Graph Theory: Breadth First Search

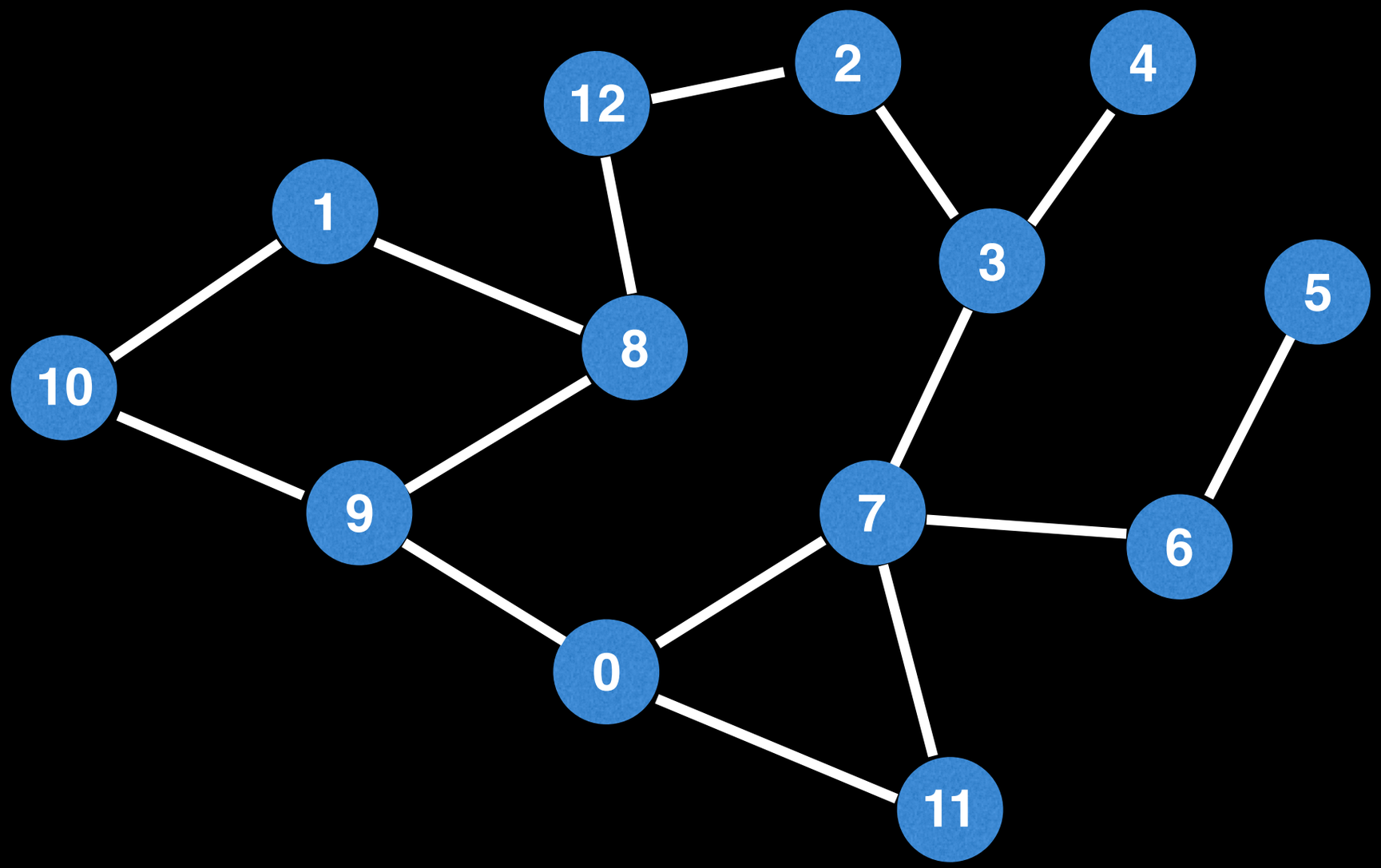
William Fiset

BFS overview

The **Breadth First Search (BFS)** is another fundamental search algorithm used to explore nodes and edges of a graph. It runs with a time complexity of **O(V+E)** and is often used as a building block in other algorithms.

The BFS algorithm is particularly useful for finding the **shortest path on unweighted graphs** and for testing connectivity.

A BFS starts at some arbitrary node of a graph and explores the neighbour nodes first, before moving to the next level neighbours.



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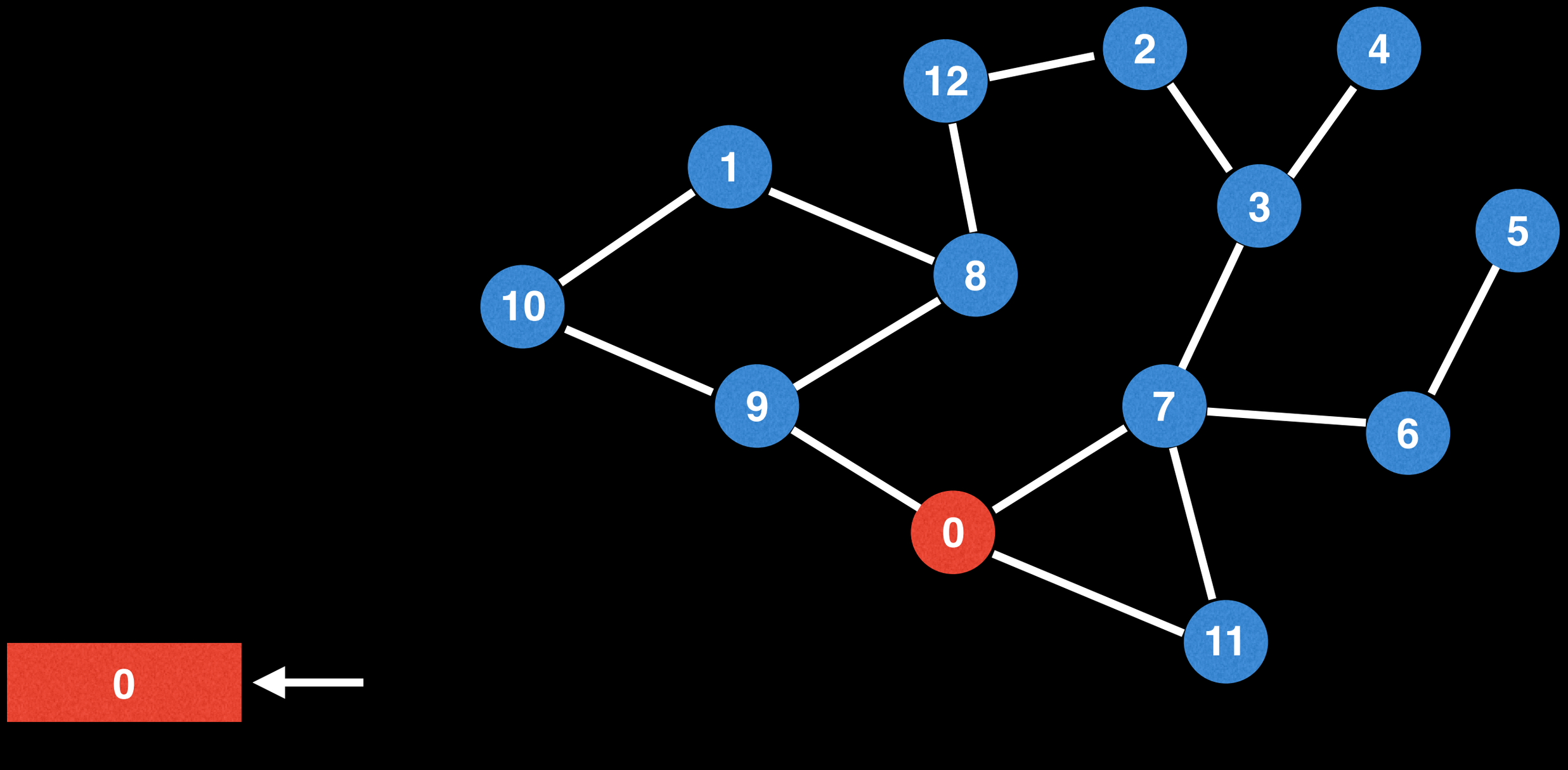

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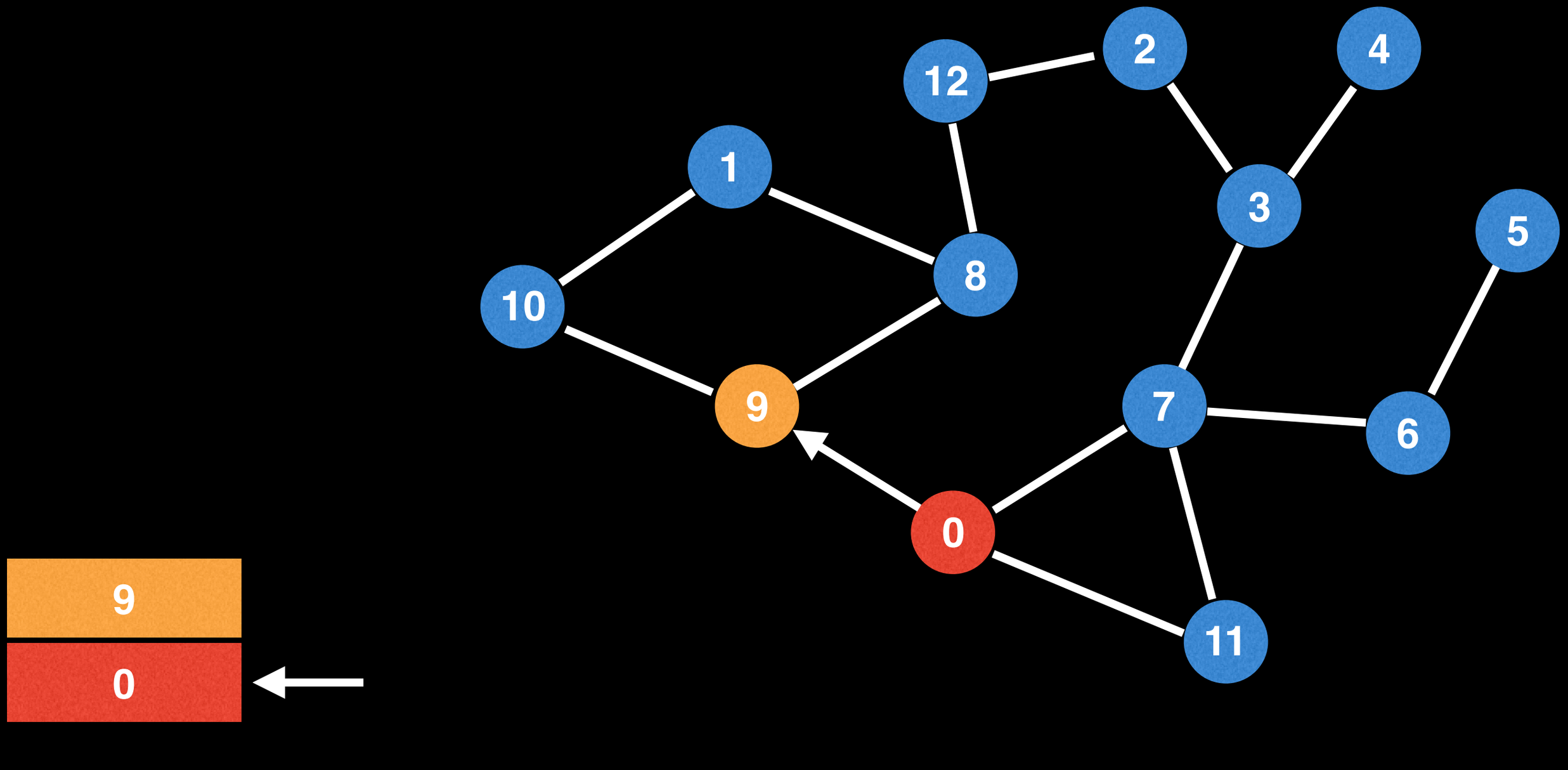

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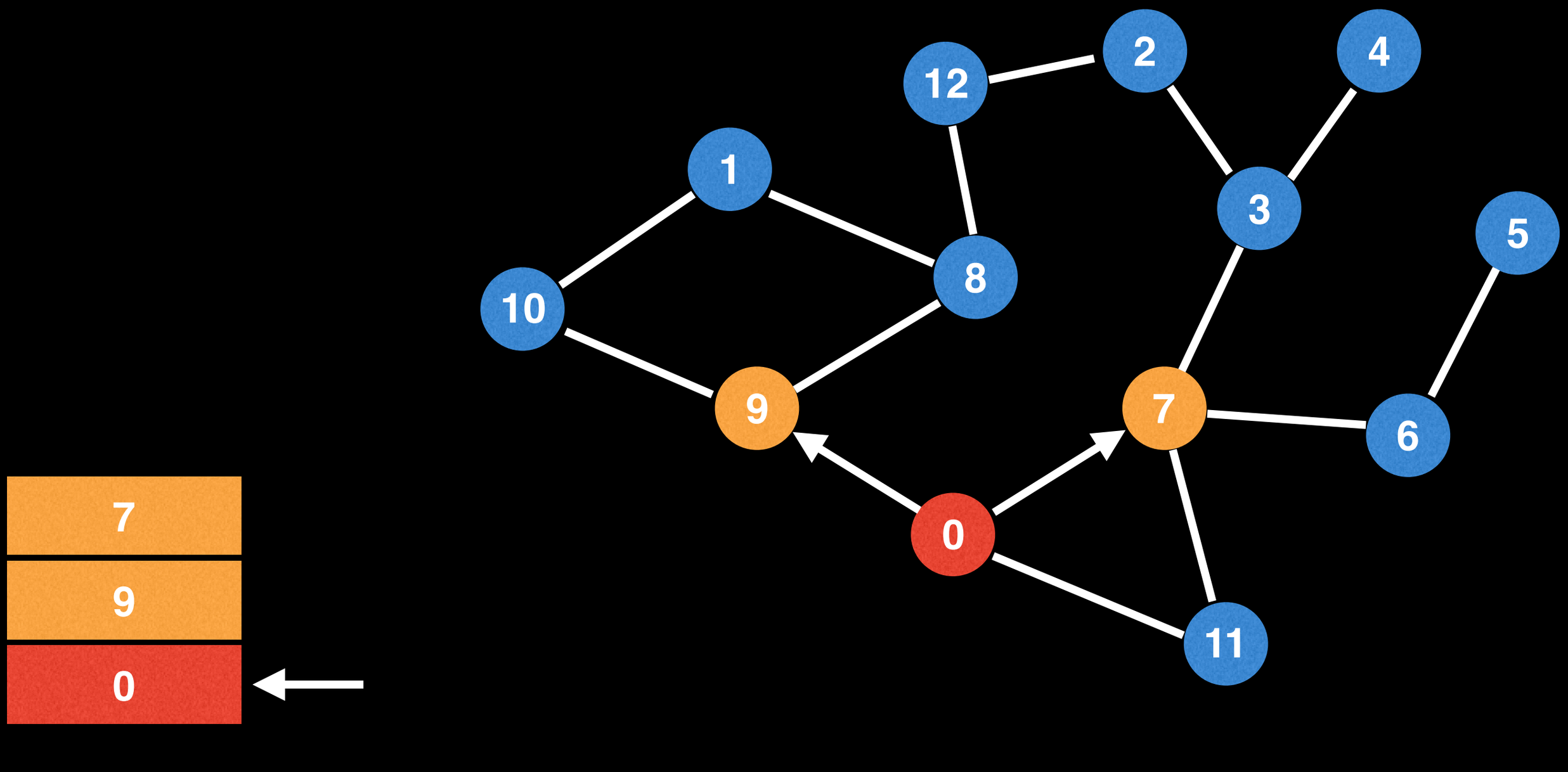

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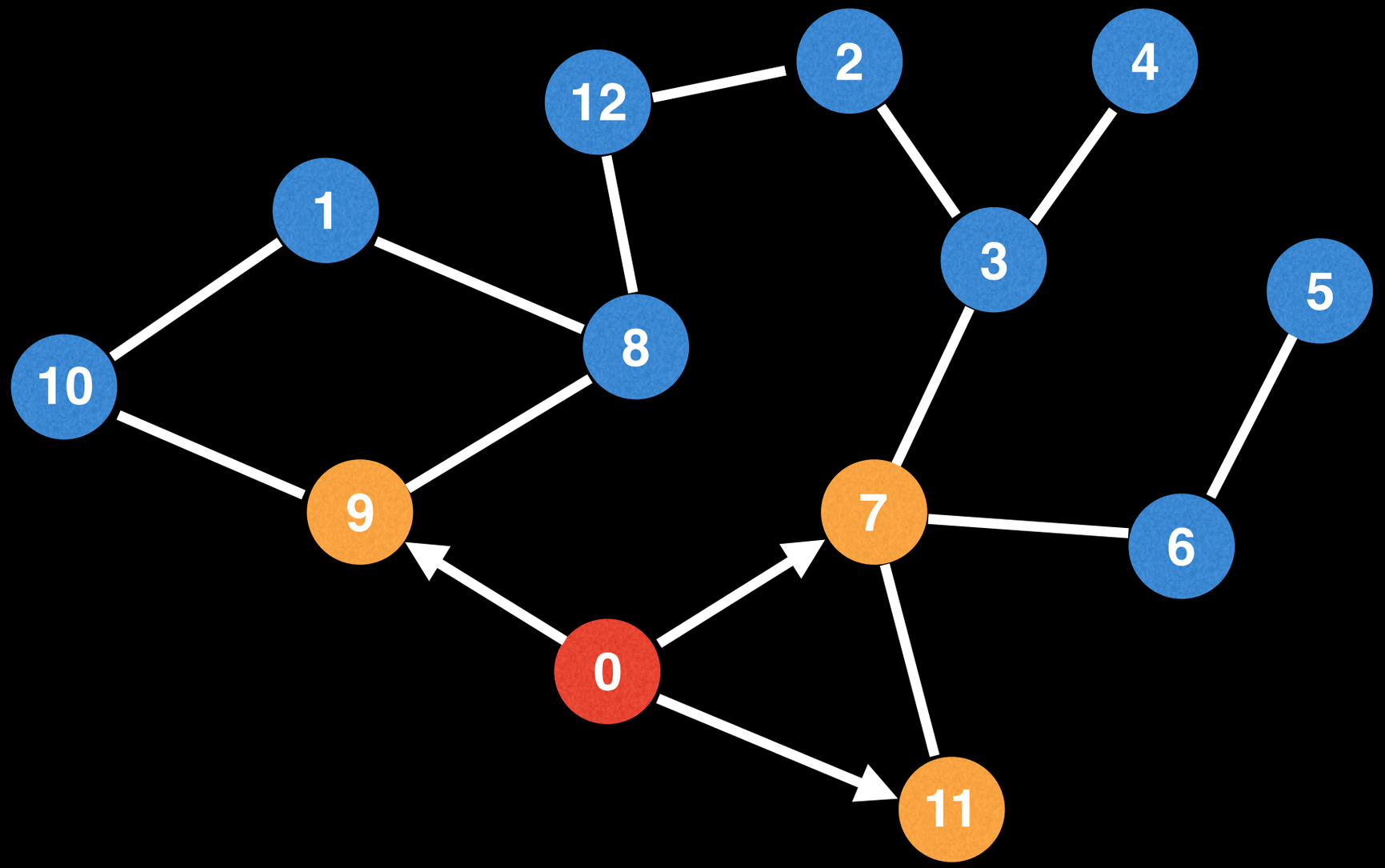

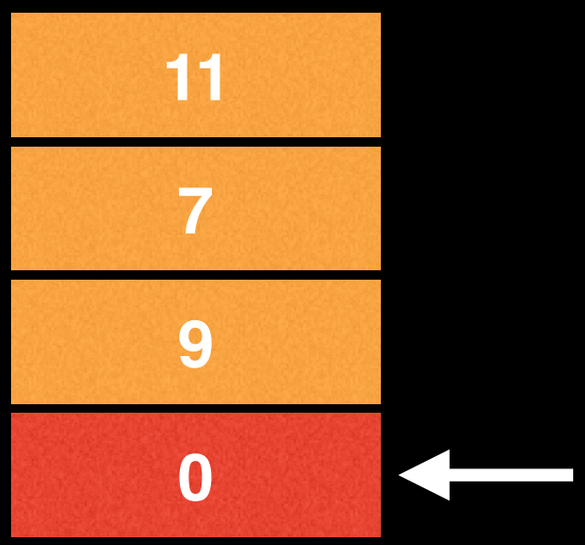

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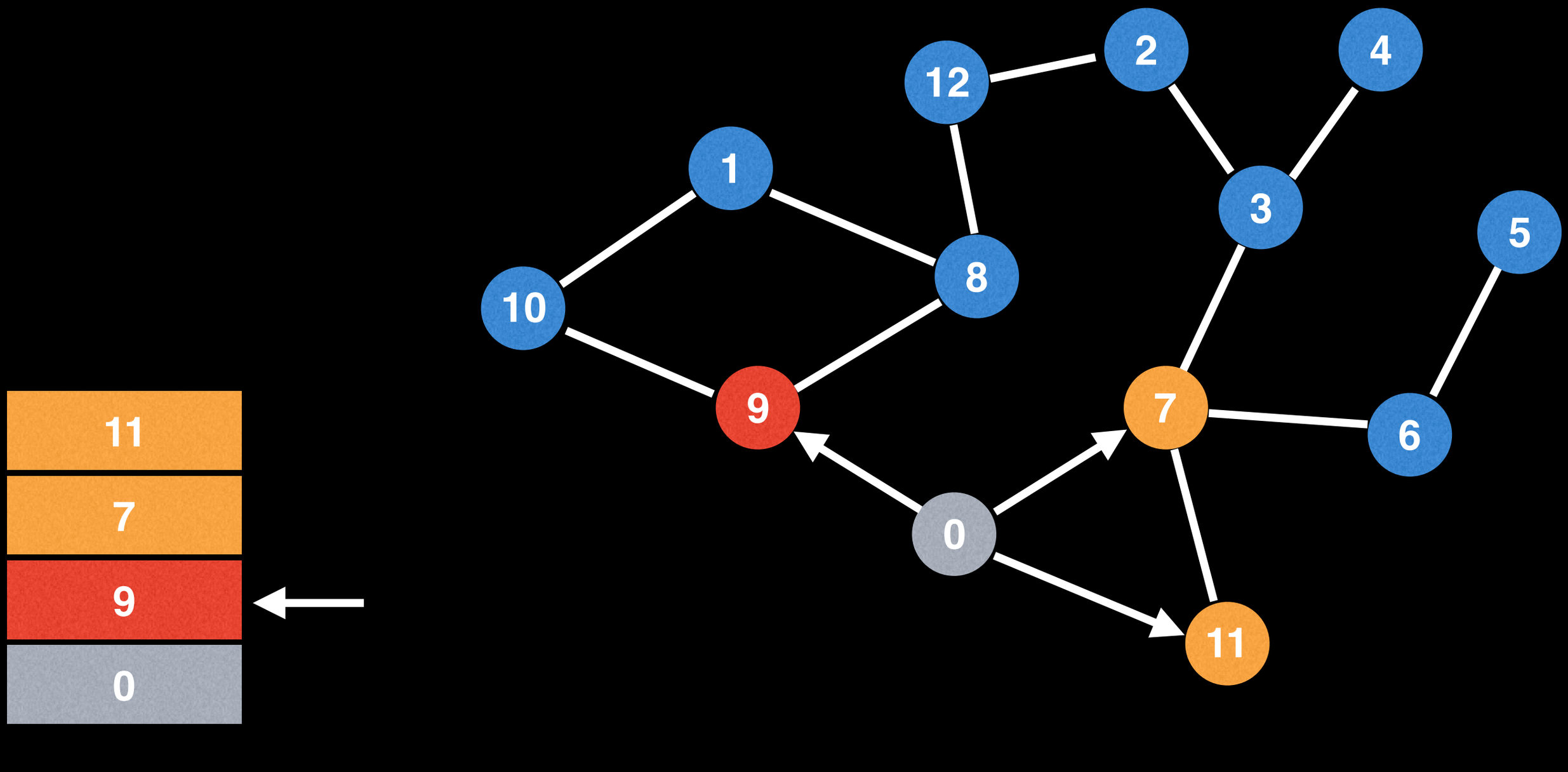

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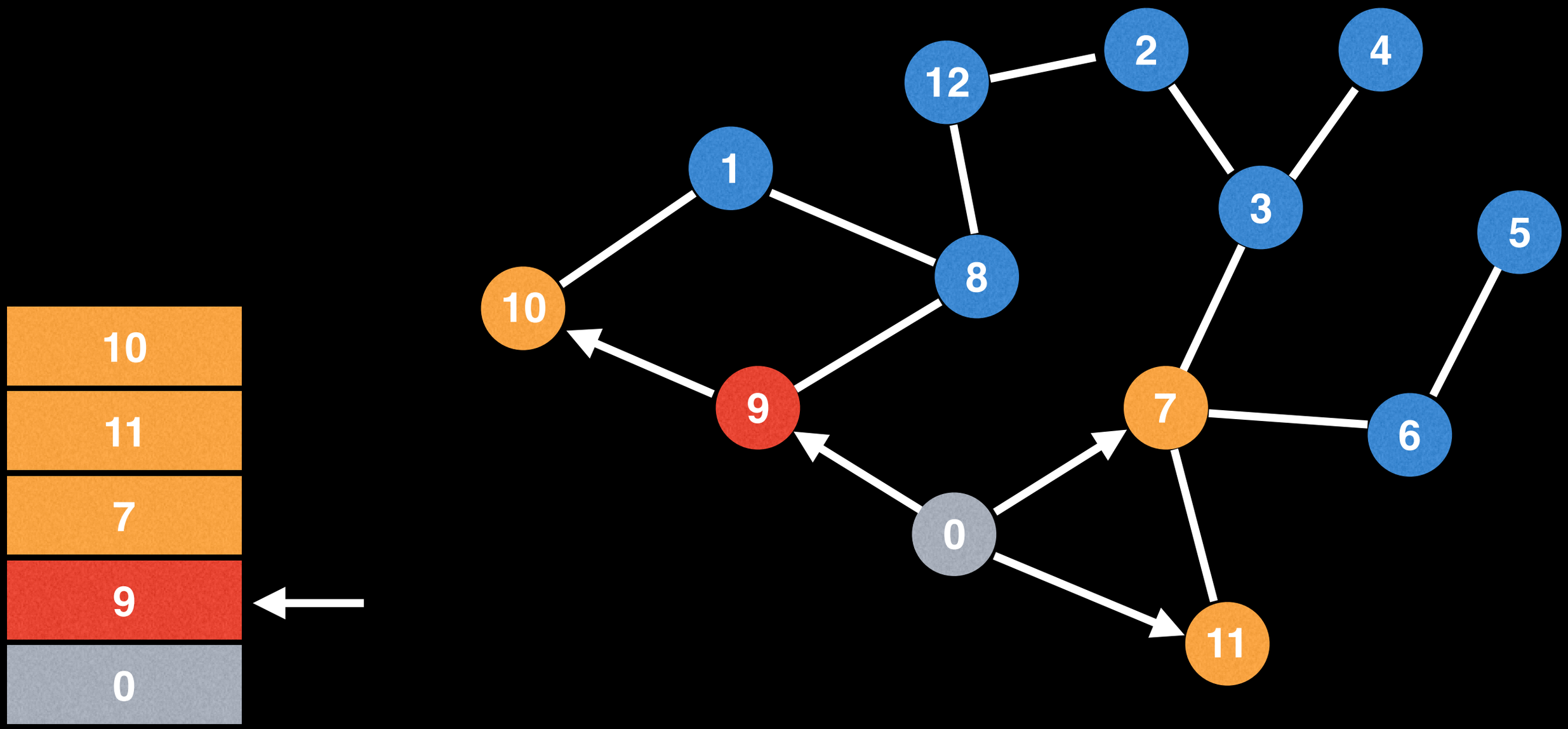

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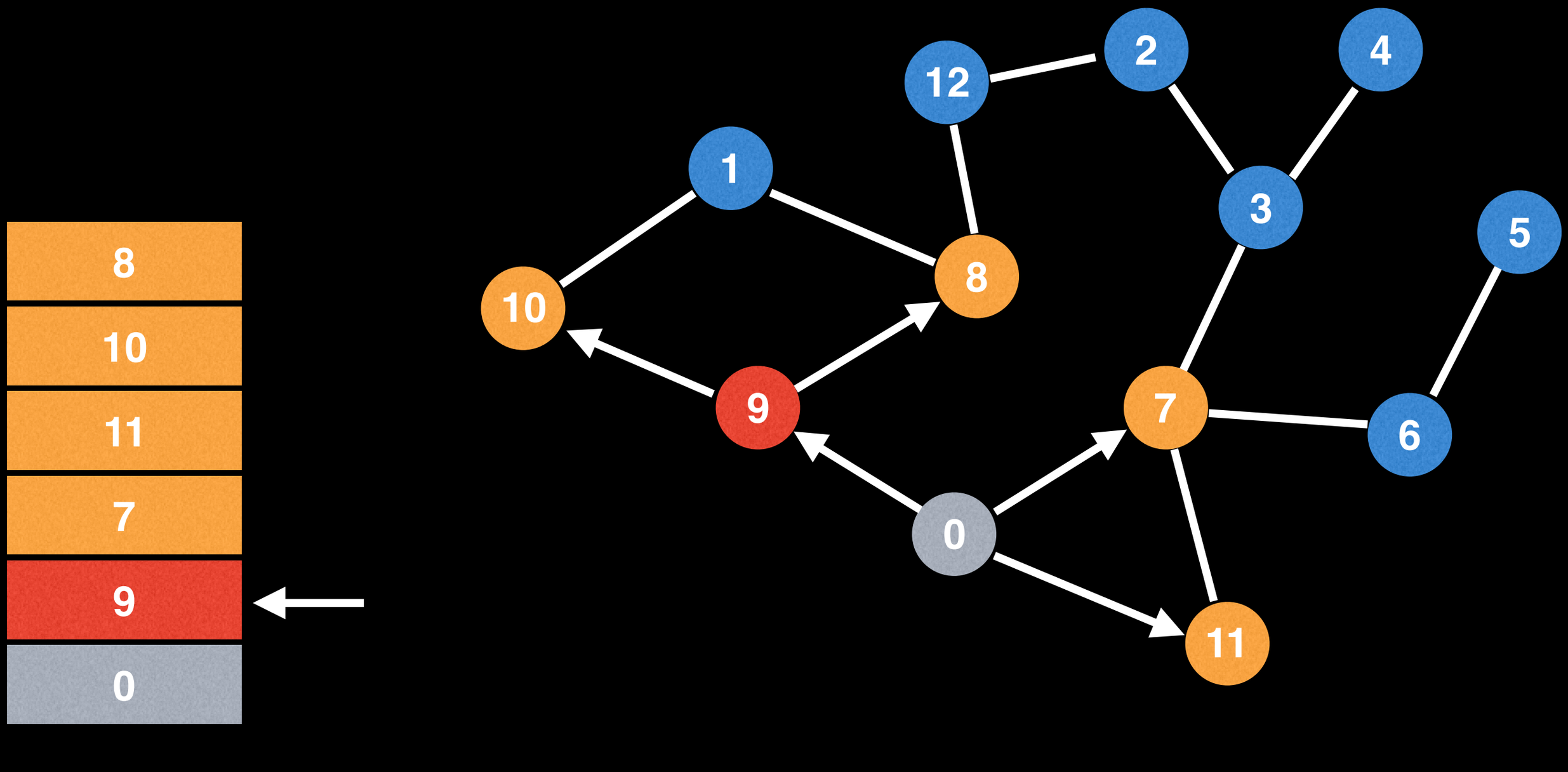

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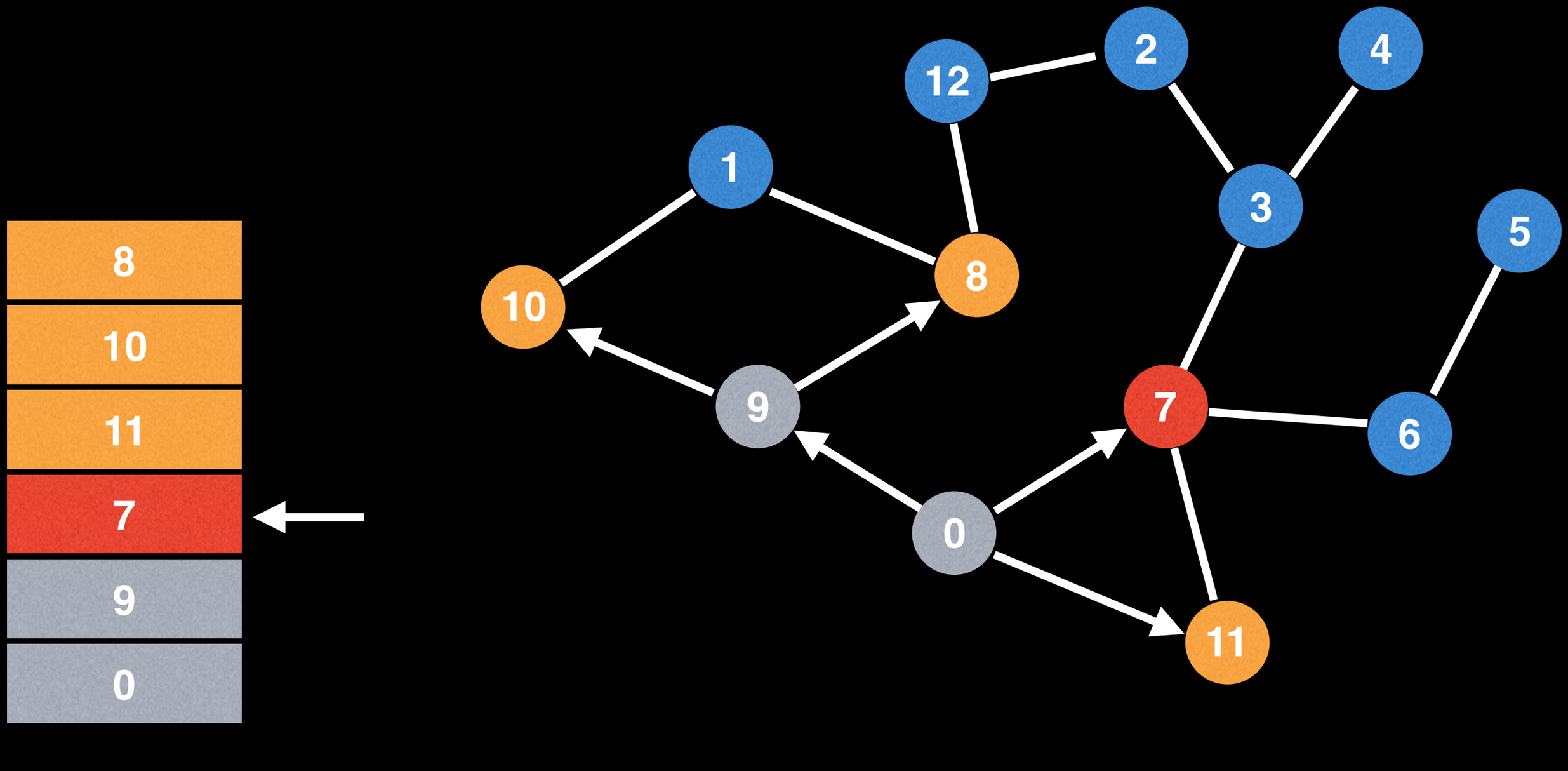

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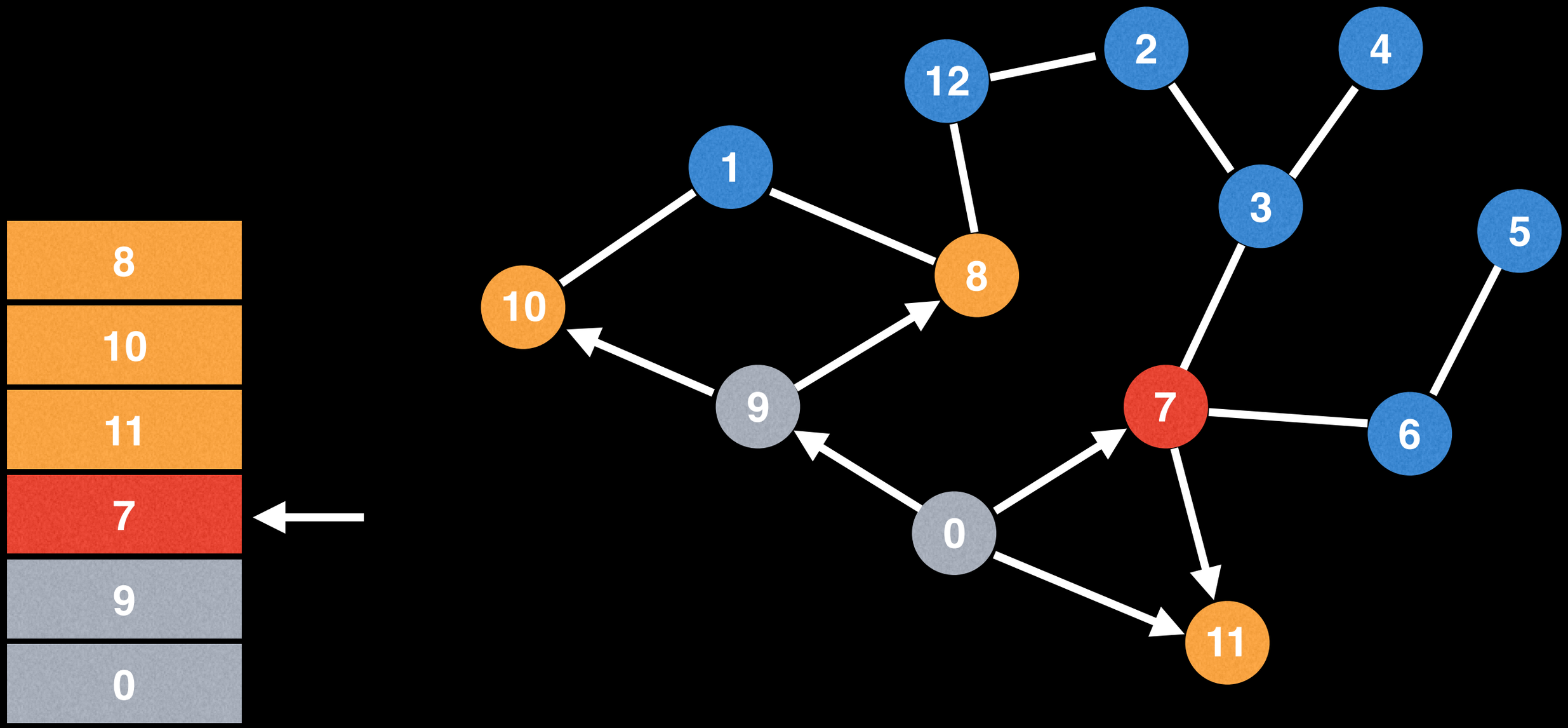

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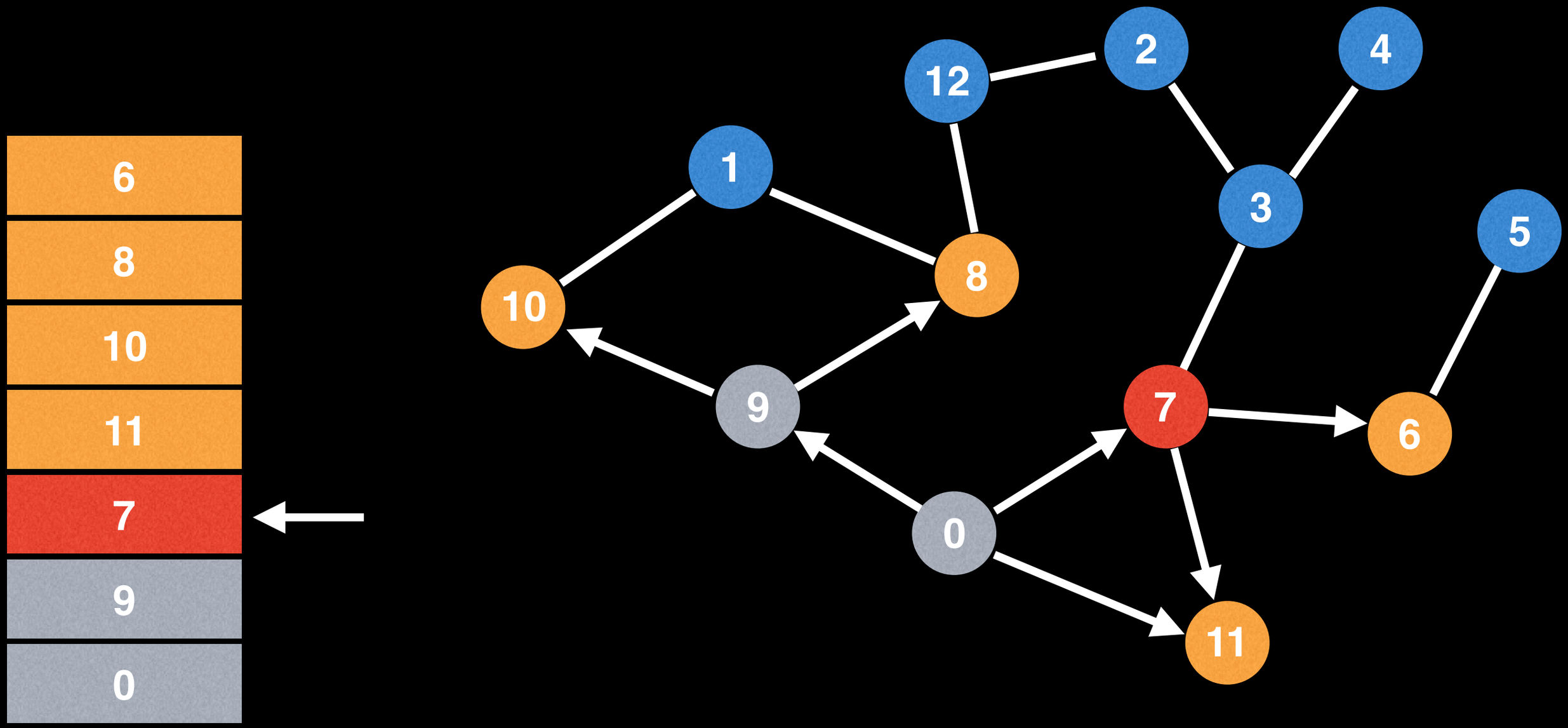

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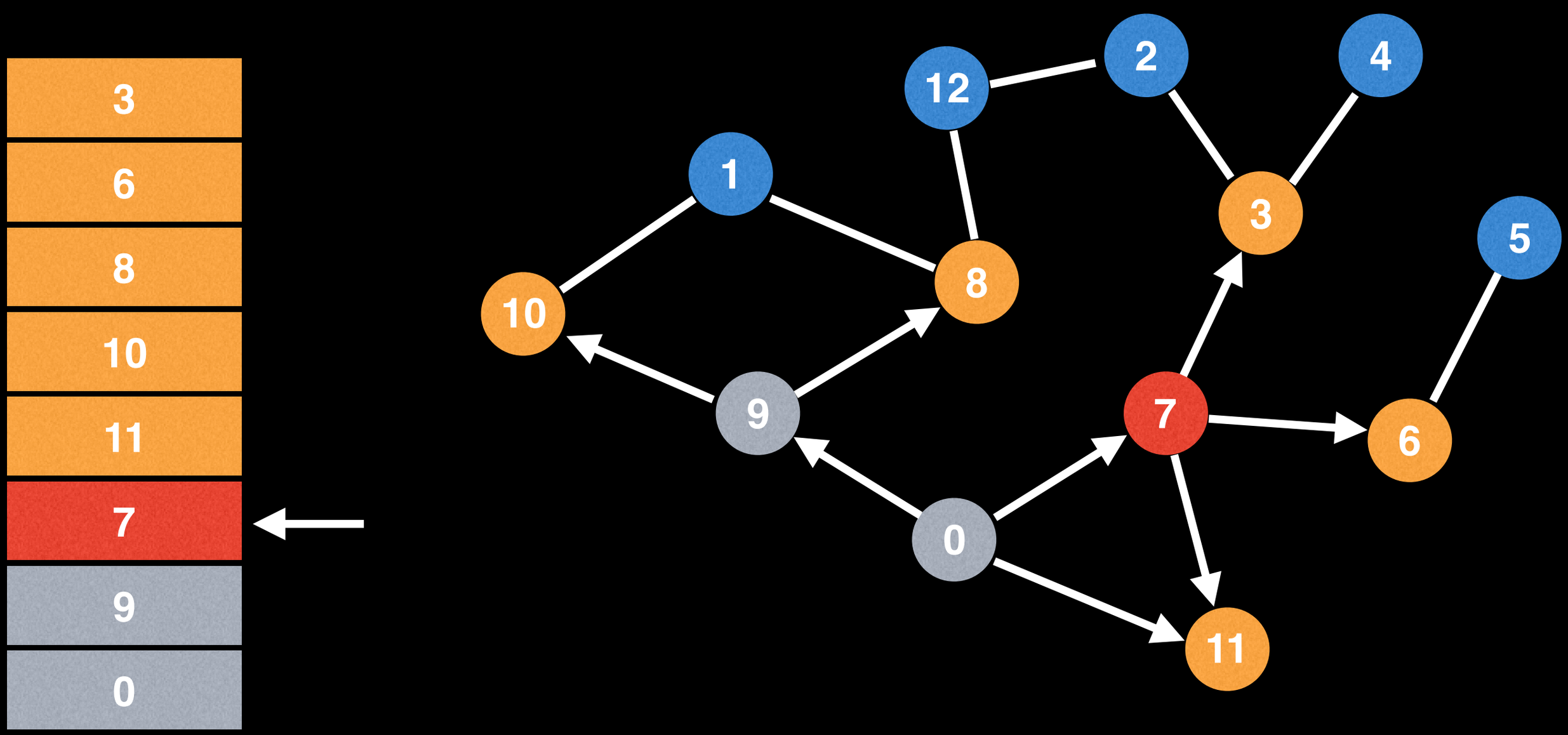

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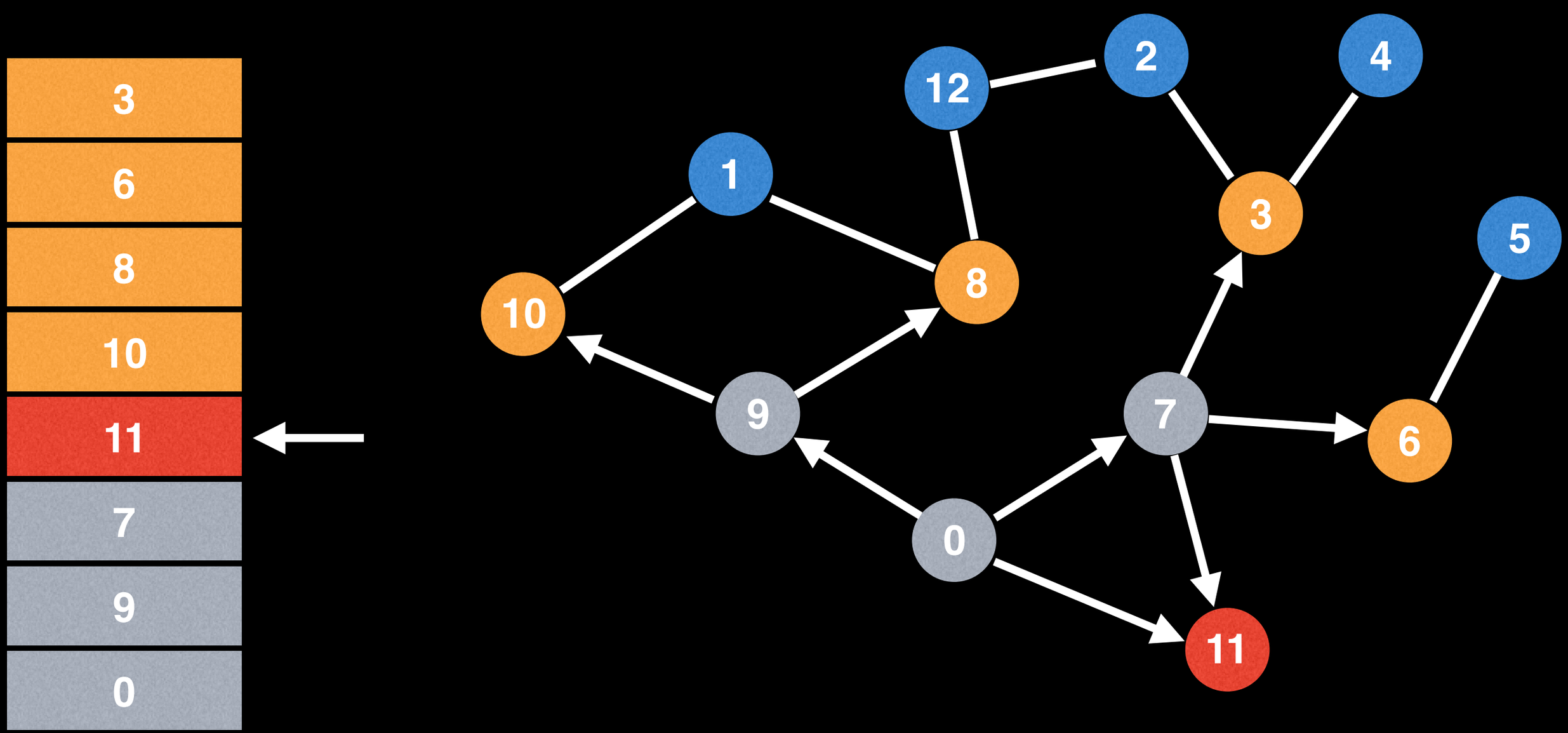

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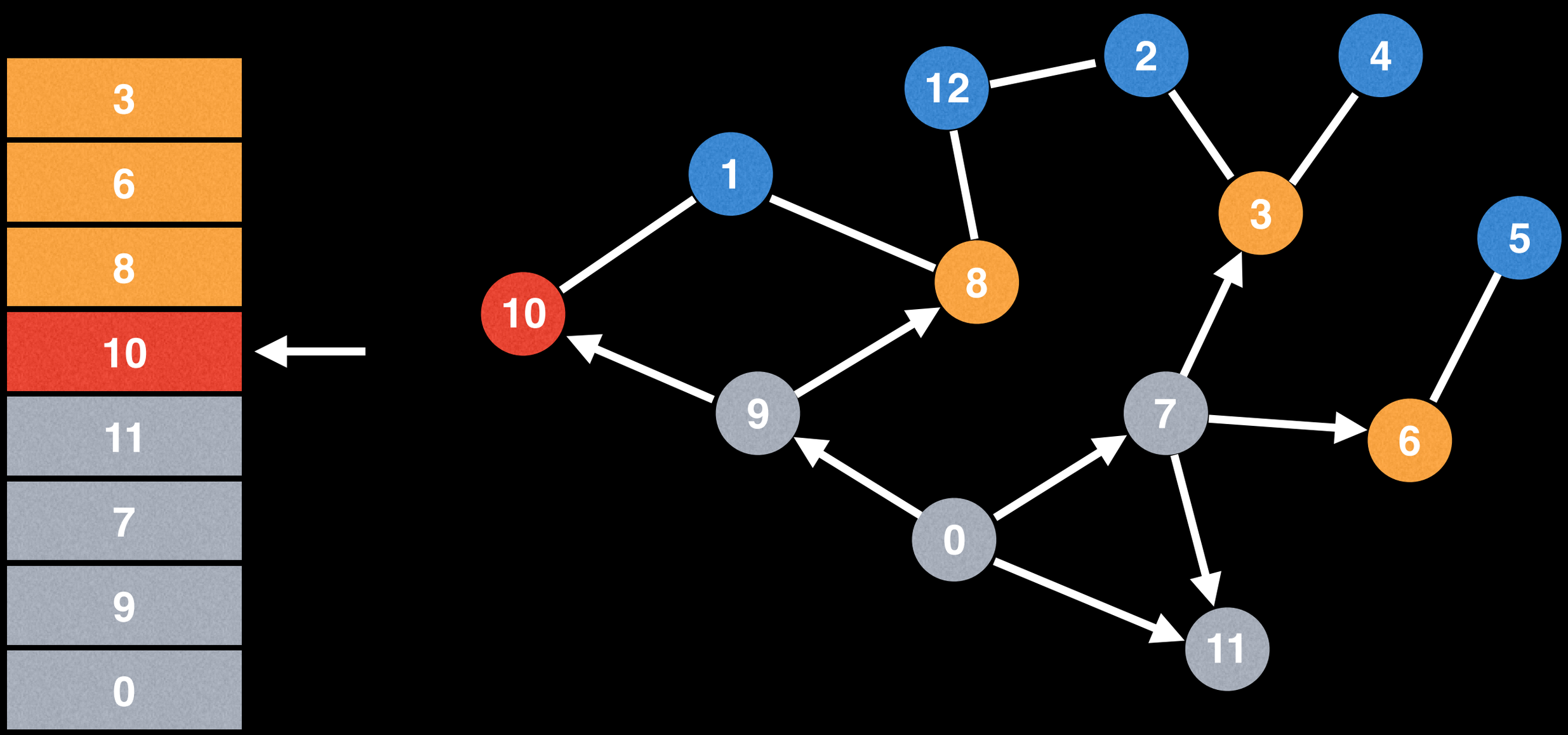

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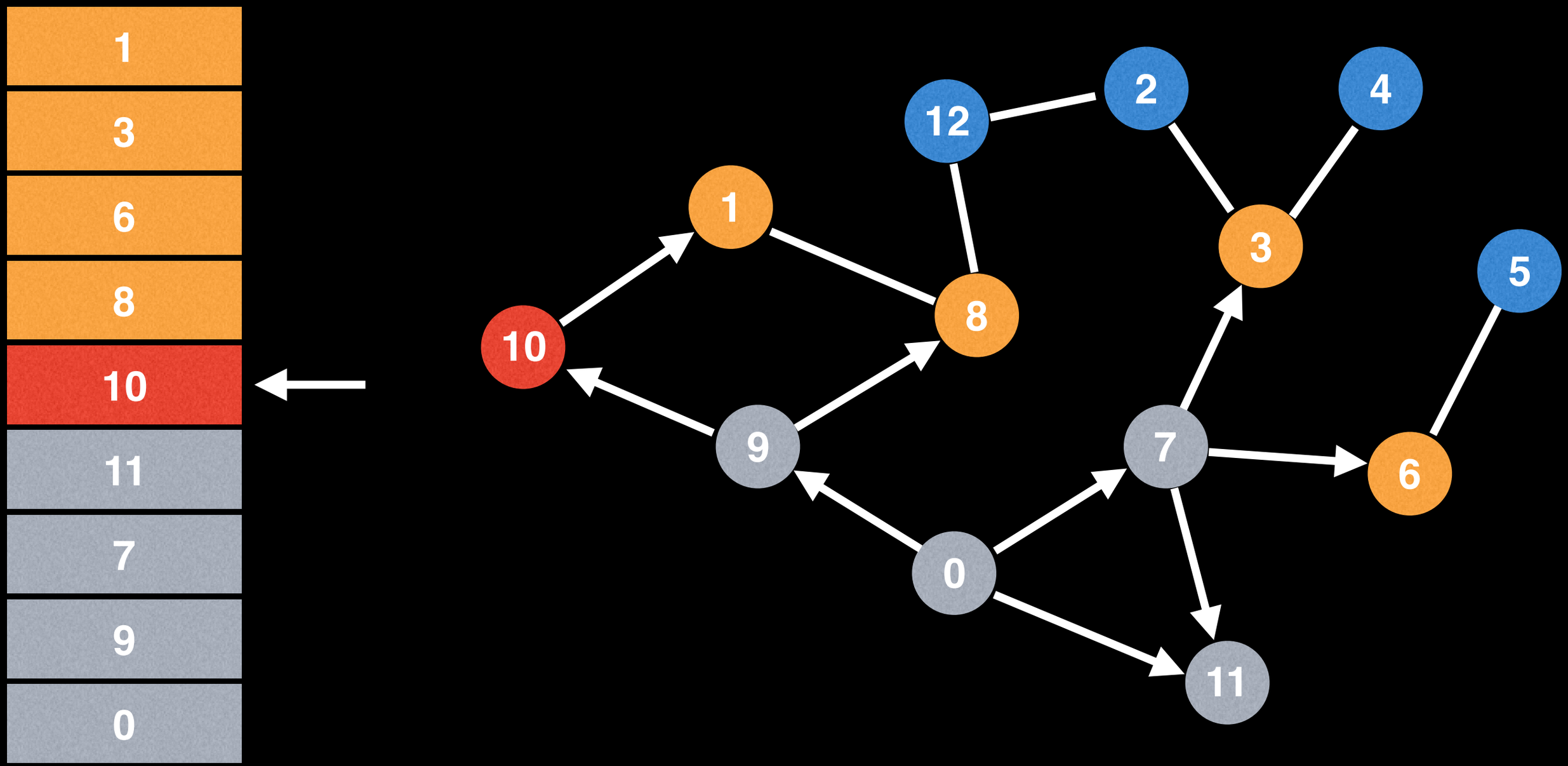

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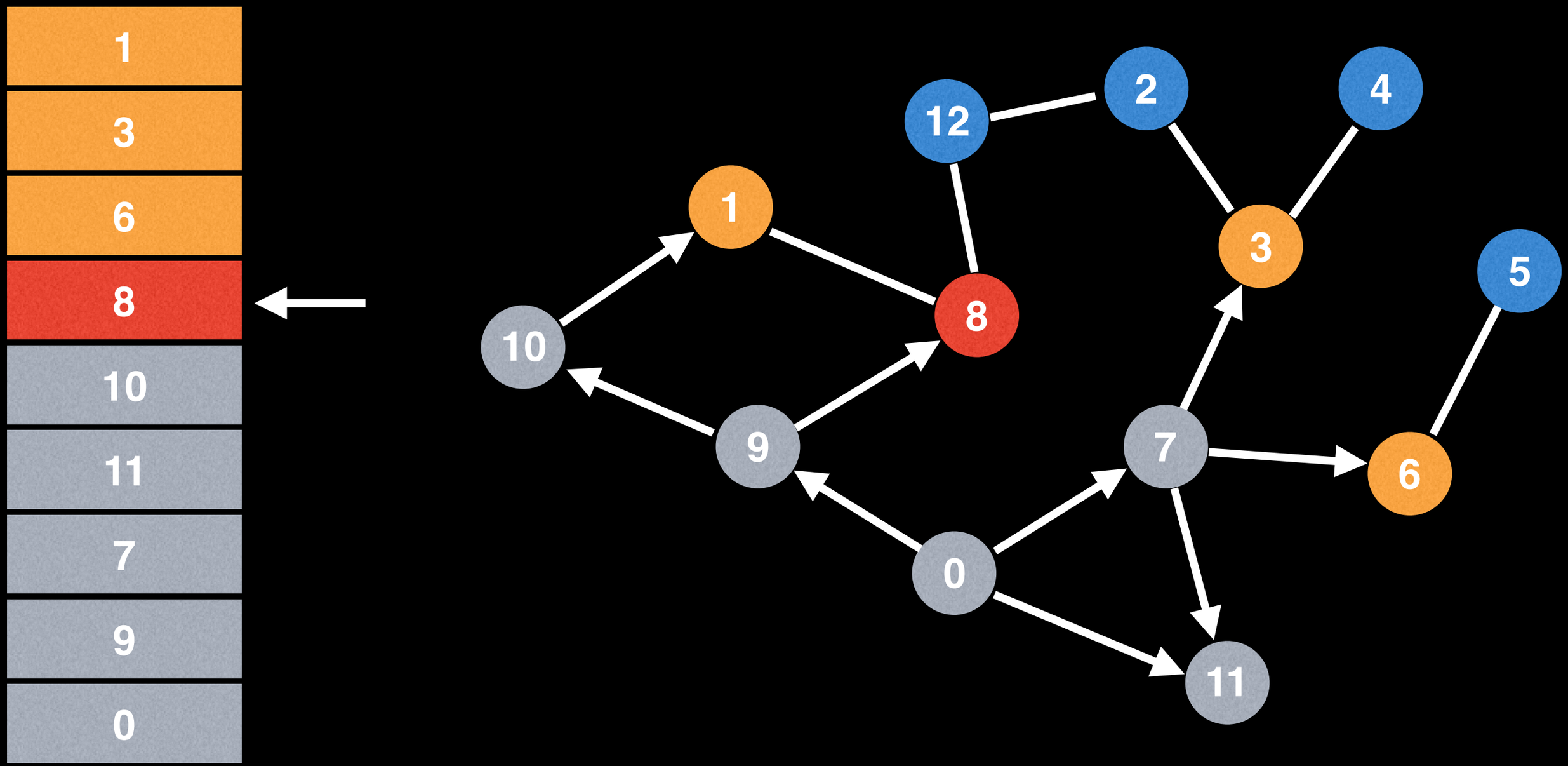

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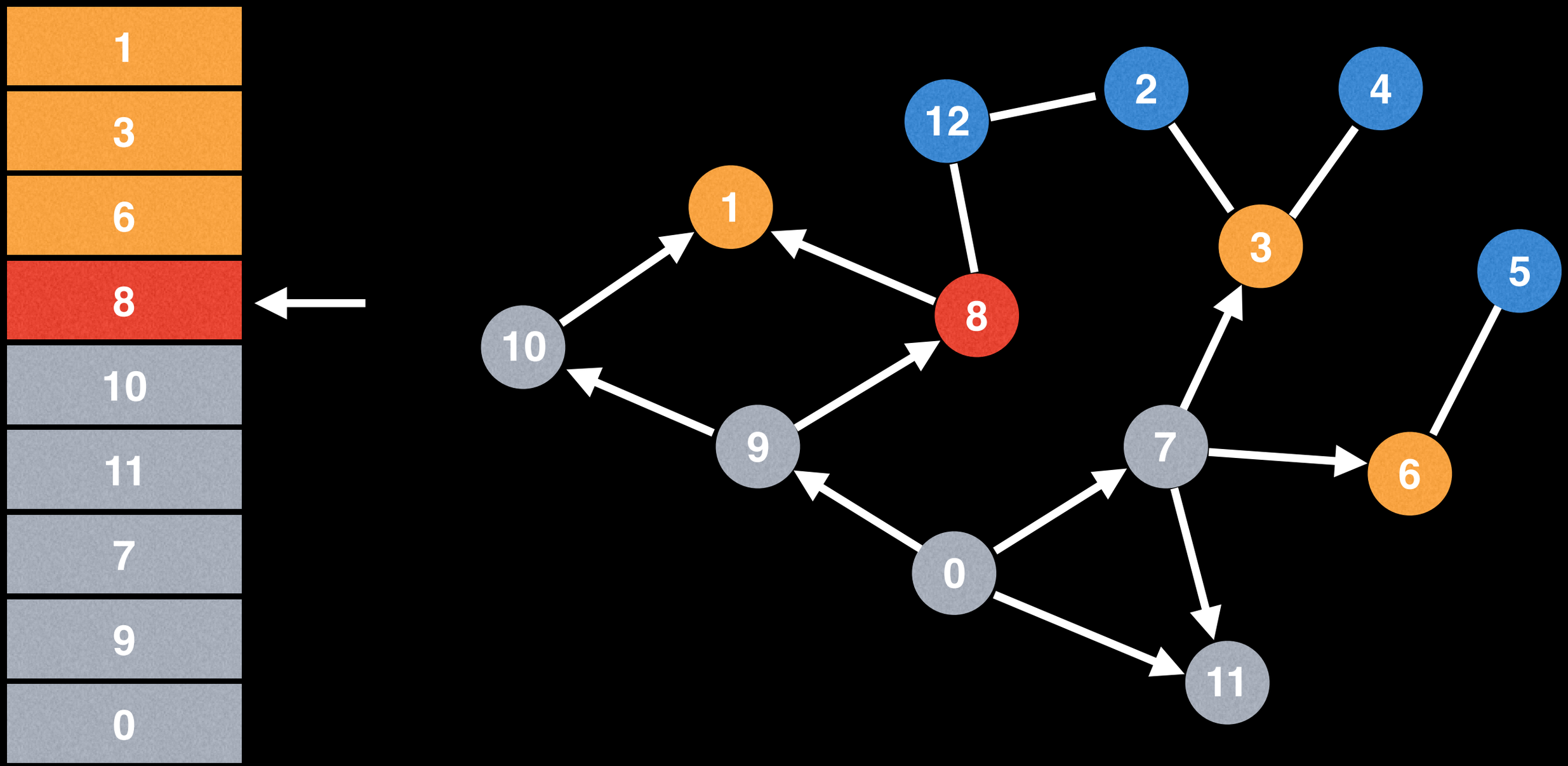

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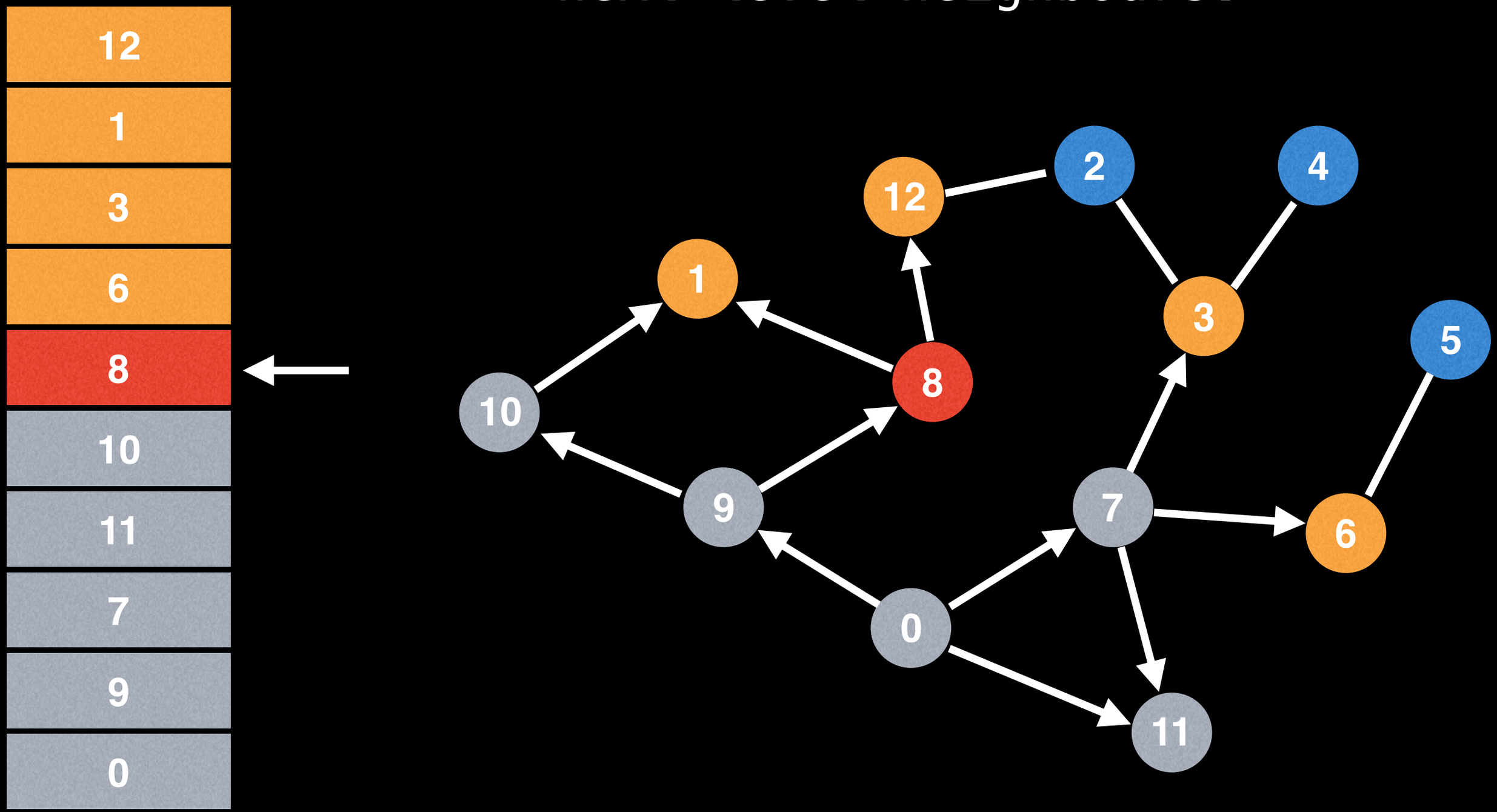

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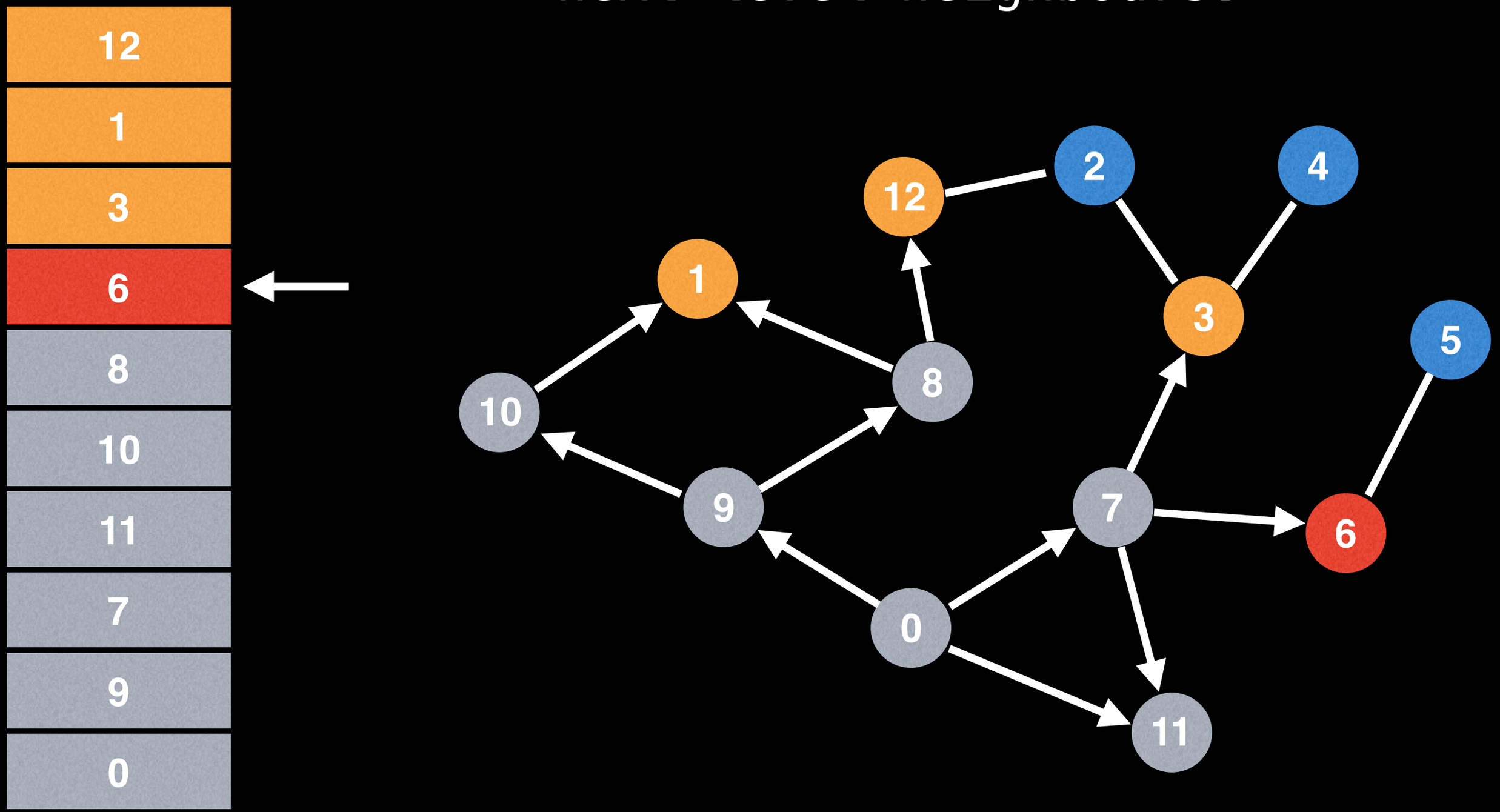

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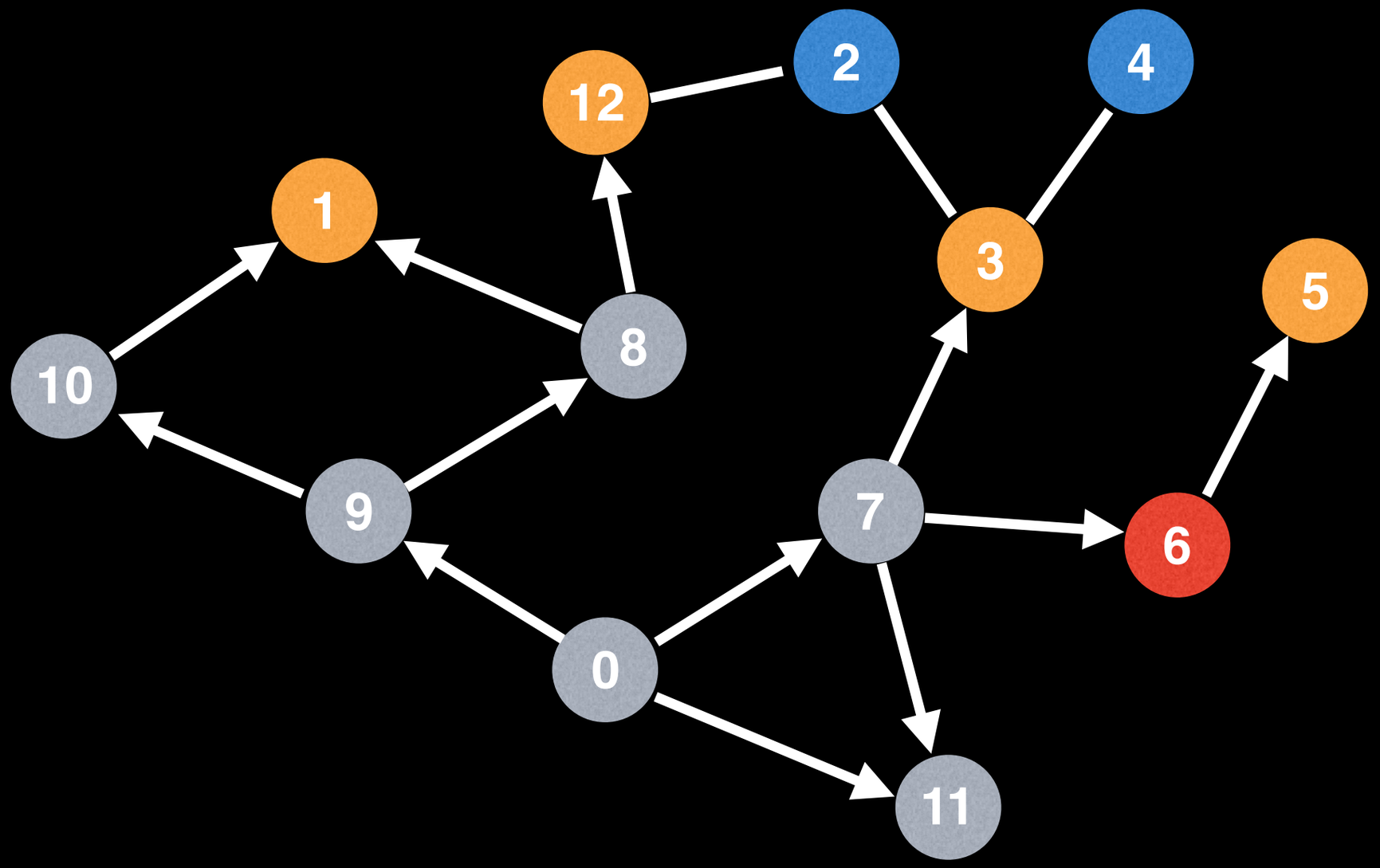

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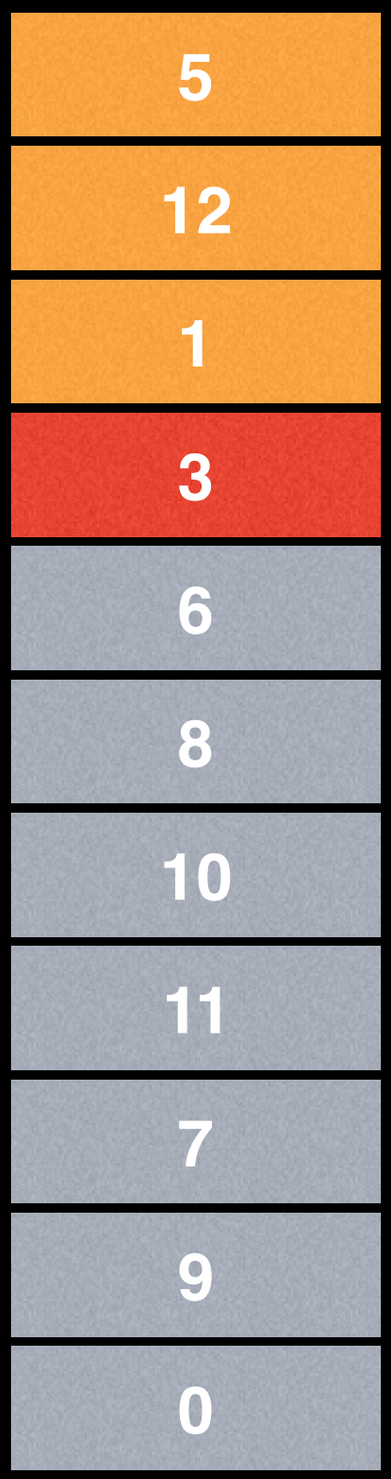

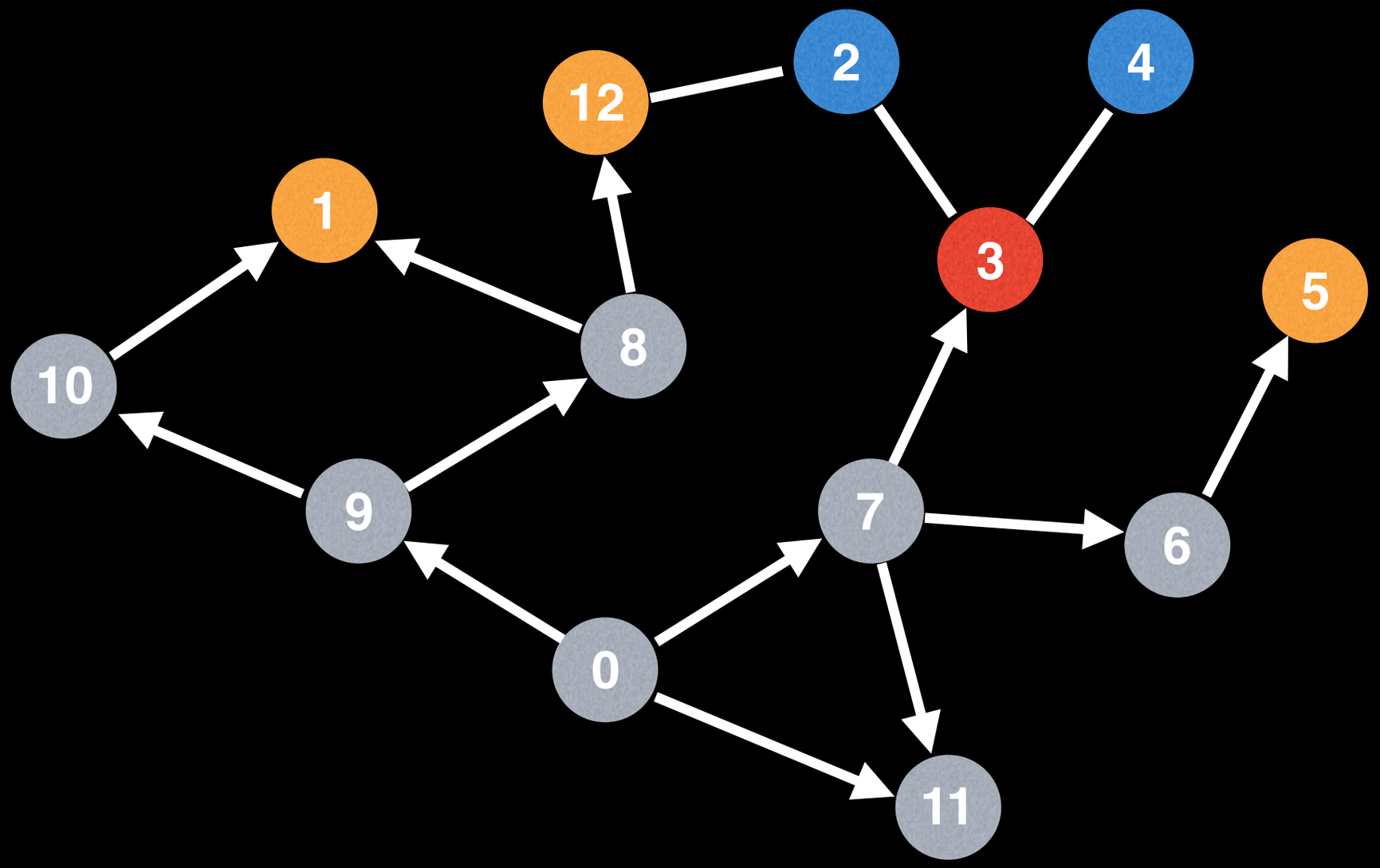

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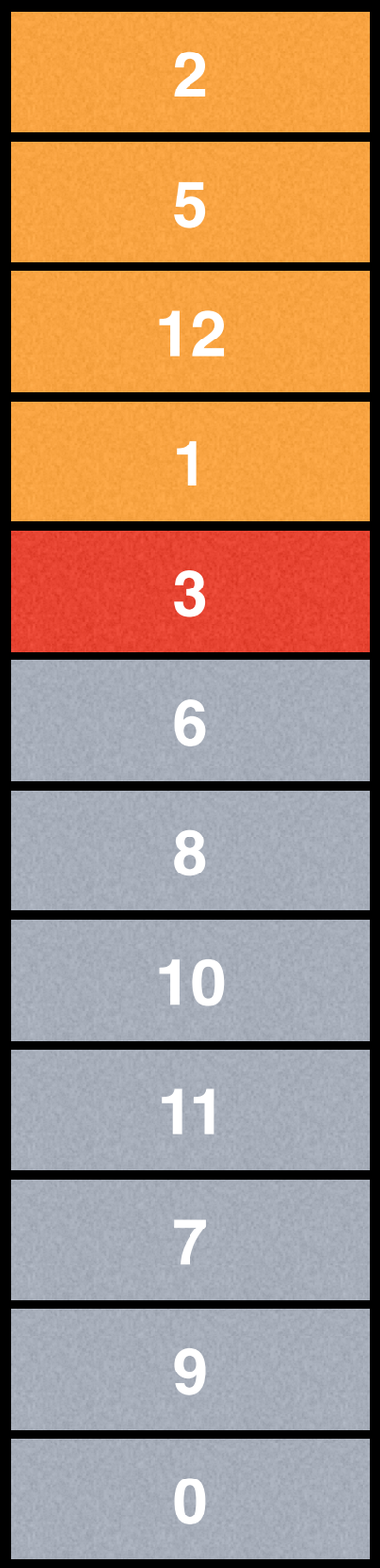

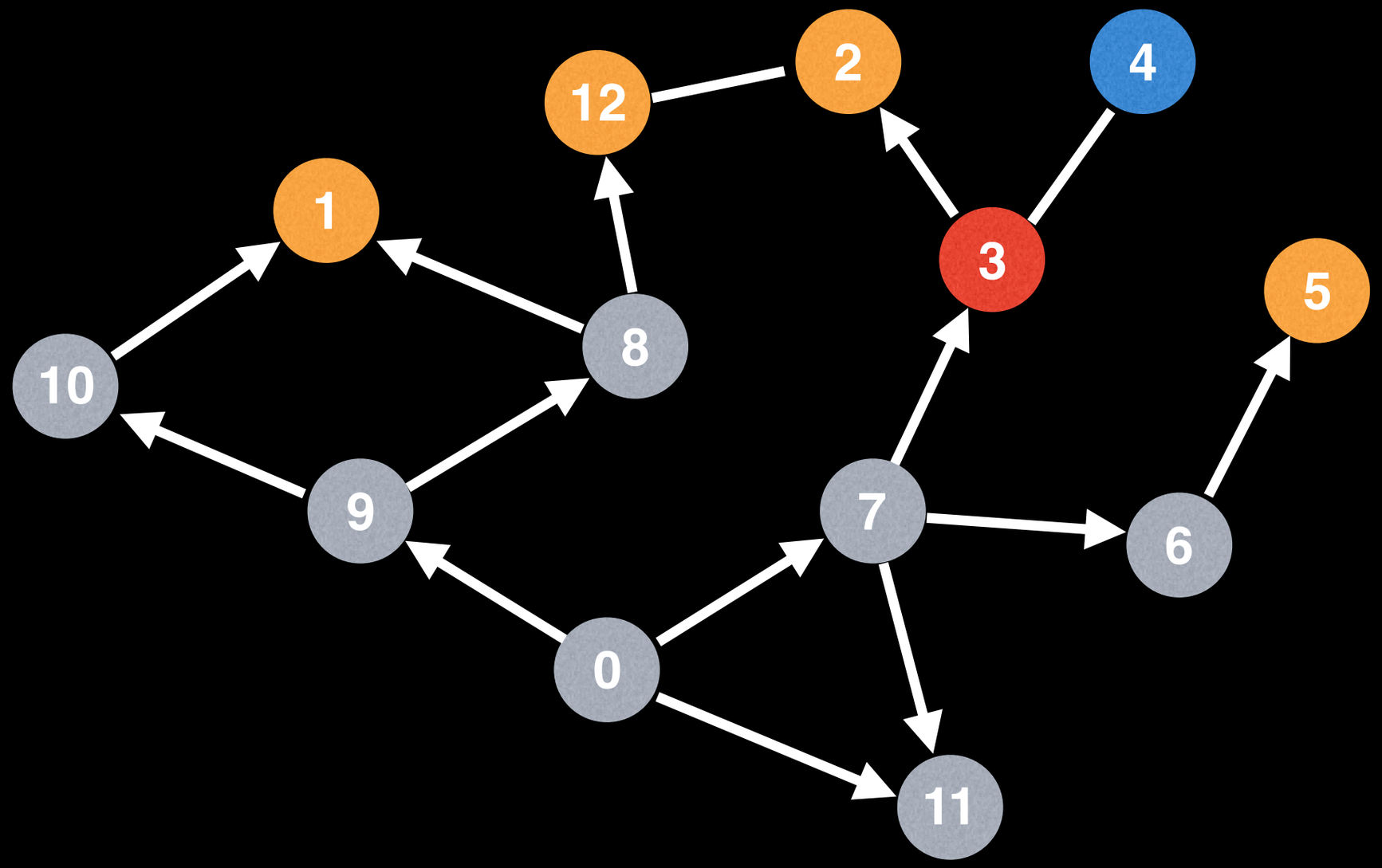

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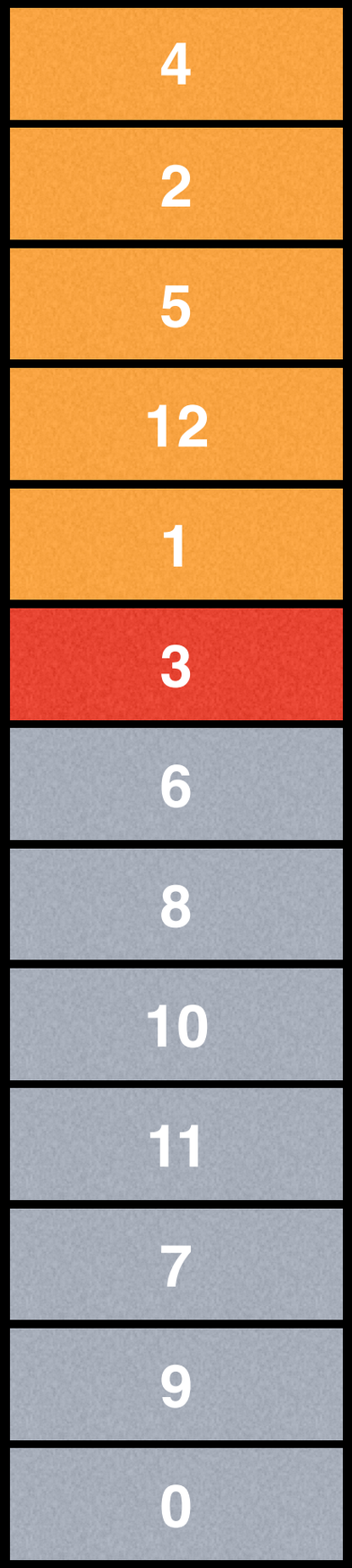

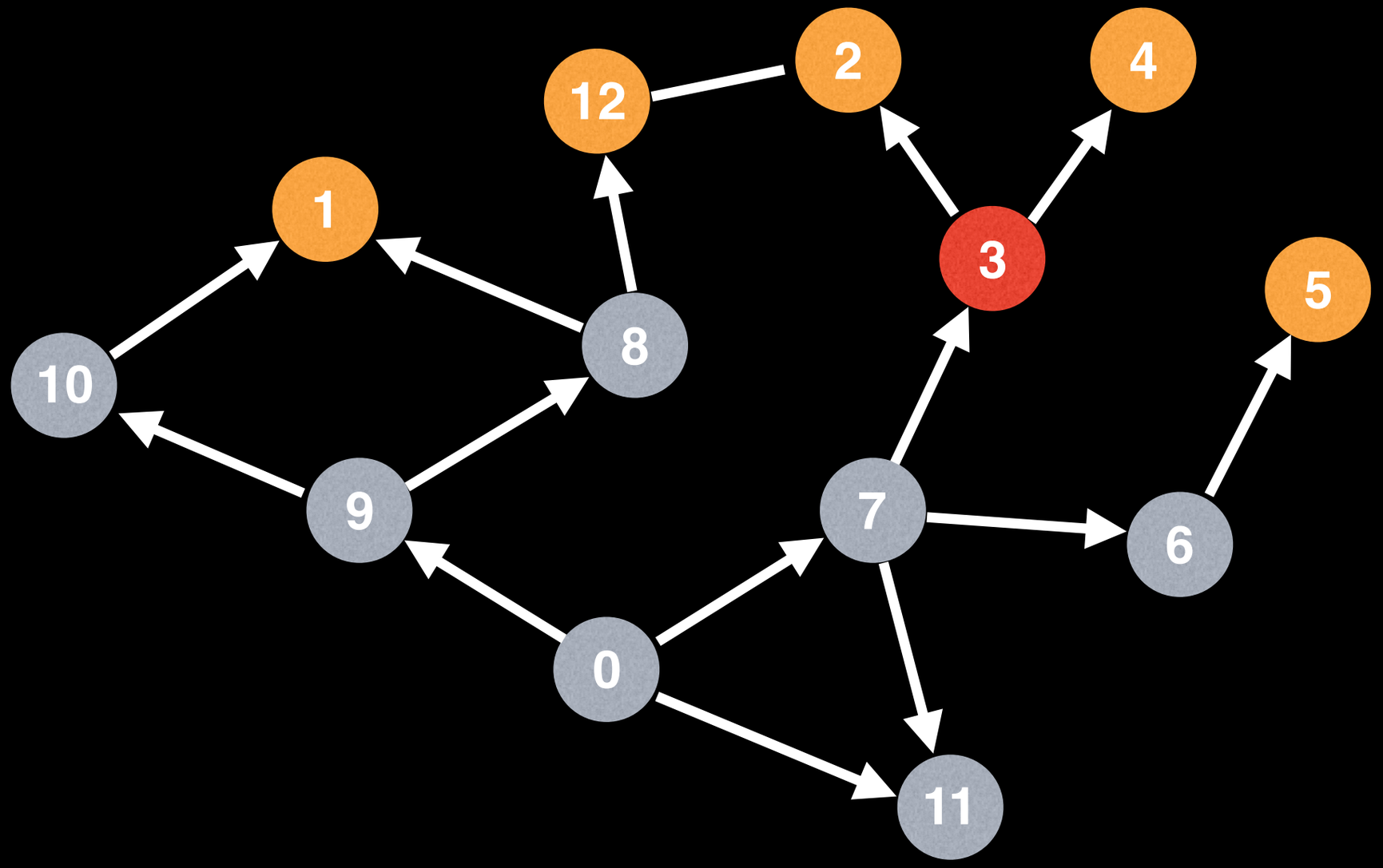

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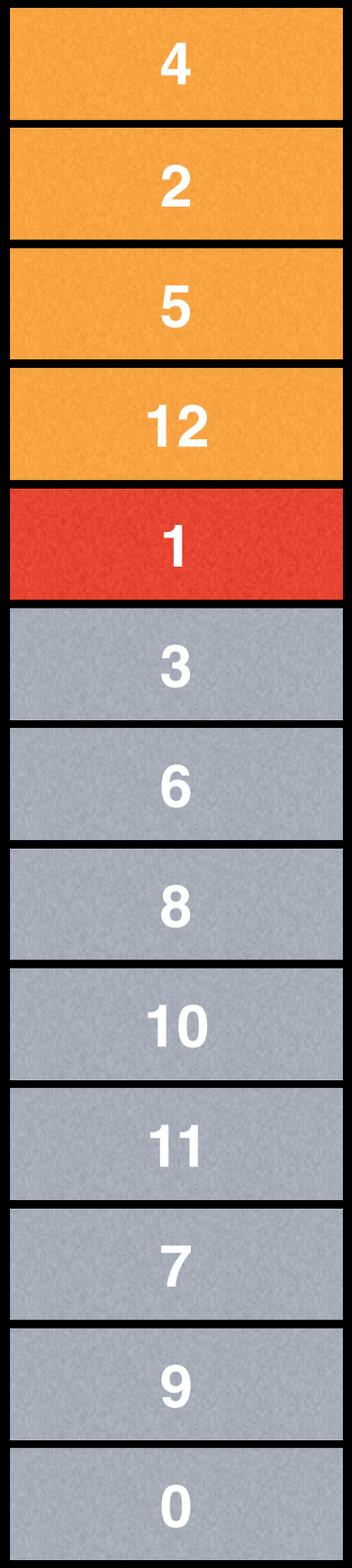

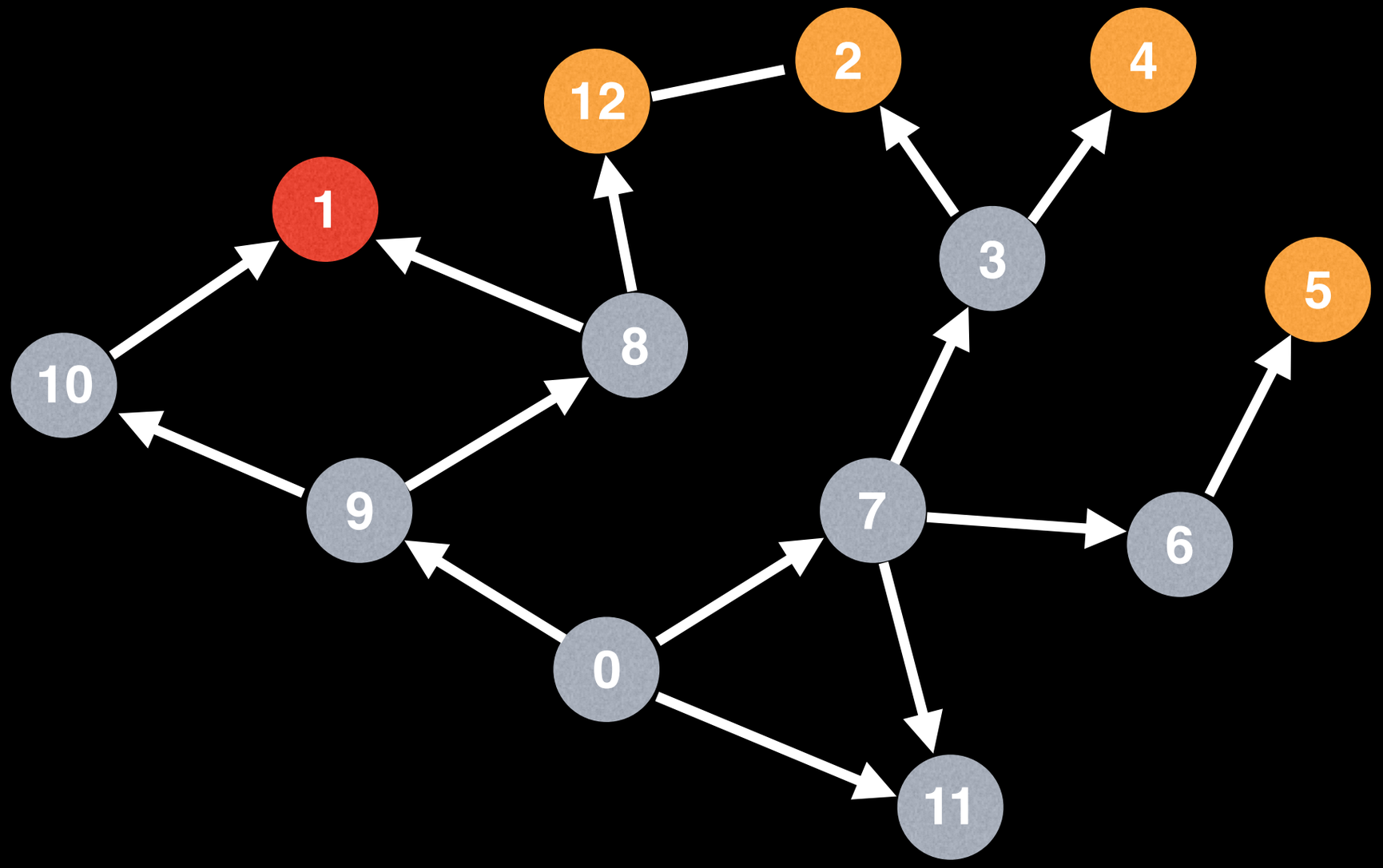

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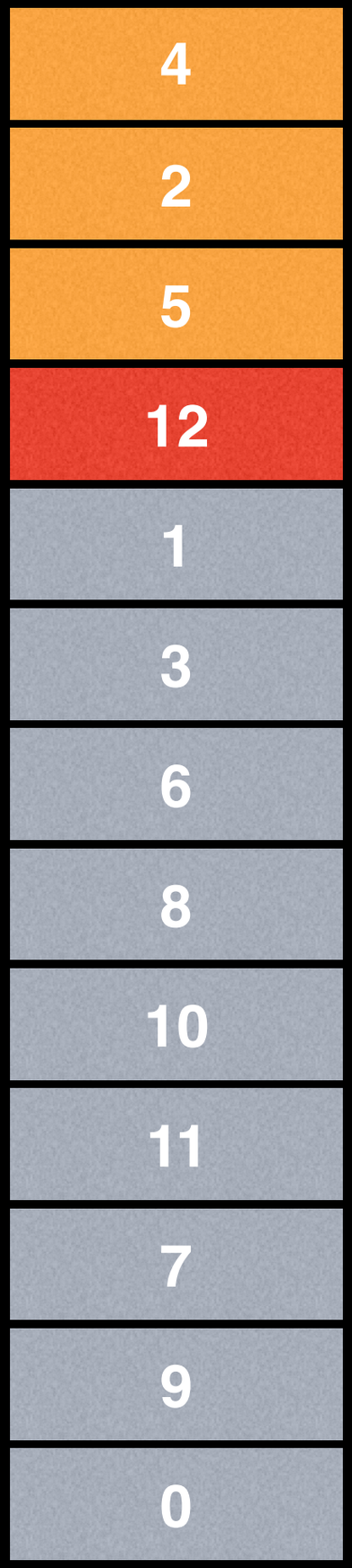

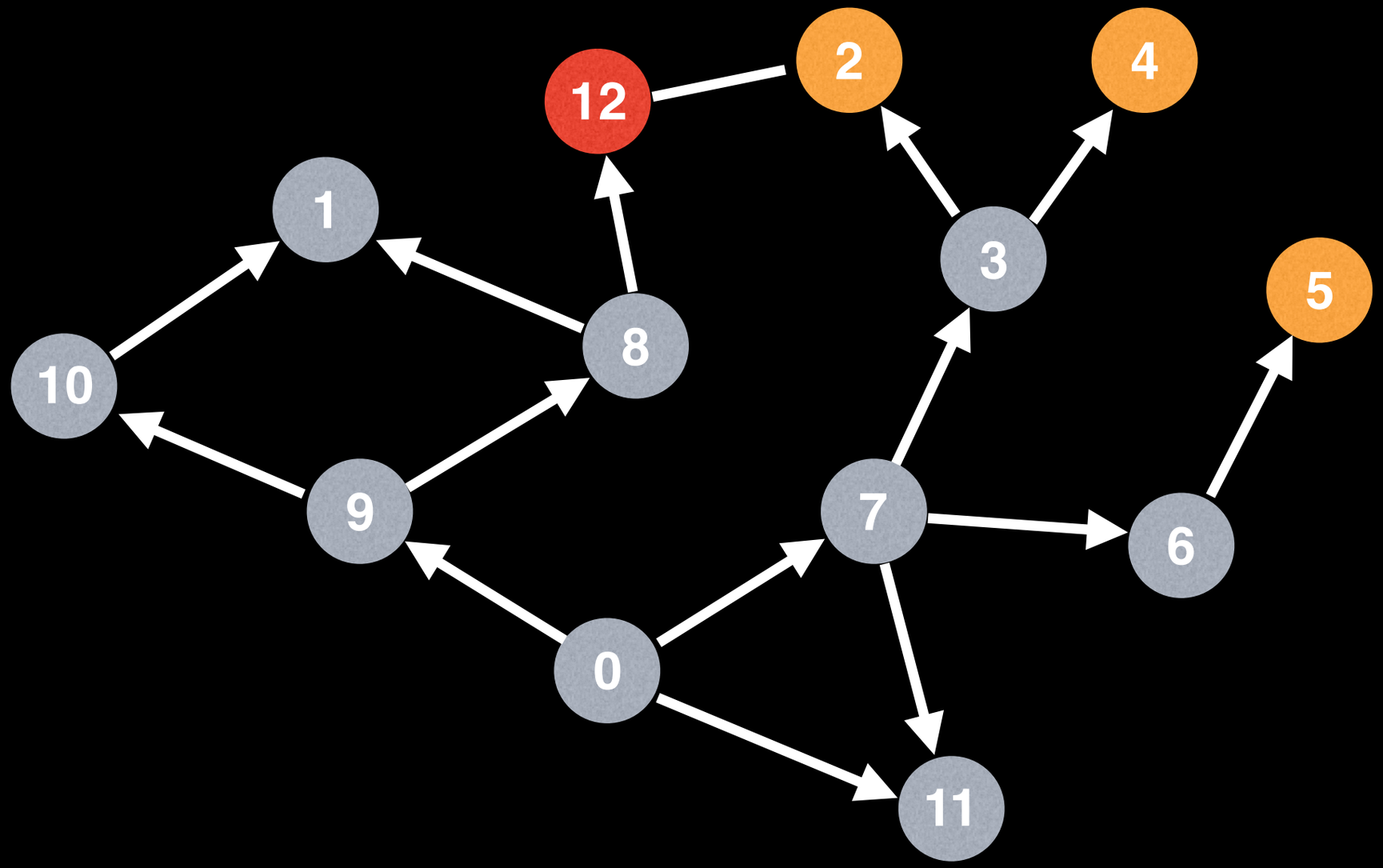

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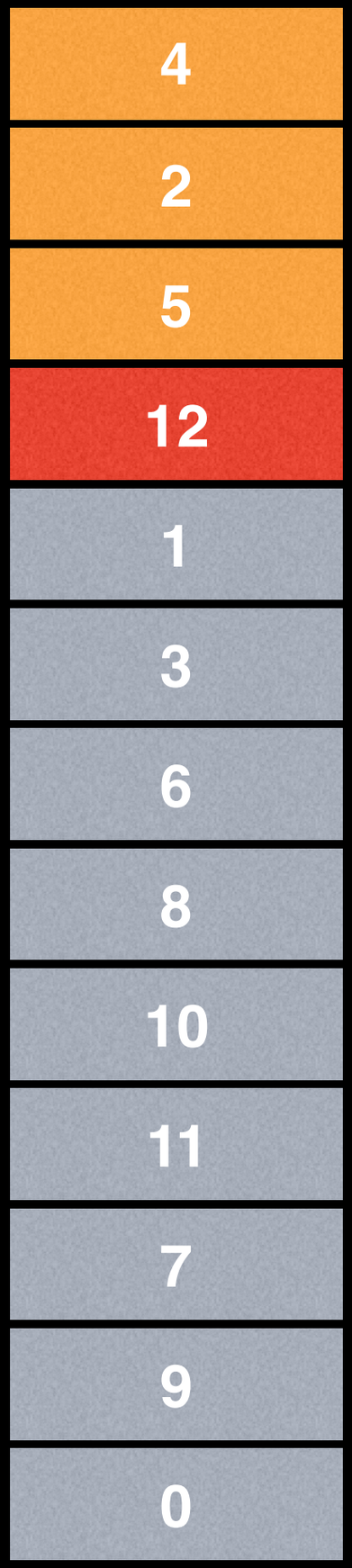

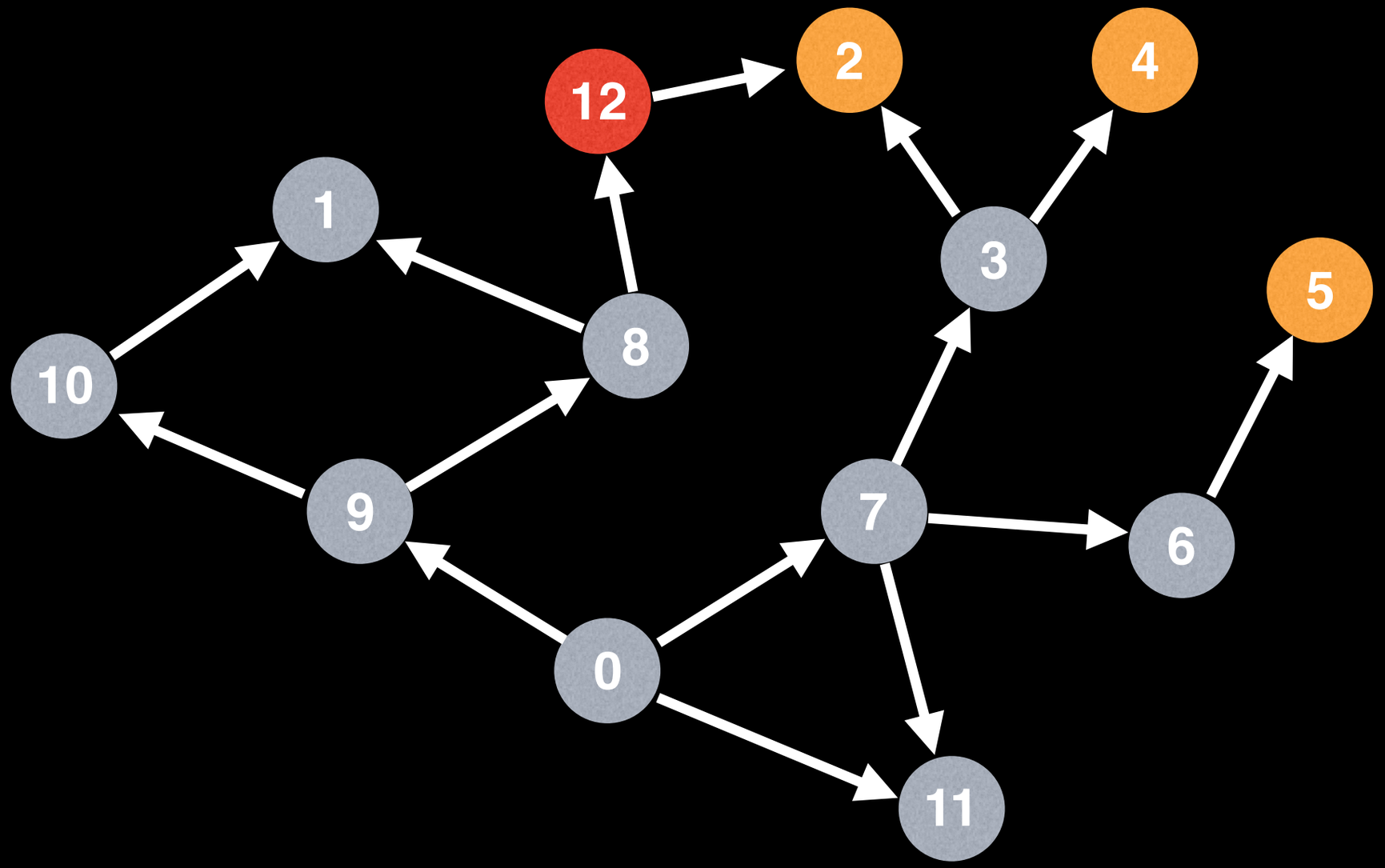

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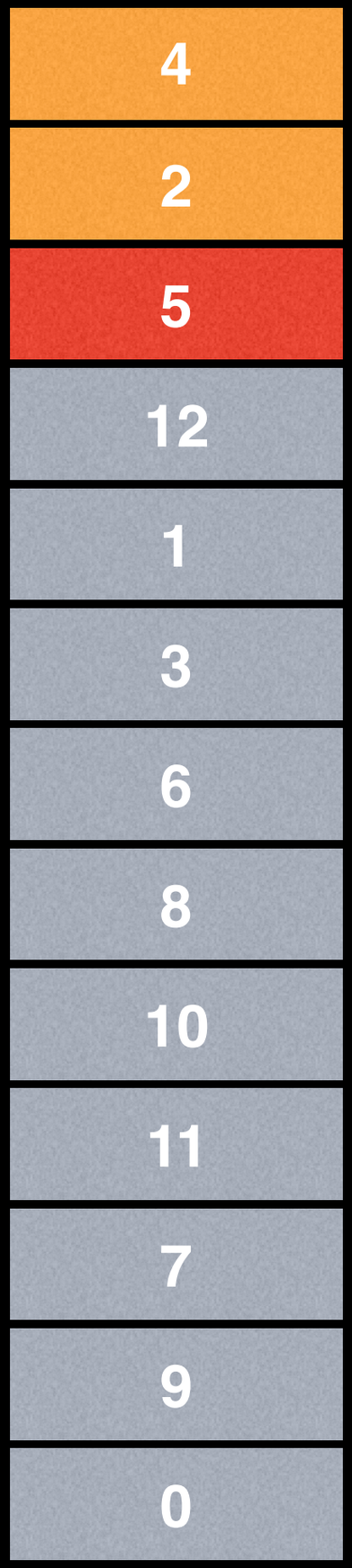

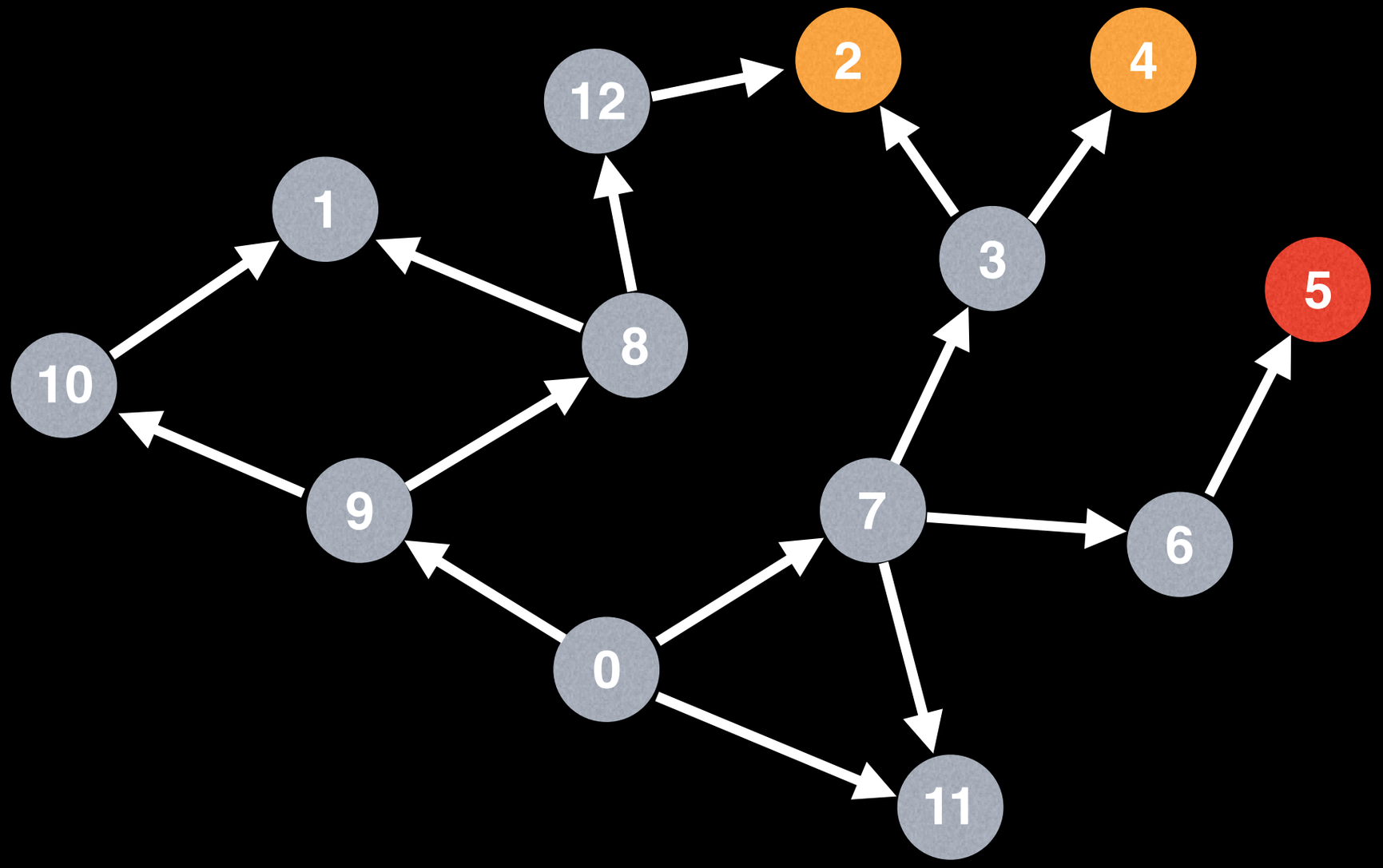

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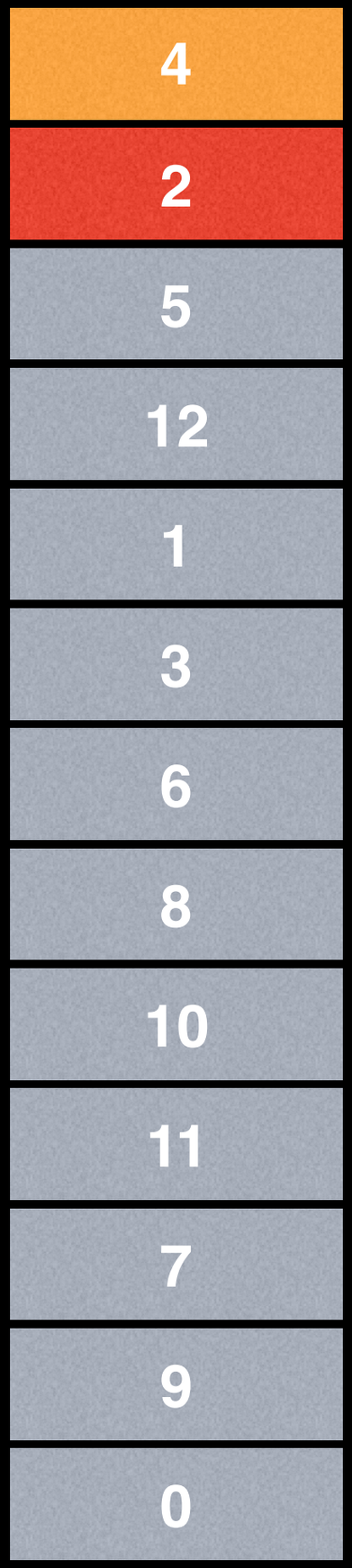

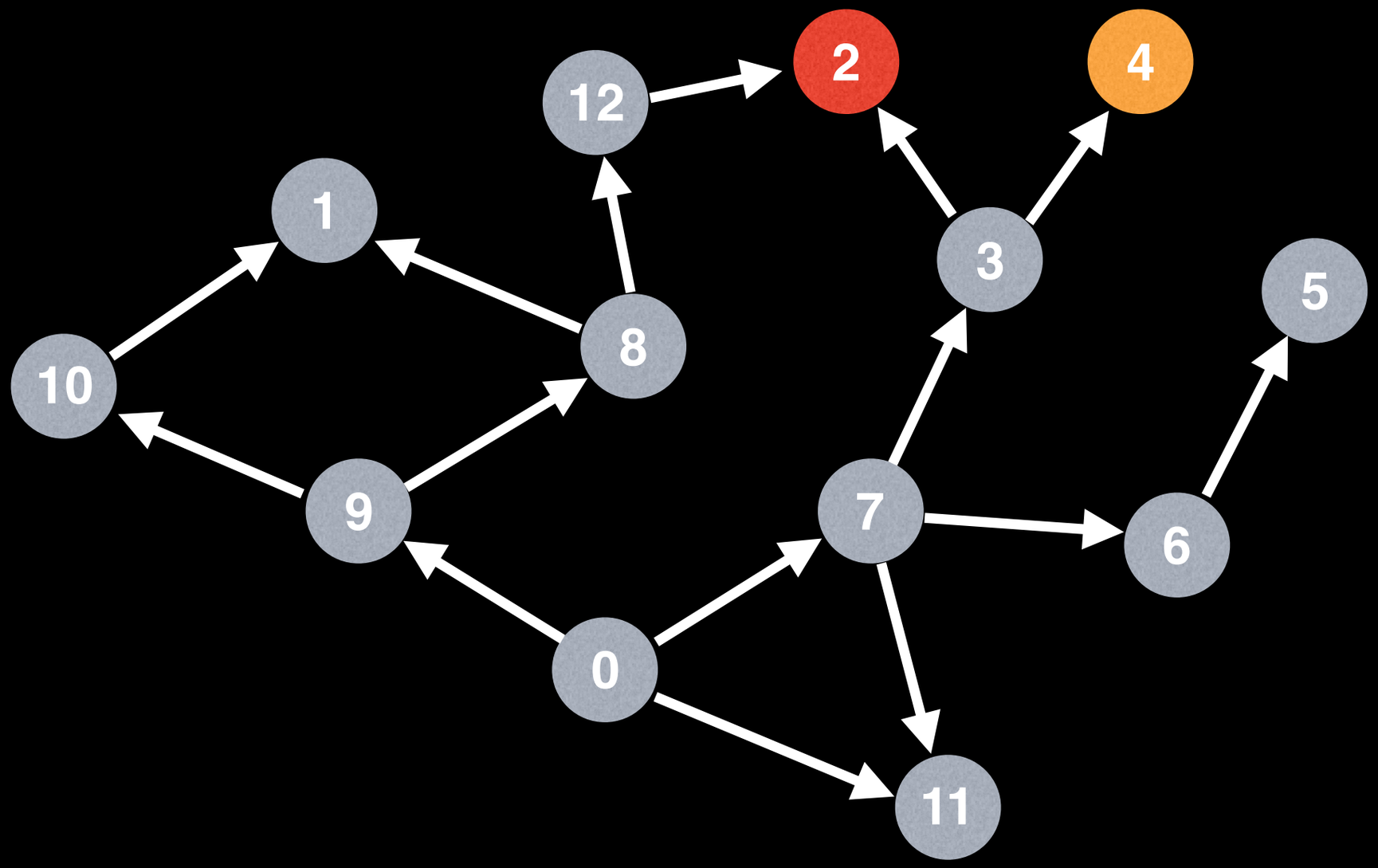

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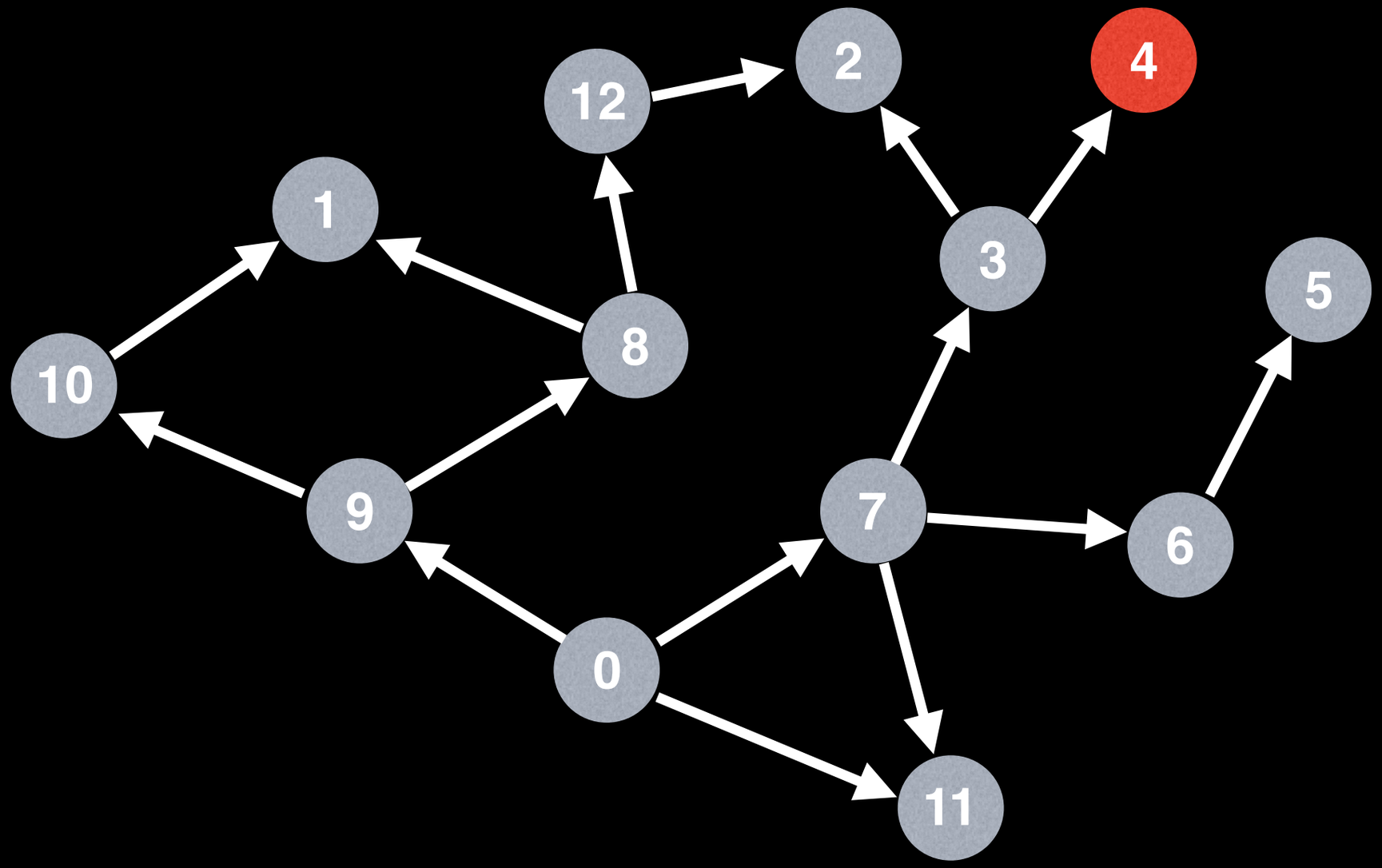

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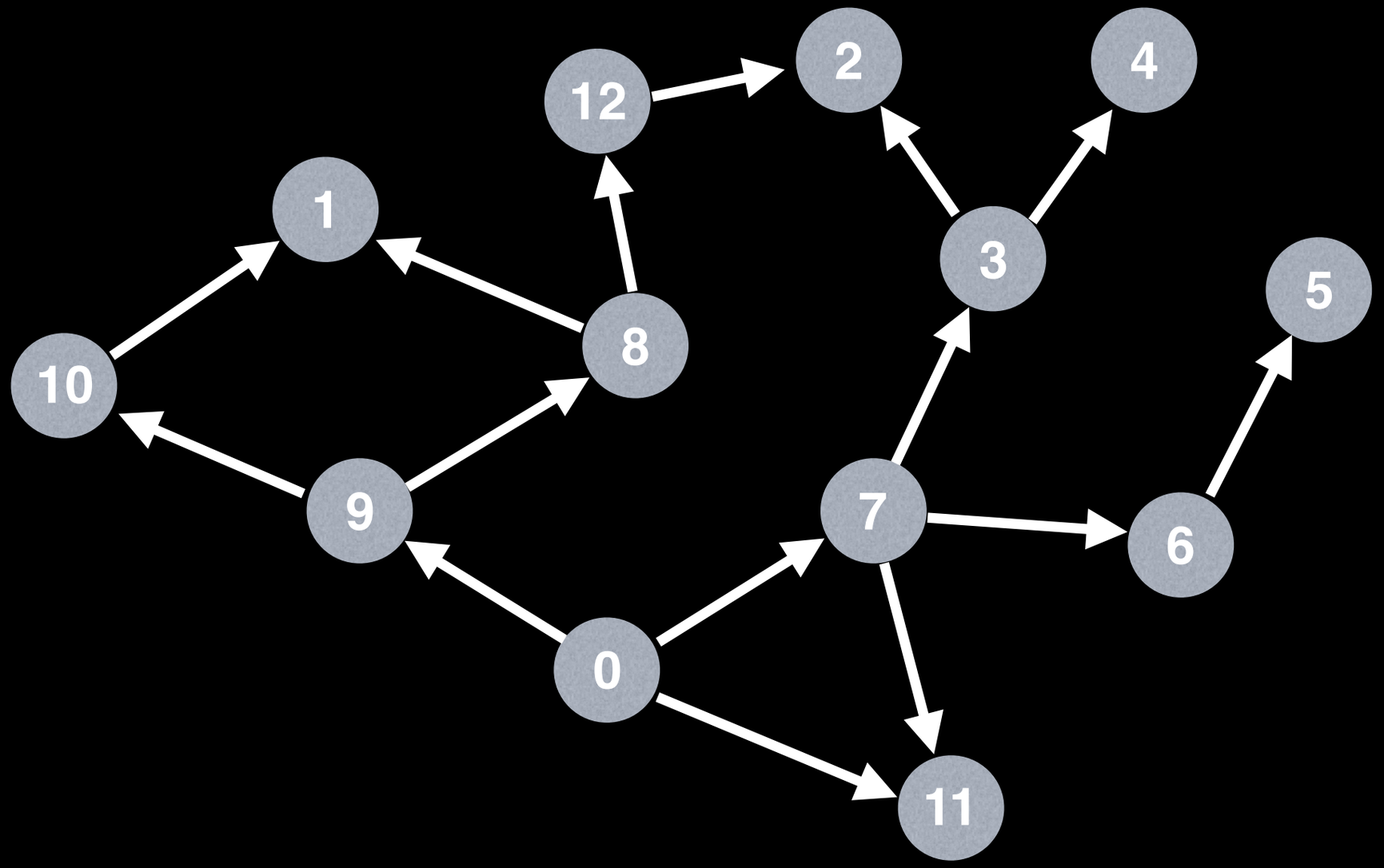
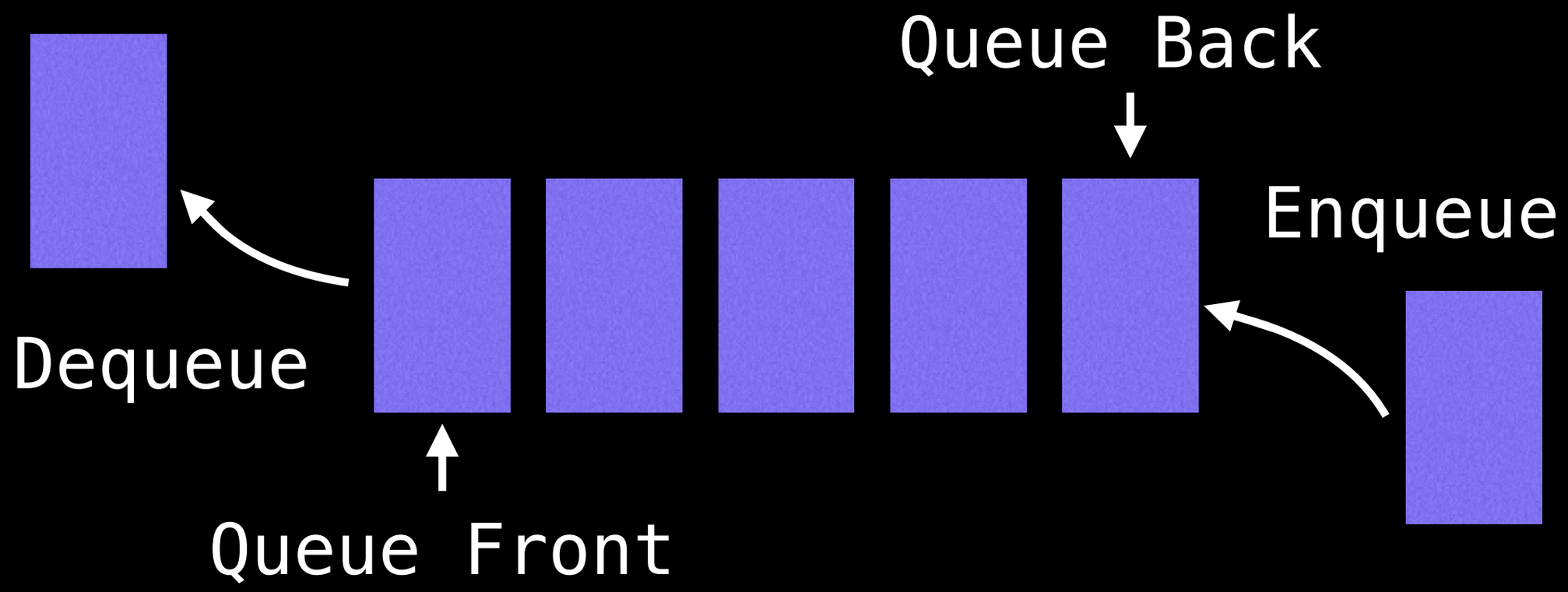


Using a Queue

The BFS algorithm uses a queue data structure to track which node to visit next. Upon reaching a new node the algorithm adds it to the queue to visit it later. The queue data structure works just like a real world queue such as a waiting line at a restaurant. People can either enter the waiting line (**enqueue**) or get seated (**dequeue**).



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# Global/class scope variables
n = number of nodes in the graph
g = adjacency list representing unweighted graph

# s = start node, e = end node, and 0 ≤ e,s < n
function bfs(s, e):

  # Do a BFS starting at node s
  prev = solve(s)

  # Return reconstructed path from s -> e
  return reconstructPath(s, e, prev)
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function solve(s):
  q = queue data structure with enqueue and dequeue
  q.enqueue(s)

  visited = [false, …, false] # size n
  visited[s] = true

  prev = [null, …, null] # size n
  while !q.isEmpty():
    node = q.dequeue()
    neighbours = g.get(node)

    for(next : neighbours):
      if !visited[next]:
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function solve(s):
  q = queue data structure with enqueue and dequeue
  q.enqueue(s)

  visited = [false, …, false] # size n
  visited[s] = true

  prev = [null, …, null] # size n
  while !q.isEmpty():
    node = q.dequeue()
    neighbours = g.get(node)

    for(next : neighbours):
      if !visited[next]:
        q.enqueue(next)
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# Global/class scope variables
n = number of nodes in the graph
g = adjacency list representing unweighted graph

# s = start node, e = end node, and 0 ≤ e,s < n
function bfs(s, e):

  # Do a BFS starting at node s
  prev = solve(s)

  # Return reconstructed path from s -> e
  return reconstructPath(s, e, prev)
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  # Reconstruct path going backwards from e
  path = []
  for(at = e; at != null; at = prev[at]):
    path.add(at)
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  # If s and e are connected return the path
  if path[0] == s:
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  return []
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