Technical Report 2

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1 Introduction

In the world of sports, it is often desirable to have some way to rank teams in order to know what the strongest teams are. This is frequently done somewhat subjectively as the concept of what constitutes a "good" team is not so well defined. However, there are some reasonable measures that can be used as the basis of a more objective ranking system. In this report, we examine two methods of ranking NHL teams. The first is the eigenranking, which finds team ratings as the dominant eigenvector of a matrix that contains head-to-head goal differentials in a particular way, and the second is the Colley ranking which is based on solving a matrix equation involving the wins and losses of each team. By computing each of these rankings and comparing them to each other and to the well-respected FiveThirtyEight Elo ranking, we can not only determine which teams are strong and which are weak, but also learn what types of teams are comparatively favored by each of the methods.

2 Eigenvector Ranking

We sorted the team names alphabetically, and assigned each team a number based on their alphabetical order. We specify the cells of our matrix as

$$a_{ij} = \frac{S_{ij}}{S_{ij} + S_{ji}} \tag{1}$$

where S_{ij} is the points earned by team i against team j.

Our data has information on the visitor team, the points earned by visitor, the home team, and the points earned by home, for each game played in the 2021-2022 season. Then we used a for loop to loop over each of the team and its opponents, where we sum up all the points team i won against team j over all of their games, and divided it over the sum of all the scores.

Let's look at the games between Pittsburgh Penguins and Tampa Bay Lightning. The number corresponding to Pittsburgh Penguins is 23, and the number corresponding to Tampa Bay Lightning is 27. In the three games played between these games, Pittsburgh Penguins won $6 + 1 + 5 = 12$ points, thus $S_{23,27} = 12$, and Tampa Bay Lightning won $2 + 5 + 1 = 8$ points, thus $S_{27,23} = 8$. $a_{23,27} = \frac{6+1+5}{(6+1+5)+(2+5+1)} = \frac{12}{20} = 0.6$, and $a_{27,23} = \frac{2+5+1}{(6+1+5)+(2+5+1)} = \frac{8}{20} = 0.4$. Note that $a_{23,27} + a_{27,23} = 1$.

	Date	Visitor	Visitor Score	Home	Home Score
1	2021-10-12	Pittsburgh Penguins	6	Tampa Bay Lightning	2
2	2021-10-26	Tampa Bay Lightning	5	Pittsburgh Penguins	1
3	2022-03-03	Pittsburgh Penguins	5	Tampa Bay Lightning	1

After generating our matrix, we subtracted the matrix with the identity matrix. Then we used the eigen function in R to calculate its dominant eigenvalue and eigenvector. Analogously to the Gould centrality index, the dominant eigenvector represents the ratings of the teams. This is because our matrix A in some way encodes how each team performed over the course of the season, so if we apply A many times to some arbitrarily chosen rating vector v, teams that fared better will have their value in v increased relative to teams that fared worse. This process is exactly the power iteration, so we know that the resulting rating is just the dominant eigenvector.

3 Colley's Ranking

The Colley's Rating Method method was developed by Dr. Wesley Colley to rank American college football teams. This method is an improvement on top of the simpler method that use the that winning rate: number of wins over total games played by a team.

$$rating_i = \frac{win_i}{total_i} \tag{2}$$

The Colley's Rating Method improvement by addressing the ties condition between teams, the strength of opponents, the unusual mathematics result such during preseason. The Colley's Rating is based on Laplace's Rule of team i given by:

$$rating_i = \frac{1 + win_i}{2 + total_i} \tag{3}$$

Follow this formula, at the beginning of the season, each team with 0 win and 0 game start with the same rating of $\frac{1}{2}$ instead of 0 given by traditional rating method. This also means that the initial accumulative rating of a team's opponents is $\sum_{j \in O} r_j = \sum_{j \in O_i} \frac{1}{2}$ where O_i is the opponent set of team i.

One team's victory is the other's defeat so the ratings between teams are interdependent. To factor this in, the winning rate of a team i is: $\frac{w_i - l_i}{2} + \sum_{j \in O_i} r_j$. Substitute this winning rate into equation (2), we have

$$r_i = 1 + \frac{\frac{w_i - l_i}{2} + \sum_{j \in O_i} r_j}{2 + t_i} \tag{4}$$

There is one equation (3) for each team. Rearrange the system of equations of all teams into a linear system $Cr = b$ format, we get:

$$\begin{aligned} r_{nx1} &: \text{the unknown Colley's rating} \\ b_{nx1} &: 1 + \frac{1}{2}(w_i - l_i) \\ w_i &: \text{total number of wins accumulated by team } i \\ l_i &: \text{total number of losses accumulated by team } i \\ n_{ij} &: \text{number of times teams } i \text{ and } j \text{ faced each other} \\ C_{nxn} &: \begin{cases} 2 + t_i, & i = j \\ -n_{ij}, & i \neq j \end{cases} \end{aligned}$$

Using the score of the visitor score and home score for each match, we calculate the winner and lose for each match and update the total number of wins, total of loses, and total game for each team.

names	count loses	count wins	net score	total game count
Anaheim Ducks	50	30	-40	80
Arizona Coyotes	57	22	-110	79
Boston Bruins	30	49	31	79
⋮	⋮	⋮	⋮	⋮
Winnipeg Jets	43	36	-12	79

Table 1: Data Summary

We construct the matrix N_{nxn} (see appendix) that keeps count of the total number of match between team i and j. The C matrix is then constructed using the information about each team total game from the Table 1 plus 2 then minus the N matrix. Using the information from Table 1 about each team's loses and wins , we calculate b vector.

The ranking then computes using *solve(C, b)* method in *R*. The result is r, the Colley's ranking vector for all team.

4 Analysis and Conclusion

As seen in the previous two sections, there is good theoretical backing for each of these two ranking systems being a valid way to gauge team strength. Consequently, one would expect the rankings to not be too dissimilar. That is, a team ranked very highly by the eigenranking should not be ranked at the bottom in the Colley ranking, and vice versa. At the same time, we don't expect the two rankings to be equal in general because the methodologies do provide distinct answers to what constitutes a "strong" team.

This is exactly what we see when comparing the results of the two ranking methods, as there are clear differences yet no team moves more than seven places up or down between the eigenranking and the Colley ranking. The rough agreement we see gives us some confidence that each of these rankings is working as expected and is indeed some measure of team strength.

To further expand on the differences between the eigenranking and the Colley ranking, let us consider what each is really measuring. In simplest terms, the Colley ranking rewards winning games against strong teams, while the eigenranking rewards outscoring strong teams over all combined matches against them. These two measures are of course correlated as teams that win frequently will tend to score many goals and teams that don't score many goals will tend to lose, but they do not correspond one-to-one. For example, we could imagine a team that loses many games by one-goal margins while winning a few games by large margins. Such a team would be more favored in the eigenranking than in the Colley ranking, and we could just as easily consider a team in the opposite circumstance that would fare better in the Colley ranking than in the eigenranking. The biggest losers and gainers when moving from the eigenranking to the Colley ranking, the Flames and the Sharks, respectively, are both explained clearly by this discrepancy in the ranking methodologies. The Flames had the second-highest goal differential in the league while only having the seventh highest number of wins, and the Stars had more wins than would be expected with their -8 goal differential.

Now, having established that the two ranking methods we investigated are roughly consistent with one another and differ in predictable fashion, it's worthwhile to compare each with a well-accepted ranking methodology. To this end, we chose to compare our results to the FiveThirtyEight NHL rankings, which are a sophisticated Elo-based model far more nuanced than either method we implemented. This method modifies each team's Elo rating on a match-by-match basis taking into account several factors including the strength of the opponent, the margin of victory or loss, the manner of victory or loss (i.e. regulation, overtime, or shootout), and the home ice advantage. We see that this ranking does not differ too dramatically from either of our rankings, and is on a team-by-team basis sometimes closer to the eigenranking and sometimes closer to the Colley ranking. This lends further confidence to our results and indicates somewhat expectedly that the nuanced approach taken by FiveThirtyEight falls somewhere between a goals-centric and wins-centric methodology.

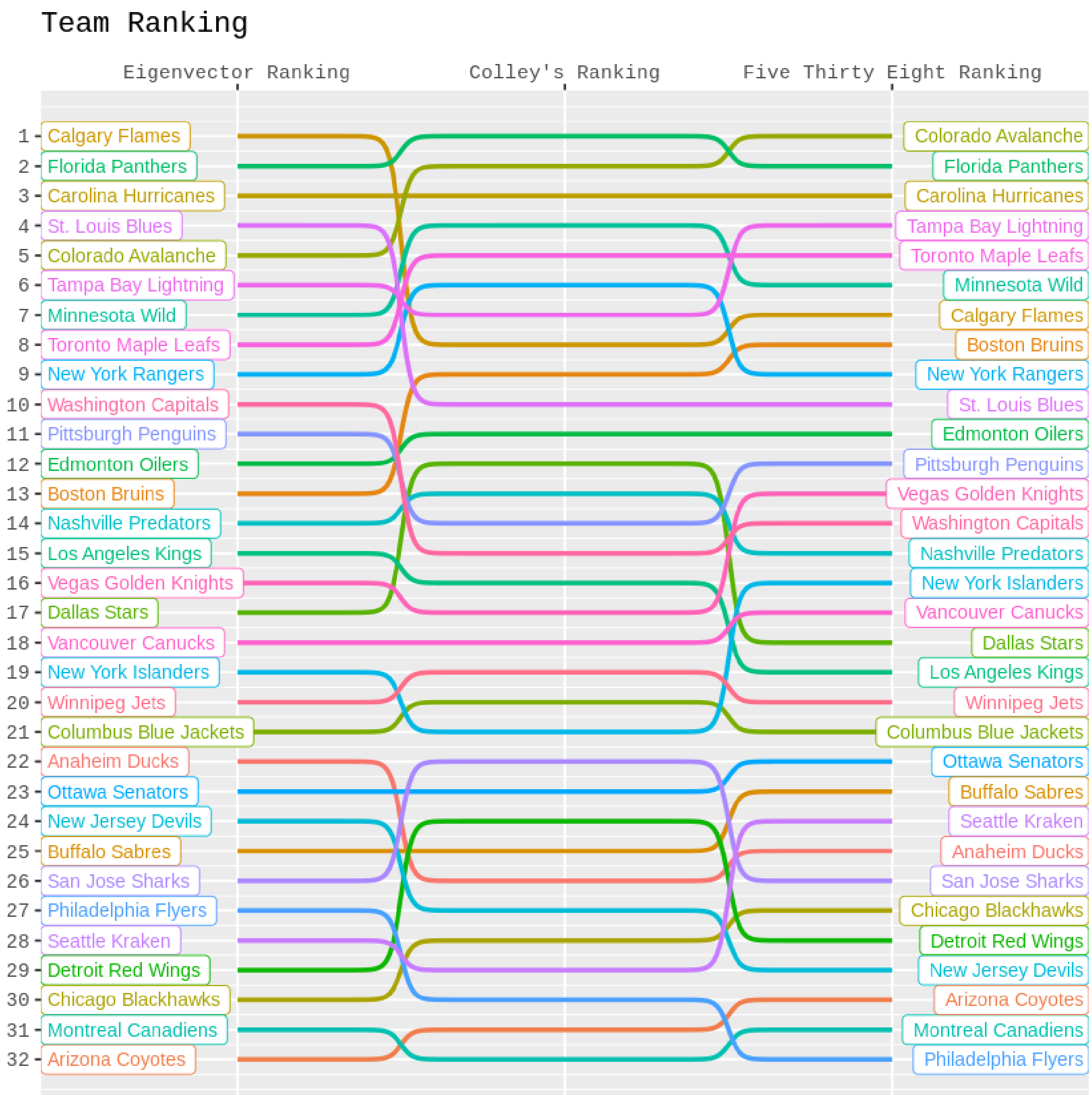

Figure 1: NHL Ranking by The Two Ranking Methods and FiveThirtyEight

5 Citation

Knight, K., 2022. The Colley matrix method for ranking. [online] Utstat.toronto.edu. Available at: ¡http://www.utstat.toronto.edu/keith/papers/colley.pdf¿ [Accessed 28 April 2022].

2021-22 NHL Schedule and Results. Hockey Reference. https://www.hockey-reference.com/leagues/NHL_2022_games.html [Accessed 26 April 2022].

2021-22 NHL Predictions. FiveThirtyEight. https://projects.fivethirtyeight.com/2022-nhl-predictions/ [Accessed 1 May 2022].

6 Appendix

6.1 Matrices

6.1.1 Eigen Ranking

$$A = \begin{bmatrix}
0.0 & 0.6 & 0.6 & 0.6 & 0.38 & 0.30 & 0.2 & 0.3 & 0.6 & 0.4 & 0.5 & 0.3 & 0.2 & 0.4 & 0.3 & 0.6 & 0.32 & 0.7 & 0.3 & 0.3 & 0.9 & 0.7 & 0.2 & 0.5 & 0.6 & 0.5 & 0.6 & 0.3 & 0.6 & 0.5 & 0.5 & 0.4 \\
0.3 & 0.0 & 0.4 & 0.3 & 0.23 & 0.36 & 0.5 & 0.4 & 0.3 & 0.4 & 0.8 & 0.2 & 0.3 & 0.4 & 0.3 & 0.7 & 0.27 & 0.5 & 0.0 & 0.3 & 0.6 & 0.3 & 0.3 & 0.5 & 0.5 & 0.4 & 0.3 & 0.6 & 0.1 & 0.3 & 0.0 & 0.4 \\
0.4 & 0.6 & 0.0 & 0.7 & 0.40 & 0.06 & 0.6 & 0.6 & 0.6 & 0.4 & 0.6 & 0.5 & 0.4 & 0.8 & 0.4 & 0.7 & 0.67 & 0.8 & 0.4 & 0.4 & 0.6 & 0.5 & 0.3 & 0.6 & 0.6 & 0.5 & 0.6 & 0.4 & 0.5 & 0.5 & 0.6 & 0.6 \\
0.4 & 0.7 & 0.3 & 0.0 & 0.17 & 0.41 & 0.5 & 0.3 & 0.4 & 0.4 & 0.3 & 0.3 & 0.3 & 0.2 & 0.6 & 0.6 & 0.67 & 0.5 & 0.5 & 0.4 & 0.3 & 0.6 & 0.5 & 0.4 & 0.3 & 0.3 & 0.3 & 0.7 & 0.7 & 0.5 & 0.4 & 0.5 \\
0.6 & 0.8 & 0.6 & 0.8 & 0.00 & 0.33 & 0.8 & 0.4 & 0.9 & 0.6 & 1.0 & 0.5 & 0.5 & 0.5 & 0.8 & 0.4 & 0.40 & 0.6 & 0.7 & 0.9 & 0.6 & 0.7 & 0.9 & 0.4 & 0.7 & 0.5 & 0.5 & 0.6 & 0.5 & 0.5 & 0.5 & 0.5 \\
0.7 & 0.6 & 0.9 & 0.6 & 0.67 & 0.00 & 0.6 & 0.5 & 0.6 & 0.3 & 0.4 & 0.7 & 0.4 & 0.7 & 0.3 & 0.9 & 0.62 & 0.5 & 0.6 & 0.6 & 0.5 & 0.6 & 0.5 & 0.5 & 0.5 & 0.7 & 0.5 & 0.5 & 0.6 & 0.6 & 0.5 & 0.7 \\
0.8 & 0.5 & 0.4 & 0.5 & 0.25 & 0.38 & 0.0 & 0.4 & 0.5 & 0.4 & 0.5 & 0.5 & 0.2 & 0.4 & 0.2 & 0.7 & 0.36 & 0.6 & 0.4 & 0.3 & 0.7 & 0.5 & 0.4 & 0.4 & 0.5 & 0.2 & 0.3 & 0.4 & 0.3 & 0.5 & 0.6 & 0.4 \\
0.7 & 0.6 & 0.4 & 0.7 & 0.62 & 0.54 & 0.6 & 0.0 & 0.4 & 0.5 & 0.7 & 0.5 & 0.4 & 0.8 & 0.6 & 0.7 & 0.50 & 0.5 & 0.6 & 0.7 & 0.5 & 0.6 & 0.6 & 0.7 & 0.6 & 0.5 & 0.6 & 0.4 & 0.7 & 0.5 & 0.4 & 0.6 \\
0.4 & 0.7 & 0.4 & 0.6 & 0.14 & 0.41 & 0.5 & 0.6 & 0.0 & 0.6 & 0.5 & 0.5 & 0.4 & 0.4 & 0.5 & 0.7 & 0.31 & 0.5 & 0.3 & 0.4 & 0.5 & 0.5 & 0.3 & 0.5 & 0.6 & 0.4 & 0.2 & 0.5 & 0.5 & 0.5 & 0.4 & 0.6 \\
0.6 & 0.6 & 0.6 & 0.6 & 0.45 & 0.67 & 0.6 & 0.5 & 0.4 & 0.0 & 0.6 & 0.6 & 0.4 & 0.4 & 0.5 & 0.5 & 0.42 & 0.6 & 0.5 & 0.4 & 0.3 & 0.7 & 0.6 & 0.5 & 0.5 & 0.4 & 0.4 & 0.3 & 0.3 & 0.4 & 0.3 & 0.5 \\
0.5 & 0.2 & 0.4 & 0.7 & 0.00 & 0.56 & 0.5 & 0.3 & 0.5 & 0.4 & 0.0 & 0.5 & 0.3 & 0.2 & 0.4 & 0.3 & 0.25 & 0.8 & 0.5 & 0.4 & 0.3 & 0.7 & 0.3 & 0.6 & 0.5 & 0.4 & 0.4 & 0.4 & 0.8 & 0.5 & 0.4 & 0.4 \\
0.7 & 0.8 & 0.5 & 0.7 & 0.48 & 0.29 & 0.5 & 0.5 & 0.5 & 0.4 & 0.5 & 0.0 & 0.3 & 0.5 & 0.2 & 0.6 & 0.75 & 0.6 & 0.6 & 0.4 & 0.4 & 0.5 & 0.7 & 0.8 & 0.6 & 0.5 & 0.5 & 0.2 & 0.6 & 0.6 & 0.6 & 0.5 \\
0.8 & 0.7 & 0.6 & 0.7 & 0.50 & 0.63 & 0.8 & 0.6 & 0.6 & 0.6 & 0.7 & 0.7 & 0.0 & 0.3 & 0.6 & 0.6 & 0.53 & 0.5 & 0.8 & 0.4 & 0.4 & 0.7 & 0.5 & 0.6 & 0.3 & 0.5 & 0.6 & 0.5 & 0.7 & 0.5 & 0.5 & 0.7 \\
0.6 & 0.6 & 0.2 & 0.8 & 0.47 & 0.33 & 0.6 & 0.2 & 0.6 & 0.6 & 0.8 & 0.5 & 0.7 & 0.0 & 0.4 & 0.7 & 0.56 & 0.6 & 0.7 & 0.6 & 0.8 & 0.6 & 0.7 & 0.3 & 0.5 & 0.3 & 0.4 & 0.5 & 0.3 & 0.5 & 0.4 & 0.5 \\
0.6 & 0.7 & 0.6 & 0.4 & 0.25 & 0.67 & 0.8 & 0.4 & 0.5 & 0.5 & 0.6 & 0.8 & 0.4 & 0.6 & 0.0 & 0.8 & 0.34 & 0.7 & 0.6 & 0.7 & 0.5 & 0.6 & 0.5 & 0.5 & 0.6 & 0.4 & 0.5 & 0.5 & 0.6 & 0.5 & 0.7 & 0.5 \\
0.4 & 0.3 & 0.3 & 0.4 & 0.60 & 0.08 & 0.3 & 0.3 & 0.3 & 0.5 & 0.7 & 0.4 & 0.4 & 0.3 & 0.2 & 0.0 & 0.56 & 0.4 & 0.3 & 0.3 & 0.5 & 0.5 & 0.4 & 0.4 & 0.3 & 0.4 & 0.5 & 0.6 & 0.4 & 0.4 & 0.3 & 0.3 \\
0.7 & 0.7 & 0.3 & 0.3 & 0.60 & 0.38 & 0.6 & 0.5 & 0.7 & 0.6 & 0.8 & 0.2 & 0.5 & 0.4 & 0.7 & 0.4 & 0.00 & 0.6 & 0.6 & 0.4 & 0.7 & 0.5 & 0.6 & 0.9 & 0.5 & 0.4 & 0.3 & 0.5 & 0.5 & 0.3 & 0.3 & 0.5 \\
0.3 & 0.5 & 0.2 & 0.5 & 0.35 & 0.53 & 0.4 & 0.5 & 0.5 & 0.4 & 0.2 & 0.4 & 0.5 & 0.4 & 0.3 & 0.6 & 0.36 & 0.0 & 0.5 & 0.5 & 0.3 & 0.5 & 0.6 & 0.4 & 0.5 & 0.6 & 0.5 & 0.3 & 0.6 & 0.5 & 0.4 & 0.3 \\
0.7 & 1.0 & 0.6 & 0.5 & 0.29 & 0.37 & 0.6 & 0.4 & 0.7 & 0.5 & 0.5 & 0.4 & 0.2 & 0.3 & 0.4 & 0.7 & 0.42 & 0.5 & 0.0 & 0.4 & 0.8 & 0.6 & 0.4 & 0.4 & 0.5 & 0.3 & 0.2 & 0.2 & 0.6 & 0.6 & 0.3 & 0.8 \\
0.7 & 0.7 & 0.6 & 0.6 & 0.08 & 0.41 & 0.7 & 0.3 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 & 0.4 & 0.3 & 0.7 & 0.60 & 0.5 & 0.6 & 0.0 & 0.7 & 0.7 & 0.7 & 1.0 & 0.7 & 0.4 & 0.7 & 0.6 & 0.3 & 0.3 & 0.5 & 0.9 \\
0.1 & 0.4 & 0.4 & 0.7 & 0.44 & 0.53 & 0.3 & 0.5 & 0.5 & 0.7 & 0.7 & 0.6 & 0.6 & 0.2 & 0.5 & 0.5 & 0.30 & 0.7 & 0.2 & 0.3 & 0.0 & 0.5 & 0.5 & 0.3 & 0.5 & 0.5 & 0.5 & 0.3 & 0.4 & 0.2 & 0.5 & 0.6 \\
0.3 & 0.7 & 0.5 & 0.4 & 0.29 & 0.39 & 0.5 & 0.4 & 0.5 & 0.3 & 0.3 & 0.5 & 0.3 & 0.4 & 0.4 & 0.5 & 0.50 & 0.5 & 0.4 & 0.3 & 0.5 & 0.0 & 0.4 & 0.4 & 0.8 & 0.5 & 0.2 & 0.3 & 0.5 & 0.6 & 0.4 & 0.8 \\
0.8 & 0.7 & 0.7 & 0.5 & 0.14 & 0.50 & 0.6 & 0.4 & 0.7 & 0.4 & 0.7 & 0.3 & 0.5 & 0.3 & 0.5 & 0.6 & 0.40 & 0.4 & 0.6 & 0.3 & 0.5 & 0.6 & 0.0 & 0.6 & 0.7 & 0.6 & 0.6 & 0.7 & 0.8 & 0.7 & 0.4 & 0.7 \\
0.5 & 0.5 & 0.4 & 0.6 & 0.58 & 0.50 & 0.6 & 0.3 & 0.5 & 0.5 & 0.4 & 0.2 & 0.4 & 0.7 & 0.5 & 0.6 & 0.08 & 0.6 & 0.6 & 0.0 & 0.7 & 0.6 & 0.4 & 0.0 & 0.5 & 0.3 & 0.2 & 0.5 & 0.4 & 0.4 & 0.4 & 0.5 \\
0.4 & 0.5 & 0.4 & 0.7 & 0.35 & 0.50 & 0.5 & 0.4 & 0.4 & 0.5 & 0.5 & 0.4 & 0.7 & 0.5 & 0.4 & 0.7 & 0.50 & 0.5 & 0.5 & 0.3 & 0.5 & 0.2 & 0.3 & 0.5 & 0.0 & 0.2 & 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.3 \\
0.5 & 0.6 & 0.5 & 0.7 & 0.50 & 0.29 & 0.8 & 0.5 & 0.6 & 0.6 & 0.6 & 0.5 & 0.5 & 0.7 & 0.6 & 0.6 & 0.62 & 0.4 & 0.7 & 0.6 & 0.5 & 0.5 & 0.4 & 0.7 & 0.8 & 0.0 & 0.5 & 0.6 & 0.7 & 0.7 & 0.8 & 0.4 \\
0.4 & 0.7 & 0.4 & 0.7 & 0.50 & 0.47 & 0.7 & 0.4 & 0.8 & 0.6 & 0.6 & 0.5 & 0.4 & 0.6 & 0.5 & 0.5 & 0.69 & 0.5 & 0.8 & 0.3 & 0.5 & 0.8 & 0.4 & 0.8 & 0.9 & 0.5 & 0.0 & 0.6 & 0.7 & 0.5 & 0.5 & 0.5 \\
0.7 & 0.4 & 0.6 & 0.3 & 0.40 & 0.47 & 0.6 & 0.6 & 0.5 & 0.7 & 0.6 & 0.8 & 0.5 & 0.5 & 0.5 & 0.4 & 0.50 & 0.7 & 0.8 & 0.4 & 0.7 & 0.7 & 0.3 & 0.5 & 0.7 & 0.4 & 0.4 & 0.0 & 0.4 & 0.7 & 0.6 & 0.5 \\
0.4 & 0.8 & 0.5 & 0.3 & 0.48 & 0.38 & 0.7 & 0.3 & 0.5 & 0.7 & 0.2 & 0.4 & 0.3 & 0.7 & 0.4 & 0.6 & 0.47 & 0.4 & 0.4 & 0.7 & 0.6 & 0.5 & 0.2 & 0.6 & 0.7 & 0.3 & 0.3 & 0.6 & 0.0 & 0.5 & 0.5 & 0.7 \\
0.5 & 0.7 & 0.5 & 0.5 & 0.50 & 0.38 & 0.5 & 0.5 & 0.5 & 0.6 & 0.5 & 0.4 & 0.5 & 0.5 & 0.5 & 0.6 & 0.68 & 0.5 & 0.4 & 0.7 & 0.8 & 0.4 & 0.3 & 0.6 & 0.7 & 0.3 & 0.5 & 0.3 & 0.5 & 0.0 & 0.6 & 0.3 \\
0.5 & 1.0 & 0.4 & 0.6 & 0.50 & 0.54 & 0.4 & 0.6 & 0.6 & 0.7 & 0.6 & 0.4 & 0.5 & 0.6 & 0.3 & 0.7 & 0.69 & 0.6 & 0.7 & 0.5 & 0.5 & 0.6 & 0.6 & 0.6 & 0.5 & 0.2 & 0.5 & 0.4 & 0.5 & 0.4 & 0.0 & 0.6 \\
0.6 & 0.6 & 0.4 & 0.5 & 0.5 & 0.3 & 0.6 & 0.4 & 0.4 & 0.5 & 0.6 & 0.5 & 0.3 & 0.5 & 0.5 & 0.7 & 0.5 & 0.7 & 0.2 & 0.1 & 0.4 & 0.2 & 0.3 & 0.5 & 0.7 & 0.6 & 0.5 & 0.5 & 0.3 & 0.7 & 0.4 & 0.0
\end{bmatrix}$$

Figure 2: A Matrix for Eigen Ranking Display of 1 Decimal Place

6.1.2 Colley's Ranking

$$N = \begin{bmatrix}
. & 3 & 2 & 2 & 4 & 2 & 3 & 3 & 2 & 2 & 2 & 3 & 2 & 4 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 2 & 2 & 4 & 42 & 3 \\
3 & . & 2 & 2 & 3 & 2 & 4 & 4 & 2 & 3 & 2 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 4 & 2 & 2 & 3 & 32 & 4 \\
2 & 2 & . & 3 & 2 & 3 & 2 & 2 & 3 & 2 & 4 & 2 & 2 & 2 & 2 & 4 & 2 & 3 & 3 & 3 & 4 & 3 & 3 & 2 & 2 & 2 & 4 & 2 & 2 & 23 & 2 \\
2 & 2 & 3 & . & 2 & 3 & 1 & 2 & 3 & 2 & 4 & 2 & 4 & 2 & 2 & 4 & 2 & 3 & 3 & 3 & 3 & 3 & 3 & 2 & 2 & 2 & 3 & 4 & 2 & 23 & 2 \\
4 & 3 & 2 & 2 & . & 2 & 3 & 3 & 2 & 3 & 2 & 4 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 4 & 3 & 2 & 2 & 4 & 32 & 2 \\
2 & 2 & 3 & 3 & 2 & . & 2 & 2 & 4 & 2 & 3 & 2 & 3 & 2 & 2 & 3 & 2 & 3 & 3 & 3 & 3 & 4 & 3 & 2 & 2 & 2 & 3 & 3 & 2 & 24 & 2 \\
3 & 4 & 2 & 1 & 3 & 2 & . & 4 & 2 & 3 & 2 & 3 & 2 & 3 & 4 & 2 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 4 & 2 & 2 & 3 & 22 & 3 \\
3 & 4 & 2 & 2 & 3 & 2 & 4 & . & 2 & 3 & 2 & 3 & 2 & 3 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 2 & 2 & 2 & 3 & 32 & 4 \\
2 & 2 & 3 & 3 & 2 & 4 & 2 & 2 & . & 2 & 3 & 2 & 3 & 2 & 2 & 3 & 2 & 4 & 4 & 3 & 3 & 3 & 3 & 2 & 2 & 2 & 1 & 3 & 2 & 24 & 2 \\
2 & 3 & 2 & 2 & 3 & 2 & 3 & 3 & 2 & . & 2 & 3 & 2 & 3 & 4 & 2 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 4 & 2 & 2 & 3 & 22 & 4 \\
2 & 2 & 4 & 4 & 2 & 3 & 2 & 2 & 3 & 2 & . & 2 & 4 & 2 & 2 & 3 & 2 & 2 & 3 & 3 & 3 & 3 & 3 & 2 & 2 & 2 & 4 & 3 & 2 & 23 & 2 \\
3 & 3 & 2 & 2 & 4 & 2 & 3 & 3 & 2 & 3 & 2 & . & 2 & 4 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 1 & 3 & 3 & 3 & 2 & 2 & 3 & 42 & 3 \\
2 & 2 & 2 & 4 & 2 & 3 & 2 & 2 & 3 & 2 & 4 & 2 & . & 2 & 2 & 3 & 2 & 3 & 3 & 3 & 3 & 3 & 3 & 2 & 2 & 2 & 4 & 3 & 2 & 23 & 2 \\
4 & 3 & 2 & 2 & 3 & 2 & 3 & 3 & 2 & 3 & 2 & 4 & 2 & . & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 3 & 3 & 2 & 2 & 2 & 42 & 3 \\
3 & 2 & 2 & 2 & 2 & 2 & 4 & 3 & 2 & 4 & 2 & 3 & 2 & 3 & . & 2 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 2 & 2 & 3 & 32 & 4 \\
2 & 2 & 4 & 4 & 2 & 3 & 2 & 2 & 3 & 2 & 3 & 2 & 3 & 2 & 2 & . & 2 & 3 & 3 & 2 & 4 & 3 & 3 & 2 & 2 & 2 & 3 & 4 & 2 & 23 & 2 \\
3 & 2 & 2 & 2 & 2 & 2 & 4 & 3 & 2 & 4 & 2 & 3 & 2 & 3 & 4 & 2 & . & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 4 & 2 & 2 & 3 & 32 & 3 \\
2 & 2 & 3 & 3 & 2 & 3 & 2 & 2 & 4 & 2 & 2 & 2 & 3 & 2 & 2 & 3 & 2 & . & 4 & 4 & 2 & 3 & 4 & 2 & 2 & 2 & 3 & 3 & 2 & 23 & 2 \\
2 & 2 & 3 & 3 & 2 & 3 & 2 & 2 & 4 & 2 & 3 & 2 & 3 & 2 & 2 & 3 & 2 & 4 & . & 4 & 3 & 4 & 3 & 2 & 2 & 2 & 2 & 3 & 2 & 22 & 2 \\
2 & 2 & 3 & 3 & 2 & 3 & 2 & 2 & 3 & 2 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 4 & 4 & . & 3 & 4 & 4 & 2 & 2 & 2 & 3 & 3 & 2 & 22 & 2 \\
2 & 2 & 4 & 3 & 2 & 3 & 2 & 2 & 3 & 2 & 3 & 2 & 3 & 2 & 2 & 4 & 2 & 2 & 3 & 3 & . & 2 & 3 & 2 & 2 & 2 & 4 & 4 & 2 & 23 & 2 \\
2 & 2 & 3 & 3 & 2 & 4 & 2 & 2 & 3 & 2 & 3 & 2 & 3 & 2 & 2 & 3 & 2 & 3 & 4 & 4 & 2 & . & 4 & 2 & 2 & 2 & 3 & 3 & 2 & 24 & 1 \\
2 & 2 & 3 & 3 & 2 & 3 & 2 & 2 & 3 & 2 & 3 & 1 & 3 & 2 & 2 & 3 & 2 & 4 & 3 & 4 & 3 & 4 & . & 2 & 2 & 2 & 3 & 3 & 2 & 24 & 2 \\
3 & 3 & 2 & 2 & 4 & 2 & 3 & 3 & 2 & 3 & 2 & 3 & 2 & 4 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & . & 3 & 3 & 2 & 2 & 3 & 32 & 3 \\
3 & 3 & 2 & 2 & 4 & 2 & 3 & 3 & 2 & 3 & 2 & 3 & 2 & 3 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & . & 3 & 2 & 2 & 3 & 42 & 2 \\
3 & 4 & 2 & 2 & 3 & 2 & 4 & 2 & 2 & 4 & 2 & 3 & 2 & 3 & 3 & 2 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & . & 2 & 2 & 3 & 22 & 4 \\
2 & 2 & 4 & 3 & 2 & 3 & 2 & 2 & 1 & 2 & 4 & 2 & 4 & 2 & 2 & 3 & 2 & 3 & 2 & 3 & 4 & 3 & 3 & 2 & 2 & 2 & . & 4 & 2 & 23 & 2 \\
2 & 2 & 2 & 4 & 2 & 3 & 2 & 2 & 3 & 2 & 3 & 2 & 3 & 2 & 2 & 4 & 2 & 3 & 3 & 3 & 4 & 3 & 3 & 2 & 2 & 2 & 4 & . & 2 & 23 & 2 \\
4 & 3 & 2 & 2 & 4 & 2 & 3 & 3 & 2 & 3 & 2 & 3 & 2 & 2 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 2 & 2 & . & 42 & 3 \\
4 & 3 & 2 & 2 & 3 & 2 & 2 & 3 & 2 & 2 & 2 & 4 & 2 & 4 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 4 & 2 & 2 & 2 & 4 & .2 & 3 \\
2 & 2 & 3 & 3 & 2 & 4 & 2 & 2 & 4 & 2 & 3 & 2 & 3 & 2 & 2 & 3 & 2 & 3 & 2 & 2 & 3 & 4 & 4 & 2 & 2 & 2 & 3 & 3 & 2 & 2. & 2 \\
3 & 4 & 2 & 2 & 2 & 2 & 3 & 4 & 2 & 4 & 2 & 3 & 2 & 3 & 4 & 2 & 3 & 2 & 2 & 2 & 2 & 1 & 2 & 3 & 2 & 4 & 2 & 2 & 3 & 32 & .
\end{bmatrix}$$

Figure 3: N matrix in the Colley's Ranking of NHL 2021-2022

$$C = \begin{bmatrix}
82 & -3 & -2 & -2 & -4 & -2 & -3 & -3 & -2 & -2 & -2 & -3 & -2 & -4 & -3 & -2 & -3 & -2 & -2 & -2 & -2 & -2 & -2 & -3 & -3 & -3 & -2 & -2 & -4 & -4 & -2 & -3 \\
-3 & 81 & -2 & -2 & -3 & -2 & -4 & -4 & -2 & -3 & -2 & -3 & -2 & -3 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -3 & -3 & -4 & -2 & -2 & -3 & -3 & -2 & -4 \\
-2 & -2 & 81 & -3 & -2 & -3 & -2 & -2 & -3 & -2 & -4 & -2 & -2 & -2 & -2 & -4 & -2 & -3 & -3 & -3 & -4 & -3 & -3 & -2 & -2 & -2 & -4 & -2 & -2 & -2 & -3 & -2 \\
-2 & -2 & -3 & 82 & -2 & -3 & -1 & -2 & -3 & -2 & -4 & -2 & -4 & -2 & -2 & -4 & -2 & -3 & -3 & -3 & -3 & -3 & -3 & -2 & -2 & -2 & -3 & -4 & -2 & -2 & -3 & -2 \\
-4 & -3 & -2 & -2 & 81 & -2 & -3 & -3 & -2 & -3 & -2 & -4 & -2 & -3 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -4 & -4 & -3 & -2 & -2 & -4 & -3 & -2 & -2 \\
-2 & -2 & -3 & -3 & -2 & 82 & -2 & -2 & -4 & -2 & -3 & -2 & -3 & -2 & -2 & -3 & -2 & -3 & -3 & -3 & -3 & -4 & -3 & -2 & -2 & -2 & -3 & -3 & -2 & -2 & -4 & -2 \\
-3 & -4 & -2 & -1 & -3 & -2 & 82 & -4 & -2 & -3 & -2 & -3 & -2 & -3 & -4 & -2 & -4 & -2 & -2 & -2 & -2 & -2 & -2 & -3 & -3 & -4 & -2 & -2 & -3 & -2 & -2 & -3 \\
-3 & -4 & -2 & -2 & -3 & -2 & -4 & 81 & -2 & -3 & -2 & -3 & -2 & -3 & -3 & -2 & -3 & -2 & -2 & -2 & -2 & -2 & -2 & -3 & -3 & -2 & -2 & -2 & -3 & -3 & -2 & -4 \\
-2 & -2 & -3 & -3 & -2 & -4 & -2 & -2 & 81 & -2 & -3 & -2 & -3 & -2 & -2 & -3 & -2 & -4 & -4 & -3 & -3 & -3 & -3 & -2 & -2 & -2 & -1 & -3 & -2 & -2 & -4 & -2 \\
-2 & -3 & -2 & -2 & -3 & -2 & -3 & -3 & -2 & 81 & -2 & -3 & -2 & -3 & -4 & -2 & -4 & -2 & -2 & -2 & -2 & -2 & -2 & -3 & -3 & -4 & -2 & -2 & -3 & -2 & -2 & -4 \\
-2 & -2 & -4 & -4 & -2 & -3 & -2 & -2 & -3 & -2 & 82 & -2 & -4 & -2 & -2 & -3 & -2 & -2 & -3 & -3 & -3 & -3 & -3 & -2 & -2 & -2 & -4 & -3 & -2 & -2 & -3 & -2 \\
-3 & -3 & -2 & -2 & -4 & -2 & -3 & -3 & -2 & -3 & -2 & 81 & -2 & -4 & -3 & -2 & -3 & -2 & -2 & -2 & -2 & -2 & -1 & -3 & -3 & -3 & -2 & -2 & -3 & -4 & -2 & -3 \\
-2 & -2 & -2 & -4 & -2 & -3 & -2 & -2 & -3 & -2 & -4 & -2 & 81 & -2 & -2 & -3 & -2 & -3 & -3 & -3 & -3 & -3 & -3 & -2 & -2 & -2 & -4 & -3 & -2 & -2 & -3 & -2 \\
-4 & -3 & -2 & -2 & -3 & -2 & -3 & -3 & -2 & -3 & -2 & -4 & -2 & 82 & -3 & -2 & -3 & -2 & -2 & -2 & -2 & -2 & -2 & -4 & -3 & -3 & -2 & -2 & -2 & -4 & -2 & -3 \\
-3 & -2 & -2 & -2 & -2 & -2 & -4 & -3 & -2 & -4 & -2 & -3 & -2 & -3 & 81 & -2 & -4 & -2 & -2 & -2 & -2 & -2 & -2 & -3 & -3 & -3 & -2 & -2 & -3 & -3 & -2 & -4 \\
-2 & -2 & -4 & -4 & -2 & -3 & -2 & -2 & -3 & -2 & -3 & -2 & -3 & -2 & -2 & 82 & -2 & -3 & -3 & -2 & -4 & -3 & -3 & -2 & -2 & -2 & -3 & -4 & -2 & -2 & -3 & -2 \\
-3 & -2 & -2 & -2 & -2 & -2 & -4 & -3 & -2 & -4 & -2 & -3 & -2 & -3 & -4 & -2 & 81 & -2 & -2 & -2 & -2 & -2 & -2 & -3 & -3 & -4 & -2 & -2 & -3 & -3 & -2 & -3 \\
-2 & -2 & -3 & -3 & -2 & -3 & -2 & -2 & -4 & -2 & -2 & -2 & -3 & -2 & -2 & -3 & -2 & 81 & -4 & -4 & -2 & -3 & -4 & -2 & -2 & -2 & -3 & -3 & -2 & -2 & -3 & -2 \\
-2 & -2 & -3 & -3 & -2 & -3 & -2 & -2 & -4 & -2 & -3 & -2 & -3 & -2 & -2 & -3 & -2 & -4 & 81 & -4 & -3 & -4 & -3 & -2 & -2 & -2 & -2 & -3 & -2 & -2 & -2 & -2 \\
-2 & -2 & -3 & -3 & -2 & -3 & -2 & -2 & -3 & -2 & -3 & -2 & -3 & -2 & -2 & -2 & -2 & -4 & -4 & 81 & -3 & -4 & -4 & -2 & -2 & -2 & -3 & -3 & -2 & -2 & -2 & -2 \\
-2 & -2 & -4 & -3 & -2 & -3 & -2 & -2 & -3 & -2 & -3 & -2 & -3 & -2 & -2 & -4 & -2 & -2 & -3 & -3 & 81 & -2 & -3 & -2 & -2 & -2 & -4 & -4 & -2 & -2 & -3 & -2 \\
-2 & -2 & -3 & -3 & -2 & -4 & -2 & -2 & -3 & -2 & -3 & -2 & -3 & -2 & -2 & -3 & -2 & -3 & -4 & -4 & -2 & 82 & -4 & -2 & -2 & -2 & -3 & -3 & -2 & -2 & -4 & -1 \\
-2 & -2 & -3 & -3 & -2 & -3 & -2 & -2 & -3 & -2 & -3 & -1 & -3 & -2 & -2 & -3 & -2 & -4 & -3 & -4 & -3 & -4 & 82 & -2 & -2 & -2 & -3 & -3 & -2 & -2 & -4 & -2 \\
-3 & -3 & -2 & -2 & -4 & -2 & -3 & -3 & -2 & -3 & -2 & -3 & -2 & -4 & -3 & -2 & -3 & -2 & -2 & -2 & -2 & -2 & -2 & 81 & -3 & -3 & -2 & -2 & -3 & -3 & -2 & -3 \\
-3 & -3 & -2 & -2 & -4 & -2 & -3 & -3 & -2 & -3 & -2 & -3 & -2 & -3 & -3 & -2 & -3 & -2 & -2 & -2 & -2 & -2 & -2 & -3 & 80 & -3 & -2 & -2 & -3 & -4 & -2 & -2 \\
-3 & -4 & -2 & -2 & -3 & -2 & -4 & -2 & -2 & -4 & -2 & -3 & -2 & -3 & -3 & -2 & -4 & -2 & -2 & -2 & -2 & -2 & -2 & -3 & -3 & 82 & -2 & -2 & -3 & -2 & -2 & -4 \\
-2 & -2 & -4 & -3 & -2 & -3 & -2 & -2 & -1 & -2 & -4 & -2 & -4 & -2 & -2 & -3 & -2 & -3 & -2 & -3 & -4 & -3 & -3 & -2 & -2 & -2 & 81 & -4 & -2 & -2 & -3 & -2 \\
-2 & -2 & -2 & -4 & -2 & -3 & -2 & -2 & -3 & -2 & -3 & -2 & -3 & -2 & -2 & -4 & -2 & -3 & -3 & -3 & -4 & -3 & -3 & -2 & -2 & -2 & -4 & 82 & -2 & -2 & -3 & -2 \\
-4 & -3 & -2 & -2 & -4 & -2 & -3 & -3 & -2 & -3 & -2 & -3 & -2 & -2 & -3 & -2 & -3 & -2 & -2 & -2 & -2 & -2 & -2 & -3 & -3 & -3 & -2 & -2 & 81 & -4 & -2 & -3 \\
-4 & -3 & -2 & -2 & -3 & -2 & -2 & -3 & -2 & -2 & -2 & -4 & -2 & -4 & -3 & -2 & -3 & -2 & -2 & -2 & -2 & -2 & -2 & -3 & -4 & -2 & -2 & -2 & -4 & 81 & -2 & -3 \\
-2 & -2 & -3 & -3 & -2 & -4 & -2 & -2 & -4 & -2 & -3 & -2 & -3 & -2 & -2 & -3 & -2 & -3 & -2 & -2 & -3 & -4 & -4 & -2 & -2 & -2 & -3 & -3 & -2 & -2 & 81 & -2 \\
-3 & -4 & -2 & -2 & -2 & -2 & -3 & -4 & -2 & -4 & -2 & -3 & -2 & -3 & -4 & -2 & -3 & -2 & -2 & -2 & -2 & -1 & -2 & -3 & -2 & -4 & -2 & -2 & -3 & -3 & -2 & 81
\end{bmatrix}$$

Figure 4: Colley's matrix in the Colley's Ranking of NHL 2021-2022

6.2 Code

6.2.1 Data

```
library(Matrix)
library(ggplot2)
library(tidyverse)
library(dplyr)
library(tibble)
install.packages("janitor")
library(janitor)
source('https://www.macalester.edu/~dshuman1/365/365Functions.r')
```

Figure 5: Libraries

```
# ------------------------ Read Data ------------------------#
data <- as.data.frame(read.csv('/content/nhl2122.csv')) %>%
    clean_names() %>%
    rename(visitor_score = g, home_score = g_1) %>%
    select(-c(x, att, log, notes, date))

# ------------------------ Team Name Index ------------------------#
teams <- sort(union(unique(data$visitor), unique(data$home)))

# ------------------------ Helper Function ------------------------#
INDEXOF = function ( s ) {match(s, teams)}

# ------------------------ Data Wrangling ------------------------#
data_summary = data %>% summarise(
    win = ifelse(visitor_score > home_score, visitor, home),
    lose = ifelse(visitor_score < home_score, visitor, home),
    score = abs(visitor_score - home_score))%>%
    pivot_longer(   cols=c(win, lose),
                    values_to = 'names',
                    names_to = 'result') %>%
    mutate(result = as.factor(result))%>%
    group_by(names, result) %>%
    summarise(  diff_by_total = sum(score),
                count = n()) %>%
    pivot_wider(id_cols = names,
                names_from = result,
                values_from = c(diff_by_total, count)) %>%
    mutate( net_score = diff_by_total_win - diff_by_total_lose,
            total_game_count = sum(count_lose, count_win)) %>%
    select(-c(diff_by_total_lose, diff_by_total_win)) %>%
    arrange(names)
```

Figure 6: Data Wrangling

6.2.2 Eigenvector Ranking

```
# calculate score
data_sum = data %>%
   # group to collapse the multiple games by (visitor vs home)
   group_by(visitor, home) %>%
    # sum to between  multiple games by (visitor vs home)
    summarize(  visitor_score = sum(visitor_score) ,
                home_score = sum(home_score))
#------------ A matrix ------------#
A = diag(1, nrow= length(teams))

#------------ Score of Each Team Against Each Other ------------#
#  score between team in (HOME vs VISITOR) and (VISITOR vs HOME)
team_score = matrix(0, nrow= length(teams), ncol= length(teams))
for(i in 1:nrow(data_sum)){
  visitor_index = INDEXOF(data_sum$visitor[i])
  home_index = INDEXOF(data_sum$home[i])
  # add score to visitor team
  team_score[visitor_index, home_index] =  team_score[visitor_index, home_index] +
                                            data_sum$visitor_score[i]
  # add score to visitor team
  team_score[home_index, visitor_index] =  team_score[home_index, visitor_index] +
                                            data_sum$home_score[i]
}

# ------------Total Score between team combined ------------#
total_score = t(team_score) + team_score

# ------------ Percentage s_ij ------------#
A =  team_score/total_score
```

Figure 7: A Matrix For Eigen Ranking

```
#————————— Eigen of A matrix —————————#
A.eigen = eigen(A)
A.lambda = as.numeric(A.eigen$values[1])
A.eigenvector = abs(as.numeric( A.eigen$vectors[,1]))

eigen_result = data.frame(eigen_ranking = A.eigenvector) %>%
               mutate(names = data_summary$names)%>% # add corresponding team name
               arrange(desc(eigen_ranking)) %>%      # reorder team based on ranking score
               mutate(eigen_rank = row_number())   # assign rank to teams
```

Figure 8: Eigen of A Matrix

6.2.3 Colley's Ranking

```
#————————— N matrix —————————#
# initialize matrix
n_matrix = Matrix(0, nrow = length(data_summary$names), ncol = length(data_summary$names))

# Populate count
for(i in 1:nrow(data)) {     # for-loop over rows
  team_i = data$visitor[i]   # visitor team name
  team_j = data$home[i]      # home team name

  # count the time both team meets
  n_matrix[INDEXOF(team_i), INDEXOF(team_j)] = n_matrix[INDEXOF(team_i), INDEXOF(team_j)] +
      1

  # both count are the same for N_ji and N_ji
  n_matrix[INDEXOF(team_j), INDEXOF(team_i)] = n_matrix[INDEXOF(team_i), INDEXOF(team_j)]
}
```

Figure 9: N Matrix

```
#————————— C matrix —————————#
C_matrix = diag(2 + data_summary$total_game_count, ncol = length(teams) ) - n_matrix
```

Figure 10: Colley's Matrix

```
#————————— b vector —————————#
colley_b_vector  = 1 + 0.5*(data_summary$count_win - data_summary$count_lose)
```

Figure 11: b Vector

```
#————————— r vector —————————#
colley_ranking_vector = solve(C_matrix, colley_b_vector)
colley_result = data.frame(colley_ranking = as.matrix(colley_ranking_vector)) %>%
                    mutate(names = data_summary$names) %>%   # add corresponding team names
                    arrange(desc(colley_ranking)) %>%        # reorder based on ranking score
                    mutate(colley_rank = row_number()) %>%   # assign ranking
                    arrange(names)                           # reorder by team names
```

Figure 12: Colley's Ranking Result

6.2.4 Plotting Result

```
#————————— Combine Result —————————#
# the ranking based on two systems listed by team names
ranking = inner_join(eigen_result, colley_result) %>%
        inner_join(fte_ranking) %>%
        select(-c(eigen_ranking, colley_ranking))

#————————— Data for Plotting —————————#
plot_data = ranking %>%
    pivot_longer(cols = c(eigen_rank,colley_rank, fte_rank),
                 names_to = "ranking_type",
                 values_to = "ranking_order") %>%
    mutate(ranking_type = fct_relevel(ranking_type,
                 c("eigen_rank", "colley_rank", 'fte_rank')))
```

Figure 13: Ranking Comparison

```
#————————— Library —————————#
library("ggbump")

#————————— Plotting —————————#
```

```
plot_data %>%
 ggplot(aes( x=ranking_type, y = ranking_order, color = as.factor(names), group=names)) +
 geom_bump( size = 1,
            smooth = 30,
            show.legend = T) +
 geom_label(data = subset(plot_data, ranking_type == "eigen_rank"),
            aes(     x= ranking_type,
                     label = names,
                     y = ranking_order),
            nudge_x = -0.6, hjust = 0, size =3) +
 geom_label(data = subset(plot_data,ranking_type == "fte_rank"),
            aes(     x= ranking_type,
                     label = names,
                     y = ranking_order),
            nudge_x = 0.6, hjust = 1, size =3)  +
 scale_x_discrete(labels = c("Eigenvector Ranking",
                             "Colley's Ranking",
                             "Five Thirty Eight Ranking"),
 position = "top")+
 scale_y_reverse(breaks = 1:32)+
 labs(title = "Team Ranking",
      x = element_blank(),
      y = element_blank()) +
 theme(legend.position = 'none',
        text=element_text(size=11,  family="Menlo"))
```

Figure 14: Plotting Comparison