

## Midterm 1 Practice Problems

### Part I

Thursday, February 24, 2022

- The exam is open notes and open book. You may use the internet and library to look things up, but of course you should cite any reference you use that is not the textbook.
- **It is not okay to talk to each other or anyone else about the exam! This of course includes discussing how you solve problems, but it even includes things like asking, “how it is going?” or “which problem you are working on?” or making comments like “problem 5 is easy.”**
- Please show and explain all of your work so that I can give you as much partial credit as possible.
- You are welcome to use any of the R functions in 365Functions.r
- The in-class portion of the exam (Questions 1–4) is due at the end of this class.
- The take-home portion of the exam (Questions 5–6) is due on Moodle by 9:00 am sharp on Tuesday morning.
- For the take-home portion, you should do your work in a single R Markdown file and submit the html file (and/or pdf) containing your code and your solutions. I have provided a starter file in case you would like to do that.
- Hints: I may give a small hint or a little help with R for free. If you are stuck, I may sell a hint for points so that you can move forward. I will warn you of the point value before I sell you anything.

Problem	Point Value	Your Score
1	X	
2	X	
3	X	
4	X	
⑤	X	
⑥	X	
Total	50	

Please sign the following (if you are able) and be sure to turn in this cover sheet with your exam:

*I pledge my honor that I have not participated in any dishonest work on this exam, nor do I know of dishonest work done by other students on this exam.*

\_\_\_\_\_  
(signature)

As an alternative to turning this in, you may also type out this pledge as part of your submission.

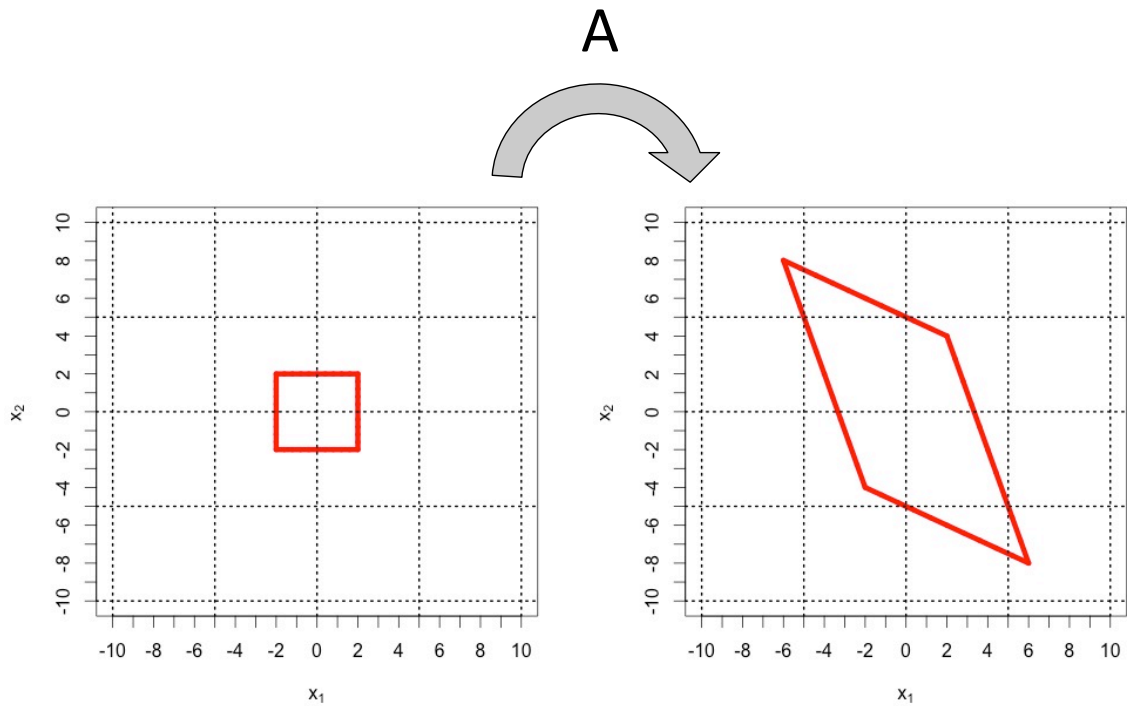
1. (6 points) You are designing your own machine that stores numbers in a way that is modeled after the 64-bit IEEE representation of floating point numbers. However, you want your machine to be retro hip, so you've decided to build **a 16-bit machine that uses one bit for the sign, 8 bits for the mantissa, and 7 bits for the exponent**:

$\pm$	m	a	n	t	i	s	s	a	e	x	p
.	.	.	.	.	.	.	.	.	.	.	.

- (a) (3 points) The decimal number -45.4375 converts to binary as -101101.0111. How will this number be stored in your machine? Make sure to show how you calculated your answer.
- (b) (3 points) The decimal number 0.1060791015625 converts to binary as 0.0001101100101. How will this number be stored in your machine? Make sure to show how you calculated your answer.

2. (4 points)  $A$  is an unknown  $2 \times 2$  matrix. The left figure below shows the set of points

$\mathcal{S} = \{x \in \mathbb{R}^2 \text{ such that } \|x\|_\infty = 2\}$ , and the right figure shows the image of this set under the linear mapping  $A$ ; i.e., the set  $\mathcal{I} = \{y \in \mathbb{R}^2 \text{ such that } y = Ax \text{ for some } x \in \mathcal{S}\}$ .



Is this enough information to determine  $\|A\|_\infty$ ? If yes, find  $\|A\|_\infty$ . If no, then explain why not.

3. (5 points)

Let  $P$  be an  $n \times n$  matrix satisfying the following two properties:

- i.  $P^\top = P$
- ii.  $P^2 = P$

Show that  $P$  is positive semidefinite.

**Extra credit:** State a non-trivial condition under which a matrix  $P$  of the type described above is strictly positive definite (i.e., not just the definition of positive definiteness or an equivalent general condition, but rather something that relies on the fact that the two extra properties above are satisfied). (2 points)