COMP/MATH/STAT 365 - Computational Linear Algebra

Tuesday/Thursday 9:40-11:10 & 1:20-2:50, THEATR 206

Zoom Link:
https://macalester.zoom.us/j/97626813127?pwd=dUZEQTNDSkMrcFNiZDJUQldBSXBnUT09

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Office Hours: M: 2:45-4:00, W: 12:00-1:00, Th: 3:00-4:00, or by appointment

Preceptors

- Henry Bell | hbell@macalester.edu | TBD | OLRI 254 / Zoom
- Daniel Chechilnitsky | dchecel@macalester.edu | TBD | OLRI 254 / Zoom
- Elizabeth Schnaubelt | eschnaub@macalester.edu | TBD | OLRI 254 / Zoom
- Yutong Wu | ywu1@macalester.edu | TBD | OLRI 254 / Zoom
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Course Topics and Objectives

Computational (or numerical) linear algebra is the study of methods for performing linear algebra computations on computers. Linear algebra is a central study in mathematics, engineering, and all of the applied quantitative sciences. It's hard (perhaps impossible) to know too much linear algebra! COMP/MATH/STAT 365 is inherently a course in *applied* linear algebra, and we will place an emphasis on practical algorithmic methods over theoretical technicalities. In MATH 236, the techniques you learn typically fail to work in advanced computations, where the problems can be enormous and approximation, round-off errors, and convergence matter. In fact, this could also be considered a course in *Numerical Analysis* or *Computational Science* (indeed, each of these are former names of 365); however we will be focusing on the linear and matrix problems from those areas (and less on numerical methods from differential equations, for example). One important emphasis this semester is on efficiency. Many "real" problems arising in applied math these days are so computationally intensive that if your algorithm isn't efficient, you are not going to be able to compute successfully.

Learning Objectives

By the end of the semester, my goals are that you will:

1) be "fluent" with vectors and matrices abstractly, notationally, and computationally
2) understand the many different computational tools available to solve problems and know why one may be better than others
3) understand the numerical subtleties behind these algorithms
4) be able to do basic R programming involving matrices and vectors
5) improve your ability to write and comprehend formal mathematical arguments and proofs
6) be able to set up, solve, and professionally write up a "case study" using linear methods
7) improve your confidence in thinking about open-ended problems with numerical tools and your confidence in self learning
8) still think linear algebra is super-cool (because I know that you think this coming in).

Learning Philosophy

I want to emphasize two philosophical points that will be critical for accomplishing these objectives:

1) Most class days as well as the homework, technical reports, and exams will feature both mathematical theory and computational programming activities. You cannot shy away from either. If you feel less confident thinking abstractly or programming, focus on that skill set and seek out help from your peers and the instructor. As I will not be able to make it around to everyone all the time, I will be counting on those of you with more experience at programming to help others debug their code. Our preceptors will also have office hours in which they can help with coding.

2) You must be an active learner in this course. This means not only participating in class activities and doing homework, but actively engaging in the material to make connections between concrete examples and the more general mathematical principles under investigation. After you've finished each problem or lab, step back to reflect on the core ideas you've learned from the exercise. **Ask questions!**

Classroom Environment

I am committed to helping you learn and succeed in this course and in your time at Macalester. Enabling this means facilitating environments that support you in the ways that you need.

Respect. Everyone comes from a different path through life, and it is our moral duty as human beings to listen to each other without judgment and to respect one another. There will be no tolerance for discrimination based on race, ethnicity, gender identity, religion, sexual orientation, disability, and other identities or life experiences, in and out of the classroom.

Empathy. Everyone has a different life situation. This will impact our personal choices during this time of pandemic and it can cause tension. I want everyone to start with empathy for each other. We all have ongoing struggles and worries and we are all trying to do our best given the circumstances.

Call me David. Students sometimes wonder what to call their professors. I prefer to be called David, but if you prefer to be more formal, I am also ok with Professor Shuman. My preferred gender pronouns are he/him/his. Please help me make sure that I call you by your preferred name and pronouns too!

Title IX. Harassment or discrimination based on sex or gender will not be tolerated. If you or anyone you know has experienced harassment or discrimination, know that you are not alone. Macalester provides staff and resources to help you find support. More information is available on the Title IX website (https://www.macalester.edu/titleix/).

Health and Wellness

Here at Macalester, you are encouraged to make your well-being a priority throughout this module and your career here. Investing time into taking care of yourself will help you engage more fully in your academic experience. Remember that beyond being a student, you are a human being carrying your own experiences, thoughts, emotions, and identities with you, particularly during a pandemic. It is important to acknowledge any stressors you may be facing, which can be mental, emotional, physical, financial, etc., and how they can have an academic impact. I encourage you to remember that you have a body with needs. Use the restroom or step out if you are upset and need a break. Please do what is necessary so long as it does not impede your or others' ability to be mentally and emotionally present in the course. Outside of the classroom, sleep, moving your body, and connecting with others can be strategies to help you be resilient at Macalester. If you are having difficulties maintaining your well-being, please don't hesitate to contact me and/or find support from the many resources available through the Laurie Hamre Center for Health & Wellness (https://www.macalester.edu/healthandwellness/about/).

Deadlines and Extensions

I set deadlines to get feedback to you as quickly as possible and because the material in this course builds from week to week. With that said, I will almost always grant extensions when you need them, as you long as you keep an open and active line of communication with me. If you are worried about finishing an assignment before a deadline, reach out in advance of the deadline to make a plan with me about when you plan to finish the work by and how to keep you on track in the course. Please, please, never skip class because you have not finished an assignment. I would rather have you attend class and then work out a plan to finish the assignment later.

Accommodations

I am committed to supporting the learning of all students in my class. If you are encountering barriers to your learning that I can mitigate, please bring them to my attention. Reasonable accommodations are available for students with documented disabilities. It is important to meet with someone from the Disability Services Office early in the semester to ensure that your accommodations are approved and in place to begin the semester successfully. Contact the office by phone at 651-696-6874, or email at disabilityservices@macalester.edu to schedule an appointment and discuss your individual circumstances. If you have official accommodations in place already, please discuss these with me soon so that you get off to a great start.

Academic Integrity

In order for you to have a fair and successful learning environment, academic honesty is an absolute necessity. See the web page of the Dean of Students for the college's policies. The definition of cheating includes (but is not limited to) copying somebody else's written assignment, copying from somebody else's exam, copying someone's computer code, or using material from previous incarnations of this course. The definition of "copying" is broader than "verbatim duplicate." In other words, taking someone else's work or ideas and in any way passing them off as your own, even if you change the exact wording or presentation, is cheating. If you have questions about academic integrity, talk to me or the office of the Dean of Students.

Course Materials

- **Recommended (not required) Text**: *Numerical Analysis*, **2nd ed,** Timothy Sauer, Pearson 2012. We will cover chapters 0, 1, 2, 3, 4, 12, and maybe some parts of 5, 10, 11 (excluded are the chapters on differential equations).

- **Computing Software**: We will use *R* via the RStudio package.
  - If you will primarily be using your own laptop, I strongly recommend that you use the desktop version of R and RStudio. You can download the latest version of RStudio here, and the latest version of R here. For more help with installation and first steps, see this R Basics tutorial.
  - If you prefer the cloud version of R, every student has an RStudio account on the RStudio server. Your ID is your email (e.g., jsmith@macalester.edu), and your password is your regular Macalester password.
  - The textbook has examples in *Matlab*, so it is a bit odd not to use it for the course. However, Matlab is expensive and we do not have a campus-wide license (R is free!). I want you to be able to use it on your own machine and on machines all around campus instead of having to come to an MSCS lab to do your work. Furthermore, R is a good, professional language to know, and it is vectorized with fully functioning linear algebra libraries under the hood. In fact they are they same libraries as are used in Matlab.
  - Once you know R, it is easy to pick up Matlab, and vice versa.

- **Typesetting Software**: During the semester we will write technical reports using LaTeX. LaTex is freely available for the Macintosh and the PC. It is the standard markup language for quantitative areas of science and is widely used in scientific publishing.

Course Work

Your course grade will be calculated using the following linear combination

$$\mathbf{G} = .10\,\mathbf{P} + .25\,\mathbf{HW} + .20\,\mathbf{TR} + .15\,\mathbf{MT}_1 + .15\,\mathbf{MT}_2 + .15\,\mathbf{F},$$

where the components are

P = Participation (10%): Participation is not the same as attendance. I expect you to come to class prepared, actively participate in class discussions, engage in the computational activities, and ASK QUESTIONS! If you are confused about something, there are probably three other people who are confused as well, so please do not be shy about asking questions, both in class and outside the class in office hours. You should strive to take risks by going beyond your intellectual comfort zone, and you should also respect and admire your peers when they take such risks.

HW = **Homework** (25%): We will often work on the "homework" activities in class. You are allowed — and in fact encouraged — to work on and discuss homework together. Discussion and shared vantage points are essential for mastering the material. In particular, a valid proof is a convincing argument. You should be able to convince your colleagues that your proof is correct. However, **you must write up your solutions separately and independently**. Furthermore, **every homework write-up must include a list of the names of your collaborators**. If you get significant help from a classmate, you should give that person credit in your solution (at no penalty to you). If you use an external source such as a website or book, you should give that source credit in your solution. **Your lowest homework score will be dropped.**

TR = **Technical Reports** (20%): You will be assigned three to four technical reports on a "case study" that is an application or illumination of the techniques we are learning in class. These will be 2-5 pages in length and are to be written using LaTeX. I will provide you with a template for writing one of these, and I will give you, ahead of time, the rubric that I will use in grading them. You will work on these in groups of size 2-3. Thanks to these technical reports, this class counts towards the Writing in Practice general education requirement at Macalester.

MT$_i$ = **Midterm Exams** (15% each): There will be two midterms. They will each have two components: (1) there will be some in-class, open-book, open-computer questions. You will have the full class period to work on this. (2) There will also be a longer take-home part that will be due at the beginning of the following class. This allows for you to have more time to work on the problems. Some of the problems will involve doing small computation and programming tasks in R. Others will be paper and pencil type problems. We will discuss these thoroughly as each draws near.

The midterm exams are tentatively scheduled for **Thursday, February 24th**, and **Thursday, April 7th**, with the take-home portion due at the beginning of class on the subsequent Tuesdays. Please inform me by January 28th if you are not able to attend class on one of these days.

F = **Final Exam** (15%): The final exam will be an open-book, computer take-home exam. I will hand it out on **Wednesday May 4th**, and it will be due on **Monday May 9th** (that's the end of the last final exam).

Approximate Course Outline

We will cover the majority of the topics in this table:

1. Numerics (Chapters 0 and 1)
    - Basic R programming: if-else, for loops, writing functions
    - Polynomial evaluation
    - Root finding
    - Numerical differentiation
    - Decimal and floating point representation of numbers
    - IEEE standards
    - Approximatation, round-off error, forward and backward error, catastrophic cancellation, sensitivity
    - loss of significance, overflow
2. Linear Algebra (Chapter 2)
    - Vector and matrix norms
    - Sparse matrices
    - LU factorizations
    - Iterative methods: Jacobi, Gauss-Seidel
    - Symmetric, positive definite matrices
    - Conjugate gradient methods
    - Condition number, preconditioning
3. Interpolation (Chapter 3)
    - Vandermonde, Lagrange, barycentric, and Newton's divided differences interpolation
    - Chebyshev interpolation
    - Cubic splines
    - Bézier curves
    - Interpolation error
4. Least Squares and Orthogonality (Chapter 4)
    - Normal equations and least squares
    - Fitting data
    - Gram Schmidt and QR factorization
    - Householder transformations
    - Krylov methods (GMRES)
    - Orthogonal polynomials
5. Eigenvalues (Chapter 12)
    - Power iteration and inverse power iteration
    - Rayleigh quotients
    - QR factorizations and eigenvectors
    - Markov chains
    - PageRank
    - SVDs and applications
    - Compression, dimension reduction, and low rank approximation
    - Principle Component Analysis
    - Netflix and the Funk SVD
6. Other Possible Topics
    - Numerical integration (Chapter 5)
        - Newton-Cotes
        - Adaptive and/or Gaussian quadrature
    - Transforms (Chapters 10,11)
        - Complex numbers
        - Fourier Transforms
        - Trigonometric interpolation
        - Discrete Fourier Transforms (DFTs) and trigonometric interpolation
        - Fast Fourier Transforms (FFTs)
        - Discrete Cosine Transforms (DCTs)
    - Matrix methods in data mining
    - Spectral clustering and network community detection
    - Common optimization problems such as Tikhonov regularization or ridge regression