Optimization on Routing Networks:
Applied case to Barcelona with Braess Paradox Detection

Catalina Odizzio
Agostina Pissinis
Vicente Lisboa
Santiago Manotas-Arroyave

Barcelona School of Economics

Networks:
Concepts and Algorithms

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Barcelona School of Economics
Spain

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Summary

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1 Abstract

By investigating Barcelona's urban transportation network, this research dives into the real-world consequences of the Braess paradox. We show that the system equilibrium differs from the optimal welfare delay using publicly accessible traffic statistics and the Frank-Wolfe Algorithm for convex optimization with linear constraints. Our research demonstrates the existence of the paradox in the real-world network, demonstrating that selective road elimination improves traffic flow over the original data. In doing so, we show proof for the paradox's reality in realistic circumstances. These findings, if expanded and examined in a wider, important policy study, may have consequences for urban planning, transportation management, and the creation of sustainable, people-centric city spaces, while also emphasizing the necessity of considering the Braess paradox in network design and optimization.

Keywords: Transportation Networks, Convex Optimization, Network Design, Traffic Flow, Urban Planning.

2 Introduction

The Braess paradox (**BP**), a counter-intuitive phenomenon in the realm of network theory, has been a topic of considerable interest and discussion among academics and practitioners alike. First introduced in (Braess, 1968), it asserts that the addition of a new link or resource to a transportation or communication network may result in a drop in overall system efficiency; resulting in longer journey times or lower information transfer under specific conditions. Because of the pervasiveness of this paradox across diverse real-world networks, there has been an explosion of research efforts targeted at understanding its underlying mechanics and prospective uses.

To date, the existing body of literature within the field of network theory, has predominantly focused on the theoretical underpinnings (Beckmann et al., 1955; Fisk, 1979; Frank, 1981; Murchland, 1970) and rigorous simulations of the paradox (Rapoport et al., 2009; Bazzan and Klügl, 2005; Morgan et al., 2009), while its prevalence in real-life networks has not been accorded the same level of exposure in road-traffic applications. Some of the studies that have addressed the BP in the context of road-traffic networks primarily concentrate on employing simulations and exploring road removal strategies as a means to optimize latency reduction; (Youn et al., 2008) do so for major cities New York, Boston, and London.

Identifying specific attributes of roads that give rise to the paradox could potentially have significant policy implications for urban planning and transportation management. Utilizing the BP as a guiding principle for selectively shutting down roads may present a viable alternative to traditional methods such as toll implementation. Furthermore, such strategic closures could yield additional benefits; including reduced air and noise pollution, improved public health, and contributions towards climate change mitigation efforts. In relation to this, the current plan to establish super-blocks within the city of Barcelona to mitigate traffic and rebuild these spaces as people-centric spaces (Mueller et al., 2020) is in itself relevant to the topic, along with its manifestation in regards to city planning and public mobility.

In this paper, we will delve into the manifestation of the BP in a real-world network, illustrating its implications for network design and optimization making use of an iterative algorithm, designed to solve convex optimization problems with linear constraints; while shedding light on possible mitigation strategies.

Focusing on publicly available transportation network information for Barcelona, we set out to calculate latencies for flows and their corresponding costs; both starting from the user equilibria and also arriving at the social optimum with the use of iterative first-order optimization. Afterwards, we set out to remove links for the original network and then recompute delays to detect the presence of the paradox in and of itself.

3 Literature Review

The appearance of the paradox revolves around the increase of transportation costs when new roads are added to a network. Without the new road (in the theoretical sphere) the Price of Anarchy (**PoA**), defined as the ratio between equilibrium routing latency and its optimum counterpart seems to increase in those scenarios where the BP manifests itself. In other words, given the gap from individual optimum to the social welfare case, negative externalities arise; resulting in sub-par equilibria. These controversies on weather or not the addition of new edges to traffic networks reduces overall delays (where it is important to highlight that these additions could also leave traffic unchanged). Overall, this phenomenon described by (Braess, 1968) is still highly debated in its applicability to real-life scenarios.

On one hand, simulations on the paradox such as the seminal one of (Rapoport et al., 2009), where researchers designed an experiment to investigate the robustness of the paradox using two topologically distinct simulated traffic networks: the original network and another with one or more links added. Participants were instructed to independently and repeatedly choose their optimal routes before and after the addition of one or more edges to the original network. Through the aggregation of players' responses, the researchers aimed to identify any systematic patterns of behavior that emerged with experience and aligned with BP's existence; players exhibited a tendency to switch to the newly introduced routes despite experiencing a significant decline in their payoffs. Thence highlighting the prevalence of the paradoxical behavior in transportation network decision-making. In another field, (Schäfer et al., 2022) provide an experimental setup that exhibits the BP in an alternating current (AC) power grid, emphasizing its significance for existing large-scale grid extension initiatives in this work. The authors present a topological theory that explains the paradox's essential mechanism and predicts *Braessian* grid extensions based on network structure. Using data from the full-scale German power grid, simulations are shown to predict blackouts due to structural changes of paradoxical nature.

As mentioned, and in regard to the policy discussion of super-blocks; (Bagloee et al., 2019) construct the BP detection problem (**BPDP**) as a bilevel issue and create a heuristic technique (namely a surrogate-based algorithm) to identify roadways in urban centers that should be closed and repurposed as green spaces, pedestrian plazas, and so on. Moreover, when solving the BPDP, the suggested formulation takes into consideration travel demand elasticity caused by road closure schemes. The approach is validated using a genuine dataset from Winnipeg's road network. Even in crowded metropolitan locations, the findings give persuasive evidence and justification for the notion of no-car zones.

As part of our approach to analyzing said network, our main consideration is the usage of the *Frank-Wolfe Algorithm* (Frank and Wolfe, 1956), which in the context of urban network studies; has been widely used to handle a wide range of transportation planning and traffic assignment issues. Because these challenges frequently involve large-scale networks, the algorithm's decreased computational complexity and lower memory needs are advantageous. Likewise, the algorithm's inclination to converge to sparse solutions is well-aligned with the intrinsic sparsity of urban traffic networks.

Many changes and extensions to the Frank-Wolfe Algorithm have been developed in recent years in order to improve its convergence features and broaden its application to a larger variety of optimization problems. For example, academics have created speedier variations of the method that use techniques like momentum (Piekniewski et al., 2016) and adjustable step sizes (Freund and Grigas, 2016). These improvements have resulted in faster convergence rates. The algorithm, which is central to our simulations, is explained later on in the Methodology section and follows a naïve implementation in accordance to that of (Peeta, 2015) and their corresponding repository.

It is also relevant to note instances where BP is not entirely present, only in specific scenarios as illustrated by (Acemoglu et al., 2018); or its complete negation such as in (Nagurney, 2010).

4 Methodology & Data

4.1 Methodology

To identify edges that could potentially decrease latency in a flow network, it is necessary to calculate an initial equilibrium. Two types of equilibrium can be defined: the user's equilibrium (also known as the Nash equilibrium) and the optimal equilibrium.

The user's equilibrium represents the flow of the network when all drivers choose the route that minimizes their own costs, given the routing decisions of all other users. On the other hand, the social optimum represents the state in which the overall welfare or efficiency of the network is maximized, taking into account both the costs of transmitting flows and the benefits received from them.

According to (Youn et al., 2008) each directed link from node i to j is associated with a delay l_{ij} , the time needed to travel along the link. Delays are assumed to follow the Bureau of Public Roads (BPR) function widely used in civil engineering:

$$l_{ij} = \frac{d_{ij}}{vij} * [1 + \alpha * (\frac{x_{ij}}{p_{ij}})^{\beta}]$$

Where d_{ij} is the distance of the link between i j, v_{ij} the speed limit (assumed 60 kph on all links for simplicity), x_{ij} the flow, and x_{ij} the capacity of the road segment. The parameters α and β have been fitted to empirical data as $\alpha = 0.2$ and $\beta = 10$.

The socially optimal flows f_{ij}^{SO} are determined by minimizing the cost to society per unit time.

$$C(x) = \sum_{link_{(ij)}} l_{ij}(f_{ij}) * f_{ij}$$

For the other hand, the equilibrium flows f_{ij}^{NE} minimize the objective function:

$$C(x) = \sum_{link_{(ij)}} \int_0^{f_{ij}} l_{ij}(f')\, df'$$

In this work, we calculated both equilibria using a Frank-Wolfe algorithm, which uses successive iterations to solve a linearized objective function via a first-order Taylor approximation, and moves towards a minimizer of this linear function. Given an equilibrium flow, we can then calculate total delays, which are proxied by the previously defined latency function.

The Frank-Wolfe algorithm can generally be broken down into five steps as described below. Then the loop of iterations continues throughout Step 2 to Step 5 until the minimum extreme point is identified.

- Step 1. Define initial solution:

  If (x is the extreme point, the initial arbitrary basic feasible solution can be considered as:

  $$x_k \in S \quad where\ k = 0$$

  k denotes the number of iterations

- Step 2. Determine search direction:

  Search direction, that is the direction vector is:

  $$p_k = y_k - x_k$$

  x_k and y_k are feasible extreme points and belong to S where S is convex. First-order Taylor expansion around x_k and problem is now reformulated to LP:

  $$\begin{aligned} &\min z_k(y) = f(x_k) + \nabla f(x_k)^T (y - x_k) \\ &s.t.\ y \in S \end{aligned}$$

- Step 3. Determine step length:

  Step size α_k is defined by the following formula where α_k must be less than 1 to be feasible:

$$f(x_k + \alpha_k p_k) < f(x_k)$$
$$min_{\alpha \in [0,1]} f(x_k + \alpha_k p_k)$$

- Step 4. Set new iteration point:

$$x_{k+1} = x_k + \alpha_k p_k$$

- Step 5. Stopping criterion:

  Check if x_{k+1} is an approximation of x, the extreme point. Otherwise, set k=k+1 and return to Step 2 for the next iteration. In the Frank-Wolfe algorithm, the value of f descends after each iteration and eventually decreases towards f(x), the global minimum.

After establishing an initial Nash equilibrium and the socially optimal solution, we tested for the Braess Paradox in the network by removing one of the edges and computing the cost of the network using the given algorithms

4.2 Data

In order to assess the manifestation of the BP, we utilized the information available in this GitHub repository. This repository provides transportation networks for various cities. For this particular investigation, we selected the transportation network corresponding to Barcelona (see Figure 1), which is composed of 110 zones (in red), 1020 nodes (in blue), and 2522 edges.

Regarding the attributes of the network, the dataset provides information on capacity, length, free flow time, speed, and toll for each of the roads. However, it should be noted that the capacity remains constant at 1 in all cases, while the speed and toll are equivalent to zero.

Given the size of the network, we opt to employ a subgraph to save computational time and enable the computation of Nash equilibrium, social optimum, and the paradox evaluation. The resulting subgraph, depicted in Figure 2, consists of 100 nodes and 257 edges.

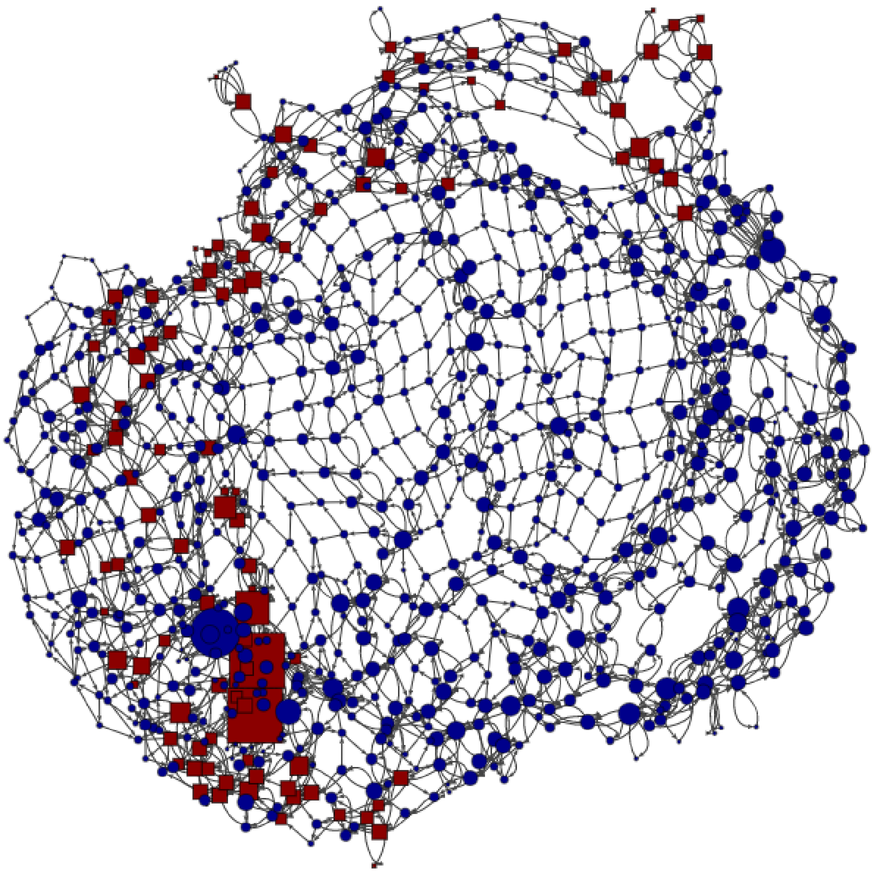

Figure 1: Barcelona's Network

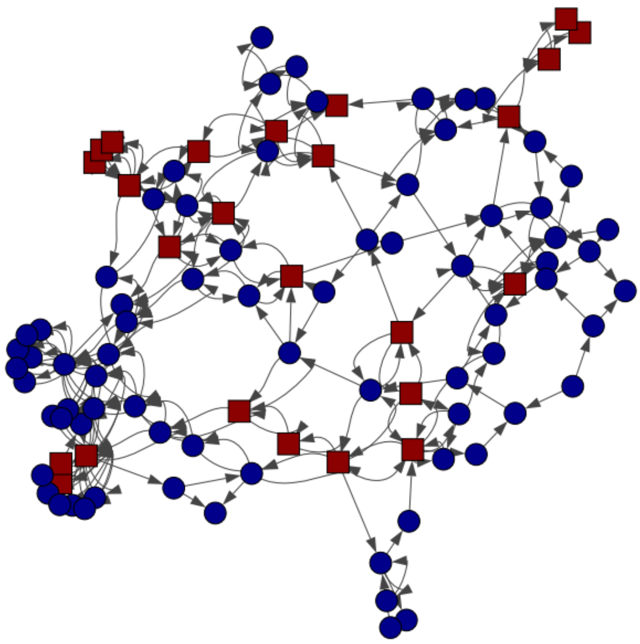

Figure 2: Subgraph

5 Results

Using the Frank-Wolfe algorithm, we computed the optimal and equilibrium flows, obtaining the traffic flow for each route for both scenarios, the distribution of which is shown in Figure 3. As can be observed, in both cases, the distribution is similar, with 75% of flows falling between 0 and 1500.

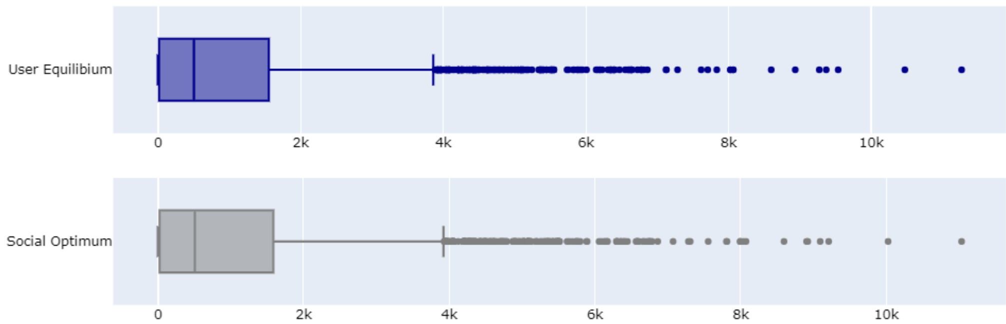

Figure 3: Flows distribution

From these flows, we calculated the delay for each road and the total cost, which were used to determine the PoA. Our analysis revealed a PoA of 1.27, indicating that the Nash equilibrium cost exceeds the social optimum, resulting in a 27% loss of time for route users due to a lack of coordination.

In light of this result, it would be reasonable to define strategies to reduce the total cost and achieve a more efficient use of the routes. Thus, we conducted the simulation by sequentially removing the different roads from the subgraph and recalculating the total cost of the network, while considering the redistribution of flows. As a result, we observed that in at least two cases, removing a route led to a marginal reduction in the total cost of 9.32e-05. This result suggests that the phenomenon of the BP is present in this network, providing evidence that road closures can occasionally mitigate travel delays. However, it cannot be asserted that the phenomenon of the BP can be visualized in the complete network of Barcelona. When taking a subgraph, various nodes and routes are lost, which can result in altered results and equilibrium calculations. Therefore, the subgraph cannot be considered a representation of the original network.

On the other hand, it is important to consider that the results of this simulation may be affected by the assumptions made and the characteristics of the data. Firstly, the parameters alpha and beta are set to 0.2 and 10, respectively. Further simulations could be performed by taking different values of these parameters to evaluate whether the phenomenon persists. Secondly, to simplify the implementation, it is assumed that the maximum speed is the same on all roads. However, it is understood that this may not reflect reality. Finally, the data indicates that all routes have a capacity of 100%, but it is not clear whether these are actual data, which may affect the calculation of the delay.

6 Conclusion

The scope of this study is limited to the analysis of a subgraph of the city of Barcelona. However, there are certain criteria that could be analyzed in a more extensive investigation with different networks.

The heterogeneity of the network appears to be a relevant characteristic to examine. Based on the initial solution obtained, the Barcelona network is quite heterogeneous since it has a high percentage of paths (24%) with a flow of less than 5, while approximately 35% of the paths have a flow greater than 1000. The high percentage of paths with low flow suggests that eliminating paths could be more feasible with a network that has lower concentration and more homogeneous paths. However, this theory would need to be empirically tested with another network.

The size of the network is also important to consider. We worked with a subgraph of 257 edges, and for future research, it is suggested to apply the algorithm to the complete network. The more possible paths there are between an origin and a destination, the more alternatives there are for redistributing flow.

Regarding the characteristics of the edges that exhibit the Braess Paradox, due to the low number of paths that meet the criteria of the paradox (<1%), it was not possible to define conclusive characteristics with respect to these edges. A possible area for further investigation is to run the algorithm on a larger network and describe the characteristics of these edges.

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