Alle 60 take 1

12. november 2024 23:04

1. True

2. $2(q+1)$ er ulige

$1=2$

3.

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(\neg P \vee \neg Q)$
1	1	0	0	1	1
0	1	1	0	0	0
1	0	0	1	0	0
0	0	1	1	0	0

$\Leftrightarrow$

4. $P: 4 > 5$ (F)

$Q: -7 < -6$ (T)

$\neg(F \wedge T) \Rightarrow T$

$\equiv$

T

P	Q	$P \Rightarrow Q$
1	1	1
0	1	0
1	0	1
0	0	1

5. $|x-7| = |x+7| + 1$

$x = -\frac{1}{2}$

6. O $x+y=0 \Rightarrow xy \leq 0$

O $x=2 \Rightarrow (x-2)(x+3)=0$

⊗ $x=2 \Rightarrow (x-2)e^x = 0$

O $x^2+y^2=0 \Rightarrow x=y$

⊗ $x^3+y^3=0 \Rightarrow x=-y$

7. $(x^2+3x-10=0) \wedge (x>0)$

$D = 3^2 - 4 \cdot 1 \cdot (-10) = 9 + 40 = 49$

$$x = \frac{-3 \pm \sqrt{49}}{2 \cdot 1} = \frac{-3 \pm 7}{2} = \begin{cases} \frac{-10}{2} \\ \frac{4}{2} = 2 \end{cases}$$

8. $0.000345 < x < 0.00345$

$x =$ f.eks. 0.001

9. $x + |x| = 21$

tallet

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$x = \frac{21}{2}$ $\qquad$ $\frac{\text{tallet}}{2}$ ...

10. $\frac{320}{128} = \frac{a}{b}$

$\frac{160}{64} = \frac{80}{32} = \frac{40}{16} = \frac{20}{8} = \frac{10}{4} = \frac{5}{2} = \frac{a}{b}$

11. $f(x) = x^3 - 9x^2 + 30x$

$(f^{-1})'(22) = ?$

$f(1) = 1 - 9 + 30 = 22$ $\qquad$?

$f^{-1} \rightarrow \quad y = x^3 - 9x^2 + 30x$

$x = y^3 - 9y^2 + 30y$

$(f^{-1})' \rightarrow 3y^2 - 18y + 30$

12. $a = 7$ $\qquad$ $A = \{a \mid a = |n| - 3,\ n \in \{-8, -9, -10, -11, -12\}$

$b = -10$ $\qquad$ $B = \{b \mid b = 7 \cdot m,\ m \in \mathbb{Z}\}$

13. $\sqrt{4}$

$\frac{17\pi}{\pi}$

$\frac{4}{17}$

$(\sqrt{17})^2$

19. $f(x) = 3e^{7x}$

$f^{-1}(x)$: $y = 3e^{7x}$

$x = 3e^{7y}$

$\frac{x}{3} = e^{7y}$

$\ln(\frac{x}{3}) = \ln(e^{7y})$

$\ln(\frac{x}{3}) = 7y$

$y = \frac{\ln(\frac{x}{3})}{7}$

$f^{-1}(6) = \frac{\ln(\frac{6}{3})}{7} = \frac{\ln(2)}{7}$

15. $f(x)=4x-5$

$g(x)=-x+1$

f^{-1}: $y=4x-5$

$x=4y-5$

$\frac{x}{4}=y-\frac{5}{4}$

$y=\frac{5}{4}+\frac{x}{4}=\frac{5+x}{4}$

$(g\circ f^{-1})=\left(-\left(\frac{5+x}{4}\right)+1\right)$

$(g\circ f^{-1})(2)=\left(-\left(\frac{7}{4}\right)+1\right)=-\frac{7}{4}+\frac{4}{4}=-\frac{3}{4}$

16. $f(x)=x^2+10$

false

false fordi $\mathbb{R}\rightarrow\mathbb{R}_+$

false

17. $f(x)=e^{kx}$

$f^{-1}(8)=\ln(2)$

f^{-1}: $y = e^{kx}$

$x = e^{ky}$

$\ln(x) = \ln(ky)$

$\ln(x) = ky$

$y = \dfrac{\ln(x)}{k}$

$\ln(2) = \dfrac{\ln(8)}{k}$

$\ln(2) = \dfrac{\ln(2^3)}{k}$

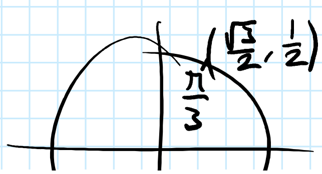
$\ln(2) = \dfrac{3\ln(2)}{k}$

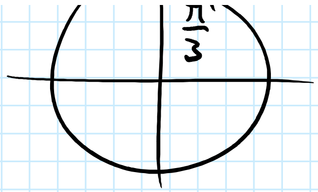
$\dfrac{\ln(2)}{3\ln(2)} = k$

$k = 3$

18. $2 \cdot 25 - 5 = 45$

19.





20. $\arcsin\left(-\frac{1}{2}\right) = \frac{1}{6}\pi$

21. $z = -i e^{\frac{\pi}{3}i}$ $\quad i = e^{\frac{\pi}{2}i}$ eulers tal

$z = e^{-\frac{\pi}{2}i} \cdot e^{\frac{\pi}{3}i}$

$= e^{\left(-\frac{\pi}{2} + \frac{\pi}{3}\right)i}$

$= e^{\left(-\frac{3\pi}{6} + \frac{2\pi}{6}\right)i}$

$= e^{-\frac{\pi}{6}i}$

Da $-\pi < -\frac{\pi}{6} < \pi$ er $\operatorname{Arg}(z) = -\frac{\pi}{6}$

22. $z = 2 \cdot \cos\left(\frac{\pi}{3}\right) + i \cdot 2 \cdot \sin\left(\frac{\pi}{3}\right)$

$= 2 \cdot \frac{1}{2} + 2i \cdot \frac{\sqrt{3}}{2}$

$= 1 + i\frac{2\sqrt{3}}{2}$

$= 1 + i\sqrt{3}$

er rigtig, men mobius vil have indtastet som $1 + i \cdot 3^{\frac{1}{2}}$

Vi have indsat som

$1+i\cdot 3^{\frac{1}{2}}$

23. $z=\sqrt{3}-i \qquad Re(z)=a=\sqrt{3} \qquad Im(z)=b=-1$

$$|z|=\sqrt{a^2+b^2}=\sqrt{\sqrt{3}^2+1^2}=\sqrt{3+1}=\sqrt{4}=2$$

$$\arg(z)=\arctan\left(\frac{b}{a}\right)=\arctan\left(\frac{-1}{\sqrt{3}}\right)=-\frac{\pi}{6}$$

24. $w_1=6+3i \qquad w_2=2-4i$

$$\frac{w_1}{w_2+1}=\frac{6+3i}{2-4i+1}=\frac{6+3i}{3-4i}=\frac{(6+3i)(3+4i)}{(3-4i)(3+4i)}=\frac{18+24i+9i-12}{9+16}=\frac{6+33i}{25}=\frac{6}{25}+\frac{33i}{25}$$

$$\left|\frac{w_1}{w_2+1}\right|=\sqrt{\left(\frac{6}{25}\right)^2+\left(\frac{33}{25}\right)^2}=\frac{\sqrt{6^2+33^2}}{25}=\frac{\sqrt{36+1089}}{25}=\frac{\sqrt{1125}}{25}$$

$$\sqrt{1125}=\sqrt{25\cdot 45}=5\cdot\sqrt{45}=5\cdot\sqrt{9\cdot 5}=5\cdot 3\cdot\sqrt{5}=15\sqrt{5}$$

$$\left|\frac{w_1}{w_2+1}\right|=\frac{15\sqrt{5}}{25}=\frac{3\sqrt{5}}{5}$$

25. $z_1=7+2i \qquad z_2=2-2i$

$$z_1 - z_2 = 7 + 2i - 2 + 2i = 5 + 4i$$

26. $z = 1 + 3i$

$$|z|^2 \cdot w + \overline{z} = \text{Re}(z)$$
$$(\sqrt{1+9})^2 \cdot w + 1 - 3i = 1$$
$$(\sqrt{10})^2 \cdot w = 3i$$
$$w = \frac{3i}{10}$$

27. $z = 1 + 7i$

$$\overline{z} = 1 - 7i$$

28. $(3i+2)(9-5i) = (2+3i)(9-5i) = 18 - 10i + 27i + 15 = 33 + 17i$

29. $i^3 = -i$

30. $|2+3i| = \sqrt{4+9} = \sqrt{13}$

31. $F(19)=4181 \quad F(20)=6765$

$F(21)=F(19)+F(20)=4181+6765=10946$

$F(22)=F(20)+F(21)=6765+10946=17711$

32. $f(1)=3$ og $f(n)=2f(n-1)-3$

$f(4)=2\cdot f(4-1)-3=2\cdot f(3)-3$

$f(3)=2\cdot f(3-1)-3=2\cdot f(2)-3$

$f(2)=2\cdot f(2-1)-3=2\cdot f(1)-3=2\cdot 3-3=3$

Dvs. $f(4)=2\cdot 3-3=3$

33. $P(z)=-3z-9$

$-3z-9=0$

$-3z=9$

$-z=3$

$z=-3$

34. $2z^2 - 8z + 10 = 0$

$D = 64 - 4 \cdot 2 \cdot 10 = -16$

$w^2 = D$

$w = 4i$

$z = \frac{-b \pm w}{2a} = \frac{8 \pm 4i}{4} = \begin{cases} 2+i \\ 2-i \end{cases}$

35. $P(z) = z^2 + nz + m$

$z_1 = 9 + 6i$ er rod.

$0 = (9+6i)^2 + n \cdot (9+6i) + m$

$0 = 81 + 54i + 54i - 36 + n(9+6i) + m$

$0 = 45 + 108i + 9n + 6ni + m$

$0 = (45 + 9n + m) + i(108 + 6n)$

Både den imaginære og reelle del skal være $=0$.

1. $45 + 9n + m = 0$

2. $108+6n=0$

$-6n=108$

$n=-18$

1. $45-9\cdot 18+m=0$

$45-162+m=0$

$m=-45+162=117$

36. $P(z)=(2-2i)\cdot z+2$

$P(z)=0$

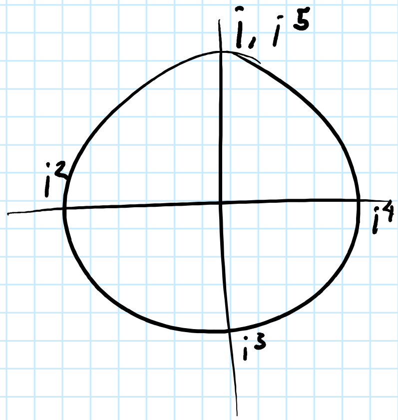
$(2-2i)\cdot z=-2$

$$z=\frac{-2}{2-2i}$$

$$z=\frac{-1}{1-i}$$

$$z=\frac{-1(1+i)}{(1-i)(1+i)}$$

$$z=\frac{-1(1+i)}{1+i-i-i^2}$$



$$z = \frac{-1(1+i)}{2}$$

$$z = -\frac{1}{2} - \frac{i}{2}$$

37. $P(z) = (2-2i)z + 3$

$$P(2i) = (2-2i)(2i) + 3$$
$$= 4i + 4 + 3$$
$$= 7 + 4i$$

38. $P(z)$ er af grad 7

- [ ] 7 forsk. komplekse rødder
- [x] 7 komplekse rødder regnet med mult.
- [x] hvis z_0 er rod er $\bar{z}_0$ også
- [ ] 7 reelle rødder
- [x] ≥ 1 reel rod

39. $P(z) = 4z^3 - 8z^2 - 11z - 3$

$z_0 = 3$ er en rod.

$a_3 = 4$, $a_2 = -8$, $a_1 = -11$, $a_0 = -3$

$b_2 = a_3 = 4$

$b_1 = a_2 + z_0 \cdot b_2 = -8 + 3 \cdot 4 = 4$

$b_0 = a_1 + z_0 \cdot b_1 = -11 + 3 \cdot 4 = 1$

$Q(z) = 4z^2 + 4z + 1$

40. $f(1) = i$ og $f(n) = f(n-1) \cdot i$

$f(172) = ?$

Går tallet op i 4?

Hvis ja $\to 1$

Hvis kun 2 $\to -1$

41. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \\ 9 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix} + s \begin{bmatrix} 6 \\ 7 \\ -9 \end{bmatrix}$

$$\begin{bmatrix} 9 \\ 8 \\ c \end{bmatrix} \quad c = ?$$

$$\begin{bmatrix} 9 \\ 8 \\ c \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \\ 9 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix} + s \begin{bmatrix} 6 \\ 7 \\ -9 \end{bmatrix}$$

konstantleddet $\begin{bmatrix} -2 \\ 7 \\ 9 \end{bmatrix}$ sættes til 0, da det ikke optræder i det homogene system

① $9 = t + 6s$

② $8 = 2t + 7s$

③ $c = -4t - 9s$

① $t = 9 - 6s$

② $8 = 2 \cdot (9 - 6s) + 7s$

$8 = 18 - 12s + 7s$

$8 = 18 - 5s$

$-10 = -5s \Rightarrow s = 2$

① $t = 9 - 6 \cdot 2$

$t = 9 - 12$

$t = -3$

③ $c = -4 \cdot (-3) - 9 \cdot 2$

$c = 12 - 18$

$c = 12 - 18$

$c = -6$

43. $B = \begin{bmatrix} 1 & -10 & -4 \\ 4 & -40 & -16 \\ -4 & 35 & 12 \end{bmatrix}$

$R_2 \leftarrow R_2 + k_1 R_1$

$R_3 \leftarrow R_3 + k_2 R_1$ $\quad k_1 = -4 \ , k_2 = 4$

$B = \begin{bmatrix} 1 & -10 & -4 \\ 0 & 0 & 0 \\ 0 & -5 & -4 \end{bmatrix}$

45. $\begin{bmatrix} i & i & -1 \\ i & i & i \\ 1 & 1 & i \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & -1-i \\ 0 & 0 & i+1 \\ 1 & 1 & i \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & i+1 \\ 1 & 1 & i \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & i \\ 0 & 0 & i+1 \\ 0 & 0 & 0 \end{bmatrix}$

rang = 2

46. Samme løsningsmengde?

46. Samme løsningsmengde?

- [ ] $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$
- [x] $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$
- [x] $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$
- [ ] $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

47. $-x_1 + 1x_2 = 2$

$3x_1 = -3$

$x_1 + x_2 = 0$

$$T = \begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & 3 \\ 1 & 1 & 0 \end{bmatrix}$$

48. $-x_1 + 3x_2 - 2 = 0$

$5x_1 + 8 = 7$

$x_1 = x_2 + 1$

$b = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} -1 & 3 & 2 \\ 5 & 0 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

49. Partikulær løsning til

$2x - 2y - 2z = 2$

$w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

50. $-2x + 2y - 5z = -10$

$2x - y + 6z = 13$

$x - y + 3z = 6$

$$T = \begin{bmatrix} -2 & 2 & -5 & -10 \\ 2 & -1 & 6 & 13 \\ 1 & -1 & 3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & -1 & 3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 3 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 7 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

51.

$$A = \begin{bmatrix} 2 & 3 & -3 \\ -2 & 1 & -2 \\ -2 & -4 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & -2 & 2 \\ 2 & 2 & 6 \\ 2 & 4 & -1 \end{bmatrix}$$

$$A+B=\begin{bmatrix}5 & 1 & -1\\ 0 & 3 & 4\\ 0 & 0 & 1\end{bmatrix}$$

$$\det(A+B) = 5\cdot\det\left(\begin{bmatrix}3 & 4\\ 0 & 1\end{bmatrix}\right) = 5\cdot 3 = 15$$

53. Hvis $\det \neq 0$

- [x] $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$
- [ ] Der findes en egentlig vektor v som opfylder $A\cdot v = 0$.
- [x] $A\cdot A^T$ har fuld rang.
- [ ] $A - A^T$ er invertibel.
- [x] Rangen af A = Rangen af A^T

54.

To matricer A og B er givet ved:

$$A=\begin{bmatrix}3 & 0 & 0\\ -2 & 2 & 0\\ -2 & -4 & -1\end{bmatrix} \text{ og } B=\begin{bmatrix}2 & -2 & 2\\ 0 & 2 & 2\\ 0 & 0 & 2\end{bmatrix}$$

Angiv de 4 nedenstående determinanter:

$\det(A) =$ -6

54.

To matricer A og B er givet ved:

$$A = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 2 & 0 \\ -2 & -4 & -1 \end{bmatrix} \text{ og } B = \begin{bmatrix} 2 & -2 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

Angiv de 4 nedenstående determinanter:

$\det(A) =$ -6

$\det(B) =$ 8

$\det(A \cdot B) =$ -48

$\det\left(A^{-1}\right) =$ -1/6

55.

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & -1 \end{bmatrix} \qquad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\det(A) = -1 \cdot \det\left(\begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}\right) - 1 \cdot \det\left(\begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}\right)$$

$$= -1 - 7 = -8$$

56.

$$a = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 5 \\ -7 \end{bmatrix} \quad c = \begin{bmatrix} 3 \\ 2 \\ x \end{bmatrix}$$

$a=\begin{bmatrix}0\\1\\0\end{bmatrix}$ $b=\begin{bmatrix}4\\5\\-2\end{bmatrix}$ $c=\begin{bmatrix}3\\2\\x\end{bmatrix}$

$$A=\begin{bmatrix}0 & 4 & 3\\1 & 5 & 2\\0 & -2 & x\end{bmatrix}$$

det(A) skal være 0 for at de er lineært afhængige.

$$-1\cdot\det\begin{bmatrix}4 & 3\\-2 & x\end{bmatrix}=-(4x+6)$$

$$4x=-6$$
$$x=-\frac{6}{4}$$
$$x=-\frac{3}{2}$$

58. $A=\begin{bmatrix}2 & 2\\-1 & 1\end{bmatrix}$ $B=\begin{bmatrix}1 & 2\\3 & 0\end{bmatrix}$

$$A\cdot B=\begin{bmatrix}2\cdot1+2\cdot3 & 2\cdot2+2\cdot0\\-1\cdot1+1\cdot3 & -1\cdot2+1\cdot0\end{bmatrix}$$
$$=\begin{bmatrix}8 & 4\\2 & -2\end{bmatrix}$$

59.

$$A = \begin{bmatrix} 6 & 1 & 5 \\ 1 & 7 & -1 \end{bmatrix}$$

$$b = \begin{bmatrix} 6 \\ 1 \\ -4 \end{bmatrix}$$

$$A \cdot b = \begin{bmatrix} 6 \cdot 6 + 1 \cdot 1 + 5 \cdot (-4) \\ 1 \cdot 6 + 7 \cdot 1 - 1 \cdot (-4) \end{bmatrix}$$

$$= \begin{bmatrix} 36 - 19 \\ 13 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 17 \\ 17 \end{bmatrix}$$

60.

$$A = \begin{bmatrix} 8 & \frac{1}{4} \\ \frac{1}{2} & 0 \end{bmatrix}$$

$$A^{-1} = A | I$$

$$= \begin{bmatrix} 8 & \frac{1}{4} & 1 & 0 \\ \frac{1}{2} & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{4} & 1 & -16 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 4 & -64 \end{bmatrix}$$