A New Six Parameter Model to Estimate the Friction Factor

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Significance

A new explicit formula for estimating the friction factor using six parameters is proposed. The model was set up by considering the effect of residual stresses in the flow by two distinct contributions: the first is attributed to the flow velocity (Reynolds number) and the second to the duct roughness. Compared to other models, this new equation gives the best fit with Nikuradse's results. A new model to calculate the friction is proposed. The model is based on assuming the residual stresses due to the laminar to turbulent flow transition by two distinct contributions: the first is attributed to the flow velocity (Reynolds number) and the second to the duct roughness. Compared to other models, this new equation gives the best fit with respect of Nikuradse's results. The model does not consider the effect of pipe wall on the velocity distribution. © 2019 American Institute of Chemical Engineers *AIChE J*, 65: 1144–1148, 2019

Keywords: friction factor, flow in ducts, Colebrook equation, Moody chart, Reynolds number

Background

Friction factor is used to estimate loses in mechanical energy in flows flowing through pipelines. Friction factor is commonly computed using either Colebrook equation[1] or Moody's chart.[2] However, in spite of its usefulness, these calculation methods are limited due to their own nature.

Colebrook equation is an implicit model that requires numerical methods to determine the value of the friction factor. Moody's chart is a graphical representation of Colebrook equation, which use is limited to Reynolds numbers at around or below 10.[3]

Because of the complications cited above, several authors[3-11] have proposed different models to estimate the friction factor. The majority of the models proposed are explicit, i.e., they allow to estimate values for the friction factor directly as a function of both, the Reynolds number and the relative roughness of the pipe conducting the flow. Some good reviews on this particular subject matter are also available in the literature.[12-18]

Table 1 shows some of the correlations proposed to calculate the friction factor in ducts. The table also shows the relative error of each of these correlations with respect of Nikuradse data.[19] The error was calculated as follows:

$$\%\text{Error} = 100 \times \frac{f_{\text{Nikuradse}} - f_{\text{model}}}{f_{\text{Nikuradse}}} \tag{1}$$

Proposed Model

Moody's chart[2] clearly shows that under the laminar flow regime, the friction factor decreases linearly with the inverse of Reynolds number; when Re is about 2000, there is a break in this lineal behavior, and the friction factor increases. For rougher pipe walls, the friction factor exhibits higher values than smoother pipes. As the flow approaches full turbulent behavior (increase in Re), the friction factor tends to be constant for every value of relative roughness of the pipe. We assume that a second transition in the flow occurs at the point where the friction factor stabilizes, irrespective of the Reynolds value. Specific details on the development of this model are going to be fully disclosed in an upcoming publication.

In this report, we present the following model to determine the friction factor in pipe networks:

$$f = \frac{64}{\text{Re}} + \frac{\lambda_1}{1 + \exp\left(\frac{\tau_1 - \text{Re}}{100}\right)} + \frac{\lambda_2}{1 + \exp\left(\frac{\tau_2 - \text{Re}}{600} \times \frac{\varepsilon}{D}\right)} \tag{2}$$

For smooth pipes, Eq. (2) reduces to

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Table 1. Summary of Models Available in Literature to Compute the Friction Factor in Ducts

Ref.	Model	Validity	Error [%]
3	$\frac{1}{\sqrt{f}}=-1.8\log\left[\frac{6.9}{\mathrm{Re}}+\left(\frac{\varepsilon}{3.7D}\right)^{1.11}\right]$	$4\times10^{3}<\mathrm{Re}<10^{8}$ $10^{-6}<\varepsilon/D<0.05$	43.86
4	$\frac{1}{\sqrt{f}}=-2\log\left[\frac{\varepsilon}{3.7065D}-\frac{5.0272}{\mathrm{Re}}\log\left[\frac{\varepsilon}{3.827D}-\frac{4.567}{\mathrm{Re}}\log\left[\left(\frac{\varepsilon}{7.7918D}\right)^{0.9924}+\left(\frac{5.3326}{208.815+\mathrm{Re}}\right)^{0.9345}\right]\right]\right]$	$3\times10^{3}<\mathrm{Re}<1.5\times10^{8}$ $0<\varepsilon/D<0.05$	10.94
5	$\frac{1}{\sqrt{f}}=-2\log\left[\frac{\varepsilon}{3.7D}-\frac{5.02}{\mathrm{Re}}\log\left[\frac{\varepsilon}{3.7D}-\frac{5.02}{\mathrm{Re}}\log\left[\frac{\varepsilon}{3.7D}+\frac{13}{\mathrm{Re}}\right]\right]\right]$	N/A	10.91
6	$\frac{1}{\sqrt{f}}=-2\log\left[\frac{\varepsilon}{3.7065D}-\frac{5.0452}{\mathrm{Re}}\log\left[\frac{1}{2.8257}\left(\frac{\varepsilon}{D}\right)^{1.1098}+\frac{5.8506}{\mathrm{Re}^{0.8981}}\right]\right]$	$4\times10^{3}<\mathrm{Re}<10^{8}$ $10^{-6}<\varepsilon/D<0.05$	11.17
7	$f=8\times\left[\left(\frac{8}{\mathrm{Re}}\right)^{12}+\frac{1}{\left[\left(2.457\ln\left(\frac{1}{\left(\frac{7}{\mathrm{Re}}\right)^{0.9}+0.27\frac{\varepsilon}{D}}\right)\right)^{16}+\left(\frac{37530}{\mathrm{Re}}\right)^{16}\right]^{\frac{3}{2}}}\right]^{\frac{1}{12}}$	$4\times10^{3}<\mathrm{Re}<10^{8}$ $10^{-6}<\varepsilon/D<0.05$	11.64
8	$f=1.613\left[\ln\left(0.34\left(\frac{\varepsilon}{D}\right)^{1.1007}-\frac{60.525}{\mathrm{Re}^{1.1105}}+\frac{56.291}{\mathrm{Re}^{1.0712}}\right)\right]^{-2}$	$3\times10^{3}<\mathrm{Re}<10^{8}$ $0<\varepsilon/D<0.05$	17.15
9	$\frac{1}{\sqrt{f}}=0.8686\ln\left(\frac{0.4587}{S^{\left(\frac{S}{S+1}\right)}}\mathrm{Re}\right);\ S=0.124\frac{\varepsilon}{D}\mathrm{Re}+\ln(0.4587\,\mathrm{Re})$	$4\times10^{3}<\mathrm{Re}<10^{8}$ $10^{-6}<\varepsilon/D<0.05$	11.18
10	$\frac{1}{\sqrt{f}}=-2\log\left[\frac{2.18}{\mathrm{Re}}\ln\left(\frac{\mathrm{Re}}{1.816\ln\left(\frac{1.1\mathrm{Re}}{\ln(1+1.1\mathrm{Re})}\right)}\right)+\frac{\varepsilon}{3.71D}\right]$	N/A	12.62
11	$\frac{1}{\sqrt{f}}=1.930\log(\mathrm{Re}\sqrt{f})-0.537$	N/A	12.78
12-14	$f=\left[\left(\frac{64}{\mathrm{Re}}\right)^{12}+\frac{1}{\left[\left(0.8687\ln\left(\frac{1}{\frac{0.883(\ln(\mathrm{Re}))^{1.282}}{\mathrm{Re}^{1.007}}+0.27\frac{\varepsilon}{D}-\frac{110}{\mathrm{Re}}\frac{\varepsilon}{D}}\right)\right)^{16}+\left(\frac{13269}{\mathrm{Re}}\right)^{16}\right]^{\frac{3}{2}}}\right]^{\frac{1}{12}}$	N/A	11.75

$$f=\frac{64}{\mathrm{Re}}+\frac{\lambda_1}{1+\exp\left(\frac{\tau_1-\mathrm{Re}}{100}\right)} \tag{2a}$$

Equations 2 and 2a were developed after close inspection of McKeon et al,[11] Nikuradse,[19] and Swanson et al[20]data sets.

However, in spite of being considered by many as the benchmark for explaining flow phenomena over rough pipe walls, Nikuradse work has been severely criticized.[21,22] Among the main arguments against Nikuradse is that it is strongly believed[22] that he manipulated his experimental data in order to fit it to von Karman analytical expression for the friction factor and velocity distribution near the pipe wall. However, later on, it was demonstrated[23] that indeed there was some data handling but not to disprove his work as a whole.[21,23]

In any case, it is observed in Nikuradse's data set that at lower r/ε ratios, more abrupt transitions on the friction factor occur as Re increases. Such transitions are due to the presence of stresses developed between the pipe wall and the flow itself. Our model explains such transitions by virtue of four (λ_1, λ_2, τ_1, τ_2) parameters in Eq. 2. This model is not the result of any statistical data fitting, rather than that, this model is built upon three basic premises:

1. Flows with $\mathrm{Re} < 2000$ are laminar and the friction factor varies linearly with the inverse of Re, according to $f = 64/\mathrm{Re}$.
2. When the flow's Reynolds number is around 3000, the flow transitions from laminar to turbulent; the friction factor tends to increase as Re does so. This behavior is due to the appearance of velocity fluctuations in the turbulent regime that produce collisions within the flow, consequently increasing the friction factor. It is proposed[24,25] that the increase in friction factor follows a sigmoidal type of function; it has not been developed any mathematical expression that relates the friction factor to the flow collision frequency. However, it would be expected that these two quantities were proportional; if this is true, then it is proposed that the second term in Eq. 1 would be sigmoidal in nature. Therefore, it is likely that at certain regions the model proposed here does not fit precisely the reported data; this mainly due to this first approach.
3. At Reynolds numbers considerable higher than 3000, the turbulent boundary layer thins enough that irregularities in the pipe surface produce another set of collisions within the flow, therefore it is necessary to include another term considering this effect in the description of the friction factor. This model considers that this new term can be modeled using a sigmoidal function. As the pipe surface is more irregular, the number of

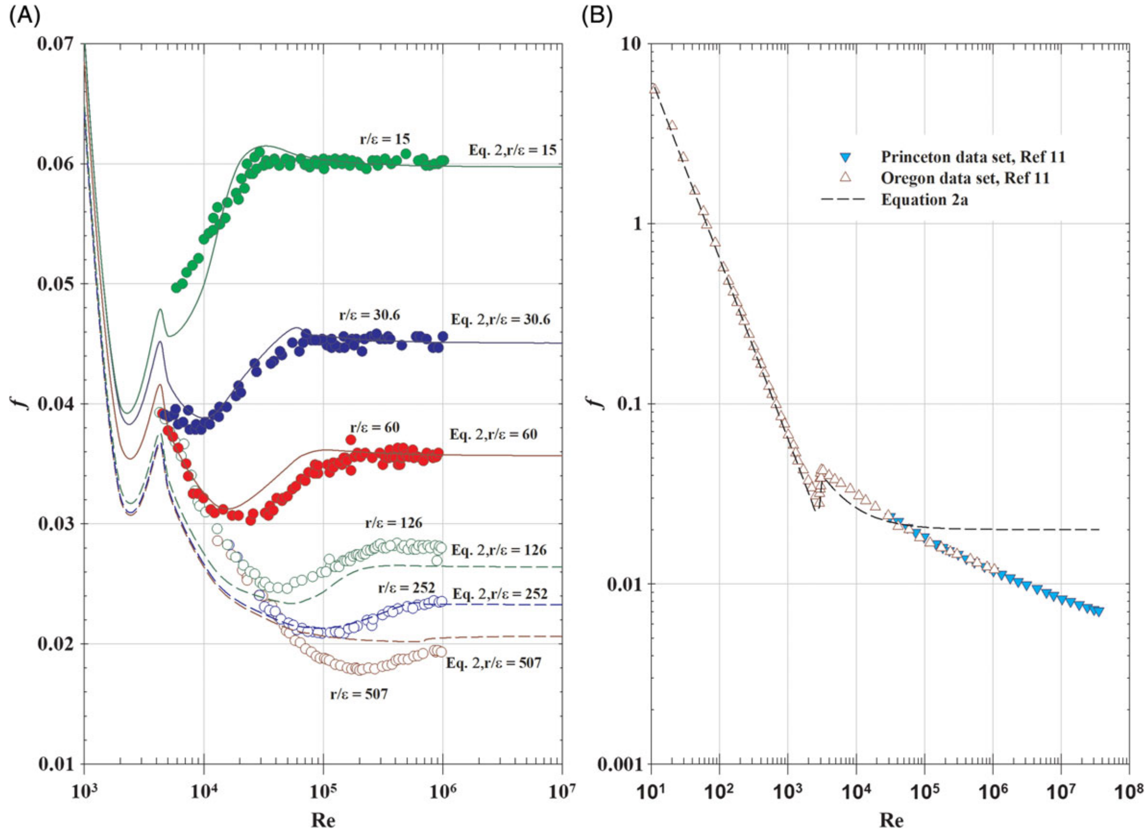

Figure 1. (A) Comparison between Nikuradse[19] data and the predictions of the proposed model (Eq. 2). (B) Comparison between McKeon et al[11] and Swanson et al[20] data and the predictions of the proposed model (Eq. 2a).

[Color figure can be viewed at wileyonlinelibrary.com]

perturbations in the flow will occur at a faster rate, hence, the need of a sigmoidal function in accounting for this effect.

In Eq. 2, Re is Reynolds number; ε/D is the relative roughness of the pipe; λ_1 is the residual stress contribution from the laminar to turbulent transition to the friction factor; λ_2 is the residual stress contribution from the pipe roughness to the friction factor; τ_1 is Reynolds number at which occurs the first transition in the friction factor; and τ_2 is Reynolds number at which the second transition occurs.

The first (τ_1) transition refers to the change in flow regime from laminar to turbulent. Friction factor for laminar flow is proportional to the inverse of Reynolds; as the flow regime turns into turbulent, such transition occurs for every pipe roughness (even for smooth ones) and it is intrinsic to the flow itself; thus it is unavoidable its presence. The second transition (τ_2) occurs at higher Reynolds numbers, and occurs at different Re values. As pipe rugosity increases, this second transition will take place faster and the increase in friction factor will be more noticeable. This second transition is intimately related to pipe rugosity.

Of these parameters, λ_1 and τ_1 are constant for all flow and relative roughness conditions, and their numerical values are 0.02 and 3000, respectively. In contrast, both λ_2 and τ_2 they do depend on the relative roughness of the pipe. The expressions for these parameters are

$$\lambda_2 = \left| \lambda_1 - \left(\frac{1}{-2\log\left(\frac{1}{3.7065} \times \frac{\varepsilon}{D}\right)} \right)^2 \right| \tag{3}$$

$$\tau_2 = \frac{0.77505}{\left(\frac{\varepsilon}{D}\right)^2} - \frac{10.984}{\frac{\varepsilon}{D}} + 7953.8 \tag{4}$$

In spite of the different nature of these two transitions, both of them contribute similarly to the magnitude of friction factor. Figure 1A shows a graph of our model compared to Nikuradse[19] experimental points; it can be noticed in this figure that there is good agreement between our model (Eq. 2) and all of Nikuradse data sets. For every *r*/ε condition, our model accurately represents the laminar flow condition as well as the laminar–turbulent transition. The model also shows good fitting with respect of the fully turbulent flow. Additionally, in Figure 1B, we compare our model (Eq. 2a) to smooth pipe experimental friction data obtained by McKeon et al[11] and Swanson et al.[20] From this figure, it is clear that our model represents accurately the experimental values already reported up to Re ~10,[6] the transition from laminar to turbulent flow is also well depicted by the model; however, as Re increases up to 10^7 (Princeton data), considerable deviations between the model and the data set become more evident. Such deviations may arise from the construction of our model; we consider that upon transitioning to turbulent flow, given the randomness of turbulence, the flow is better depicted by a sigmoidal function rather than a continuous function as proposed by McKeon et al.[11]

Figure 2A,B compares the results of some of the models shown in Table 1 with the experimental data of Nikuradse[19] and Eq. 2 for *r*/ε = 507 and *r*/ε = 15, respectively. From these figures, it can be noticed that for very rough pipes (*r*/ε = 15),

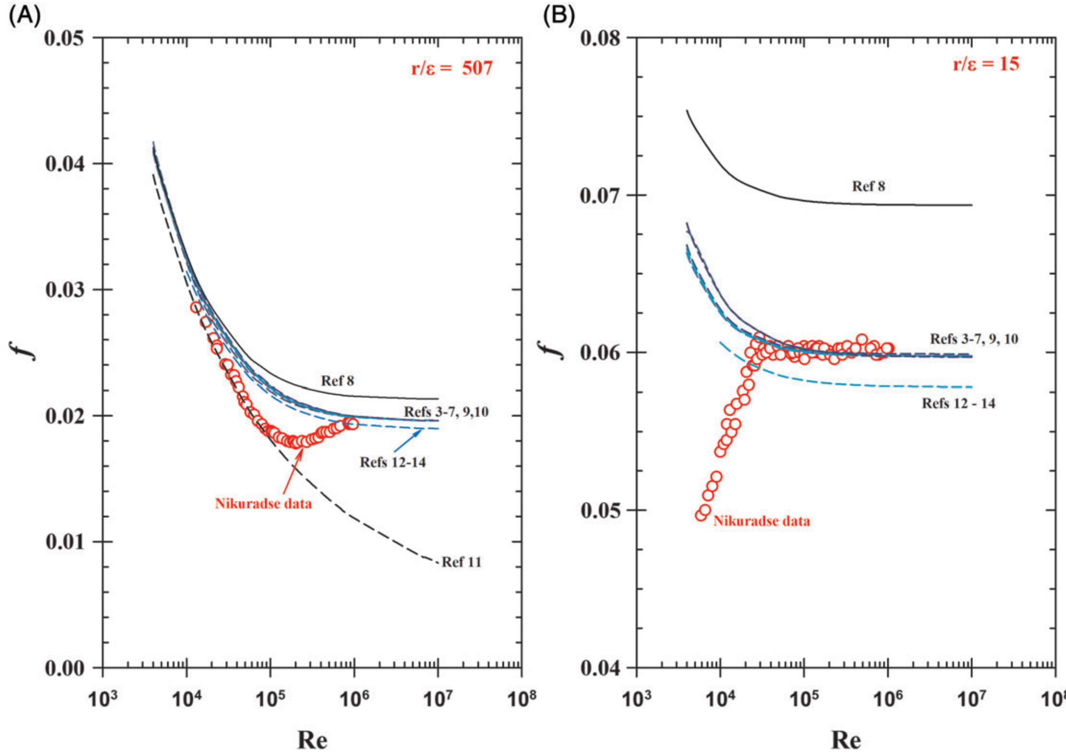

Figure 2. (A) Comparison between Nikuradse data and predictions from different models used for the condition *r*/ε = 507. (B) Comparison between Nikuradse data and predictions from different models used for the condition *r*/ε = 15.

[Color figure can be viewed at wileyonlinelibrary.com]

Fang et al model[8] and the modified Churchill model[12-14] present the most considerable deviations from Nikuradse's data. The other models show a lesser degree of discrepancy with respect of the data set used. Since McKeon model was constructed for smooth pipes, it is not even considered for comparison to the data set used. On the other hand, for the smoothest condition ($r/\varepsilon = 507$), the different models proposed show lower deviation with respect of Nikuradse's data. Even McKeon's model[11] shows excellent agreement with Nikuradse's experimental points up to Re ~10^5; as Re increases, so does the deviation between the model[11] and the experimental data.[19]

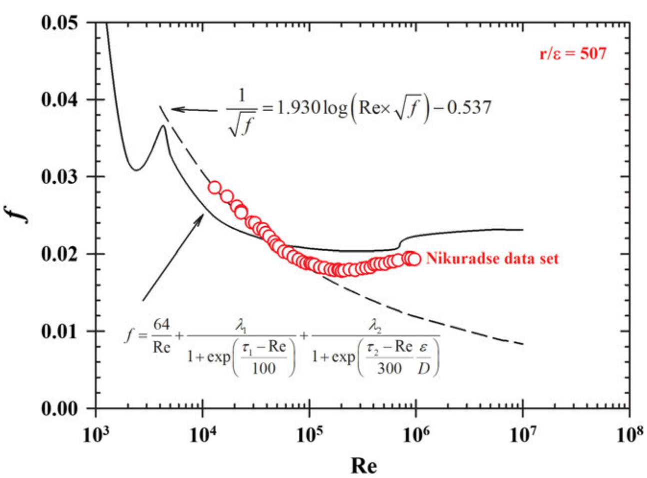

Figure 3. Comparison between Nikuradse experimental data set, Eq. (2) and McKeon et al[11] model for the friction factor using *r*/ε = 507.

[Color figure can be viewed at wileyonlinelibrary.com]

Figure 3 shows the comparison between our model (Eq. 2), McKeon et al model[11] and Nikuradse's data set[19] for the smoothest pipe wall condition ($r/\varepsilon = 507$). As seen from this figure, McKeon's model has better agreement with the experimental data than our model for Re values between 10^4 and 10.[6] However, for Re >10,[6] our model predictions are closer to the experimental data, compared to McKeon's model; that model rapidly deviates from the measured points as Re increases. This behavior can be explained in terms of the third term in our model; such term considers the effect of the pipe rugosity regardless of its numerical value as a consequence the model results in a line closer to the experimental points; in the case of McKeon's model, it does not consider at all the rugosity and thus its model continuously decreases as Re increases.

Final Remarks

A new explicit model to estimate the friction factor in flows was developed based on the two distinctive contributions to the flow in pipes: the first corresponds to the increase in Reynolds number (transition laminar–turbulent flow regime). The second transition is attributed to the actual roughness of the pipe wall and acts faster on the flow than the Reynolds contribution.

The model was validated in part by using Nikuradse experimental data[19]; even though there are certain uncertainties regarding the accuracy of that data; as Churchill and Chan[21] pointed out, in spite of its imprecisions, Nikuradse data still are the most complete results available to us. Beattie[23] confirms the utility of Nikuradse results although is well aware of the possible manipulation of them. In spite of the criticism received by Nikuradse work, we confirm in this article that the actual results from him do not change considerably. If in the near future new experimental results become available, our model still will be valid since it captures in a simple manner the physics behind the friction factor as the sum of three distinct contributions, as shown in Eq. 2.

Acknowledgment

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