743.

ON THE NEWTON-FOURIER IMAGINARY PROBLEM.

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THE Newtonian process of approximation to the root of a numerical equation $f(u)=0$, consists in deriving from an assumed approximate root ξ a new value $\xi_1 = \xi - \dfrac{f(\xi)}{f'(\xi)}$, which should be a closer approximation to the root sought for: taking the coefficients of $f(u)$ to be real, and also the root sought for, and the assumed value ξ, to be each of them real, Fourier investigated the conditions under which ξ_1 is in fact a closer approximation. But the question may be looked at in a more general manner: ξ may be any real or imaginary value, and we have to inquire in what cases the series of derived values

$$\xi_1 = \xi - \frac{f(\xi)}{f'(\xi)},\quad \xi_2 = \xi_1 - \frac{f(\xi_1)}{f'(\xi_1)}, \ldots$$

converge to a root, real or imaginary, of the equation $f(u)=0$. Representing as usual the imaginary value ξ, $=x+iy$, by means of the point whose coordinates are x, y, and in like manner ξ_1, $=x_1+iy_1$, &c., then we have a problem relating to an infinite plane; the roots of the equation are represented by points $A, B, C, \ldots$; the value ξ is represented by an arbitrary point P; and from this by a determinate geometrical construction we obtain the point P_1, and thence in like manner the points $P_2, P_3, \ldots$ which represent the values $\xi_1, \xi_2, \xi_3, \ldots$ respectively. And the problem is to divide the plane into regions, such that, starting with a point P_1 anywhere in one region, we arrive ultimately at the root A; anywhere in another region we arrive ultimately at the root B; and so on for the several roots of the equation. The division into regions is made without difficulty in the case of a quadric equation; but in the next succeeding case, that of a cubic equation, it is anything but obvious what the division is: and the author had not succeeded in finding it.