Afledte og omvendte funktioner

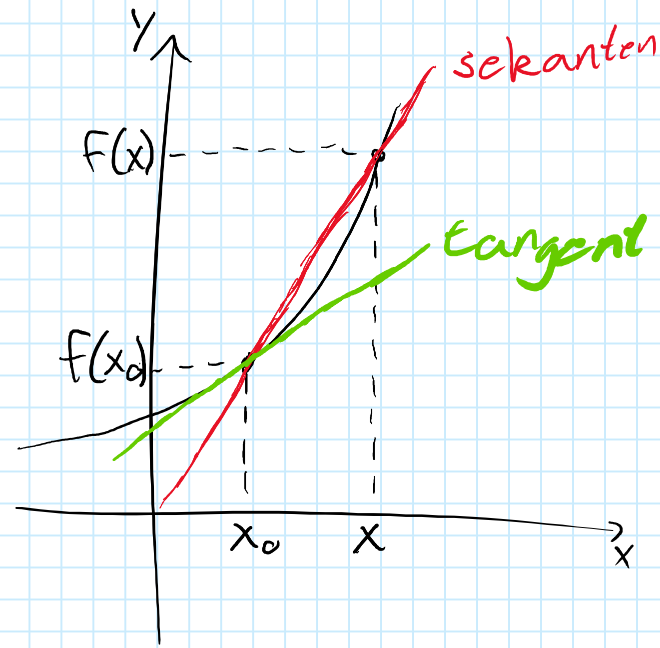
Thursday, September 16, 2021 8:05 AM

Hvis $\frac{f(x)-f(x_0)}{x-x_0} \to a$ når $x \to x_0$

så er f differentiabel i x_0

og skriver $a = f'(x_0)$.

a er hældningskoefficienten for tangenten.



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så er f diff. i x_0 og $a = f'(x_0)$.

$$f(x) = f(x_0) + a(x-x_0) + \varepsilon(x-x_0)(x-x_0)$$

diff. x^2 i x_0.

$f: \mathbb{R} \to \mathbb{C}$ $\quad f'(x_0)$ er et komplekst tal.

Afledte funktioner

$f(x)$	$f'(x)$
k	0
x	1
x^2	$2x$
x^n	$n \cdot x^{n-1}$
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
e^x	e^x
$\ln^x$	$\frac{1}{x}$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$

Regneregler:

$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$

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$$(k \cdot f(x))' = k \cdot f'(x)$$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$$(f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Eks.

$$(\cos(7x^4))' = -\sin(7x^4) \cdot 7 \cdot 4x^3$$

Eks.

$$\left(\frac{e^x}{x}\right)' = \left(e^x \cdot \frac{1}{x}\right)' = e^x \cdot \frac{1}{x} + e^x \cdot \left(-\frac{1}{x^2}\right)$$

Omvendte funktioner

$f(x)$	$f^{-1}(x)$	def. mængde $D_m(f^{-1})$
x^2	$\sqrt{x} = x^{\frac{1}{2}}$	$x \geq 0$
x^n	$\sqrt[n]{x} = x^{\frac{1}{n}}$	$x \geq 0$
e^x	$\ln(x)$	$x > 0$
$\cos x$	$\arccos x$	$[-1, 1]$
$\sin x$	$\arcsin x$	$[-1, 1]$
$\tan x$	$\arctan x$	$\mathbb{R}$

① $f(x) = y \Leftrightarrow f^{-1}(y) = x$

Løs $e^x = 5$ $\quad \underline{\ln(5) = x}$

② $f^{-1}(f(x)) = x$

Vi prøver at diff. på begge sider i ②.

$$(f^{-1})'(\underbrace{f(x)}_{y}) \cdot f'(x) = 1$$

③ $(f^{-1})'(y) = \frac{1}{f'(x)}$

Givet funktionen $y = e^x$

$$\ln'(y) = \frac{1}{(e^x)'} = \frac{1}{e^x} = \frac{1}{y}$$

Konkl:

$$\ln'(x) = \frac{1}{x}$$

Givet $y = \cos x$

$$\arccos'(y) = \frac{1}{\cos' x} = \frac{1}{-\sin x} = -\frac{1}{\sin x}$$

$$= -\frac{1}{\sqrt{1-\cos^2 x}} = -\frac{1}{\sqrt{1-y^2}}$$

$\cos^2 + \sin^2 = 1$

$\sin = \sqrt{1-\cos^2}$

Funktioner af typen $\mathbb{R} \to \mathbb{C}$

$$f(t) = g(t) + i\,h(t)$$

$$f'(t) = g'(t) + i \cdot h'(t)$$

Givet $c \in \mathbb{C}$

$$\left(e^{c \cdot t}\right)' = c \cdot e^{ct}$$

Eks.

$$f(t) = i\,e^{(1+2i)t} + i$$

$$f'(t) = i \cdot (1+2i)e^{(1+2i)t} + 0$$

$$f(0) = i + i = 2i$$

$$f'(0) = i(1+2i) = \underline{\underline{-2+i}}$$

kompleks diff.kv.