IS a defect.)
- **Antivirus software:** a "false positive" is when a normal file is thought to be a virus
- **Medical screening:** low-cost tests given to a large group can give many false positives (saying you have a disease when you don't), and then ask you to get more accurate tests.

But many people don't understand the true numbers behind "Yes" or "No", like in this example:

## Example: Allergy or Not?



Hunter says she is itchy. There is a test for Allergy to Cats, but this test is not always right:

- For people that **really do** have the allergy, the test says "Yes" **80%** of the time
- For people that **do not** have the allergy, the test says "Yes" **10%** of the time ("false positive")

Here it is in a table:

	Test says "Yes"	Test says "No"
Have allergy	<b>80%</b>	<b>20% "False Negative"</b>
Don't have it	<b>10% "False Positive"</b>	<b>90%</b>

Question: If 1% of the population have the allergy, and **Hunter's test says "Yes"**, what are the chances that Hunter really has the allergy?

Do you think 75%? Or maybe 50%?

A similar test was given to Doctors and most guessed around 75% ...  
... but they were very wrong!

(Source: "Probabilistic reasoning in clinical medicine: Problems and opportunities" by David M. Eddy 1982, which this example is based on)

There are three different ways to solve this:

- "Imagine a 1000",
- "Tree Diagrams" or
- "Bayes' Theorem",

use any you prefer. Let's look at them now:

## Try Imagining A Thousand People

When trying to understand questions like this, just imagine a large group (say 1000) and play with the numbers:

- Of 1000 people, only **10** really have the allergy (1% of 1000 is 10)
- The test is 80% right for people who **have** the allergy, so it will get **8 of those 10 right**.
- But 990 **do not** have the allergy, and the test will say "Yes" to 10% of them, which is **99 people** it says "Yes" to **wrongly** (false positive)
- So out of 1000 people the test says "**Yes**" to  $(8+99) = \mathbf{107 \text{ people}}$

As a table:

	1% have it	Test says "Yes"	Test says "No"
Have allergy	10	<b>8</b>	<b>2</b>
Don't have it	990	<b>99</b>	<b>891</b>
	1000	107	893

So 107 people get a "Yes" but only 8 of those really have the allergy:

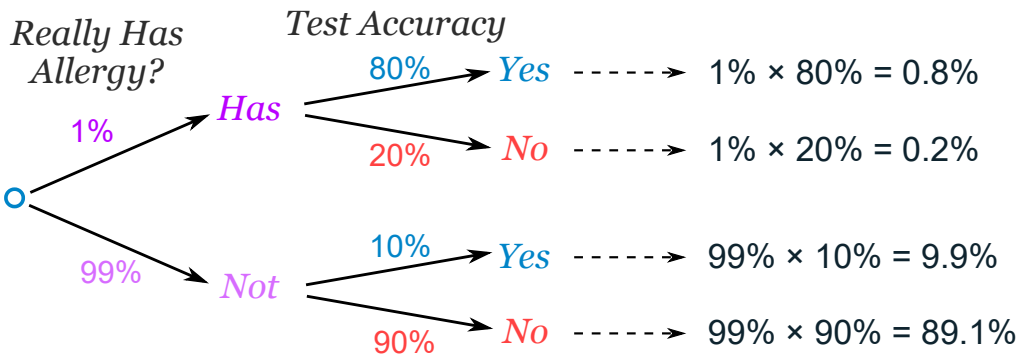
$8 / 107 = \text{about } 7\%$

So, even though Hunter's test said "Yes", it is still only **7% likely** that Hunter has a Cat Allergy.

Why so small? Well, the allergy is so rare that those who actually have it are greatly **outnumbered** by those with a false positive.

As A Tree

Drawing a tree diagram can really help:



First of all, let's check that all the percentages add up:

$0.8\% + 0.2\% + 9.9\% + 89.1\% = \mathbf{100\%}$  (good!)

And the two "Yes" answers add up to  $0.8\% + 9.9\% = \mathbf{10.7\%}$ , but only 0.8% are correct.

$0.8/10.7 = \mathbf{7\%}$  (same answer as above)

## Bayes' Theorem

[Bayes' Theorem](#) has a special formula for this kind of thing:

$$P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(\text{not } A)P(B|\text{not } A)}$$

where:

- P means "Probability of"
- | means "given that"
- A in this case is "actually has the allergy"
- B in this case is "test says Yes"

So:

**P(A|B)** means "The probability that Hunter actually has the allergy given that the test says Yes"

**P(B|A)** means "The probability that the test says Yes given that Hunter actually has the allergy"

To be clearer, let's change A to **has** (actually has allergy) and B to **Yes** (test says yes):

$$P(\text{has}|\text{Yes}) = \frac{P(\text{has})P(\text{Yes}|\text{has})}{P(\text{has})P(\text{Yes}|\text{has}) + P(\text{not has})P(\text{Yes}|\text{not has})}$$

And put in the numbers:

$$\begin{aligned} P(\text{has}|\text{yes}) &= \frac{0.01 \times 0.8}{0.01 \times 0.8 + 0.99 \times 0.1} \\ &= 0.0748... \end{aligned}$$

Which is about **7%**

Learn more about this at [Bayes' Theorem](#).

## One Last Example

### Extreme Example: Computer Virus

A computer virus spreads around the world, all reporting to a master computer.

The good guys capture the master computer and find that a million computers have been infected (but don't know which ones).



Governments decide to take action!

No one can use the internet until their computer passes the "virus-free" test. The test is 99% accurate (pretty good, right?) But 1% of the time it says you have the virus when you don't (a "false positive").

Now let's say there are **1000 million** internet users.

- Of 1 million **with** the virus 99% of them get correctly banned = about **1 million**
- But false positives are  $999 \text{ million} \times 1\% =$  about **10 million**

So a total of **11 million** get banned, but only 1 out of those 11 actually have the virus.

**So if you get banned there is only a 9% chance you actually have the virus!**

## Conclusion

When dealing with false positives and false negatives (or other tricky probability questions) we can use these methods:

- Imagine you have 1000 (of whatever),
- Make a tree diagram, or
- Use Bayes' Theorem

[Question 1](#) [Question 2](#) [Question 3](#) [Question 4](#) [Question 5](#)  
[Question 6](#) [Question 7](#) [Question 8](#) [Question 9](#) [Question 10](#)