Polynomial evaluation

$p(x) = 9x^4 + 3x^3 - 2x^2 + 5x + 7$

$p(2) = ?$

Method 1: (Halverson)

$9 \cdot 2 \cdot 2 \cdot 2 \cdot 2 + 3 \cdot 2 \cdot 2 \cdot 2 - 2 \cdot 2 \cdot 2 + 5 \cdot 2 + 7 = 177$

10 multiplications
4 add/subtract

Method 2:

$2 \cdot 2 = (2)^2$

$2 \cdot (2^2) = 2^3$

$2^3 \cdot 2 = 2^4$

7 multiplications
4 add/subtract

$9 \cdot 2^4 + 3 \cdot 2^3 - 2 \cdot 2^2 + 5 \cdot 2 + 7$

Method 3: Horner's method (nested polynomial evaluation)

$$\begin{aligned} p(x) &= 7 + 5x - 2x^2 + 3x^3 + 9x^4 \\ &= 7 + x[5 - 2x + 3x^2 + 9x^3] \\ &= 7 + x \cdot [5 + x \cdot [-2 + 3x + 9x^2]] \\ &= 7 + x \cdot [5 + x \cdot [-2 + x \cdot [3 + 9 \cdot x]]] \end{aligned}$$

4 multiplication
4 add/subtract

operations is a proxy for complexity of an algorithm