Discussion of "Friction Modeling of Flood Flow Simulations" by Vasilis Bellos, Ioannis Nalbantis, and George Tsakiris

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We acknowledge the scientific contribution of the discussed paper, but we would like to provide a few comments regarding similar models that are concurrent with the analyzed problem.

We believe that the term *flood flow simulation* from the discussed paper can be understood as open-channel gradually varied flow in natural stream systems involving floodplain or overbank flows. Otherwise, for us is difficult to imagine how the methods from the discussed paper can be realistically applied to such highly variable complex conditions that occur during flood. We perceived that the methods from the discussed paper ultimately represent a reformulation of expressions derived from the Manning equation used in the standard step method for the slope of energy grade line calculations, and therefore we doubt if they can deal with vegetated overbanks and if the presented equations are adequate for simulating complexities such as transition from nonemergent to emergent flow conditions as usually occur during flood flow. However, our comments are mostly about the form of the presented equations and not their hydraulic interpretation. The computational burden of simulating transitionally turbulent flow during extensive simulations can be reduced simply by avoiding computationally demanding functions such as those with noninteger power terms (Goldberg 1991), which require the execution of significantly more floating-point operations in the CPUs of computers compared with multiplication or integer power terms (Clamond 2009; Winning and Coole 2015; Biberg 2017). We show a few transformations that can hopefully reduce such computational burden.

Matching Models

The discussed paper presents a unified flow friction model for flood flow, which is based on the power-law unified formula for pipe flow of Cheng (2008) [Eq. (1)]. Eq. (1) includes laminar regime f_L, the smooth and rough turbulent regimes f_{TS}, f_{TR}, and transitions between them, all as a function of the Reynolds number R, and with the relative roughness of inner pipe surface ε/D, which are both dimensionless. The formula gives the Darcy (Moody) flow friction factor f, which is widely used in engineering

$$\left.\begin{aligned} f &= f_L^{\alpha} + f_{TS}^{(1-\alpha)\cdot b} + f_{TR}^{(1-\alpha)\cdot(1-b)} \\ \alpha &= \frac{1}{1+\left(\frac{R}{2.712}\right)^{8.4}} \\ b &= \frac{1}{1+\left(\frac{R}{150\cdot\frac{\varepsilon}{D}}\right)^{1.8}} \end{aligned}\right\} \qquad (1)$$

where f = Darcy (Moody) flow friction factor (dimensionless), with indexes L for laminar, TS for turbulent smooth, and TR for turbulent rough; R = Reynolds number (dimensionless); ε/D = relative roughness of inner pipe surface (dimensionless); and α and b = Cheng's power terms (Cheng 2008).

Similarly, Brkić and Praks (2018) recently developed a simpler unified flow friction model for flow through pressurized pipes that does not contain complex, resource-intensive, and computationally expensive noninteger power terms [Eq. (2)]. Switching functions y_1, y_2, and y_3 from Eq. (2), as defined by Brkić and Praks (2018), provide the Nikuradse inflectional shape for flow friction curves, and in the current form they are valid for pipe flow (they can be rearranged for flood flow). Their main purpose is to provide a smooth transition between laminar and turbulent smooth, and between turbulent smooth and turbulent rough hydraulic regimes

$$\left.\begin{aligned} f &= (1-y_1)\cdot f_L + (y_1-y_3)\cdot f_{TS} + y_2\cdot f_{TR} \\ y_1 &= 1-\frac{1048}{\frac{4.489}{10^{20}}\cdot R^6\cdot\left(0.148\cdot R-\frac{2.306\cdot R}{0.003133\cdot R+9.646}\right)+1{,}050} \\ y_2 &= 1.012-\frac{1}{0.02521\cdot R\cdot\frac{\varepsilon}{D}+2.202} \\ y_3 &= 1-\frac{1}{0.000389\cdot\left(R\cdot\frac{\varepsilon}{D}\right)^2+0.0000239\cdot R+1.61} \end{aligned}\right\} \qquad (2)$$

where f = Darcy (Moody) flow friction factor (dimensionless), with indexes L for laminar, TS for turbulent smooth, and TR for turbulent rough; R = Reynolds number (dimensionless); ε/D = relative roughness of inner pipe surface (dimensionless); and y_1, y_2, and y_3 = switching functions.

Models by Cheng (2008), Brkić and Praks (2018), and Díaz-Damacillo and Plascencia (2019) were developed for the same purpose and give similar and comparable results for pressurized pipe flow.

Floating-Point Computing

Repetitive Functions

Multiplication and integer power terms require less computing resources in terms of the execution of flow point operations in the CPUs of computers. The same is true of logarithmic functions. Eqs. (9) and (38) of the discussed paper give an approximation

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of the Lambert W-function: $W(1.35 \cdot R_h)$, which requires numerous repetitions of the logarithmic function. This is easily avoided by using precomputed values for $v = \ln(1.35 \cdot R_h)$ and $V = \ln(v)$ [Eq. (3)], so only two logarithmic functions are executed instead of 11, which results in a significant saving of CPU time during extensive simulations

$$\left.\begin{aligned} v &= \ln(1.35 \cdot R_h) \\ V &= \ln(v) \\ W(1.35 \cdot R_h) &= v - V + \frac{V}{v} + \frac{V^2 - 2 \cdot v}{2 \cdot v^2} \end{aligned}\right\} \quad (3)$$

where R_h = Reynolds number, assuming that the characteristic length of the flow is the water depth (dimensionless); v and V = auxiliary terms defined in text; and W = Lambert function.

Overflow Errors

Transcendental equations can be solved by numerical means, e.g., by the Newton iteration (Praks and Brkić 2018a). However, the possibility of obtaining the exact analytical solution of such equations, e.g., through the Lambert W-function, is appealing. The reason is that a closed, analytical form of a function is invaluable because it provides the necessary asymptotic forms both at small and large values of the independent variable (Belkić 2019). In that sense, Eq. (37) of the discussed paper uses the Lambert W-function to relate f, the unknown flow friction factor, with the known R_h, the water depth as the characteristic length Reynolds number, while Eq. (36) relates f using implicit relation in a classical way, where both equations give the same final results [Eq. (4)]

$$\underbrace{\frac{1}{\sqrt{f}} = 0.86 \cdot \ln(1.16 \cdot R_h \cdot \sqrt{f})}_{(36)} \leftrightarrow \underbrace{\frac{1}{\sqrt{f}} = \frac{R_h}{0.86 \cdot e^{W(1.35 \cdot R_h)}}}_{(37)} \quad (4)$$

where f = Darcy (Moody) flow friction factor (dimensionless); R_h = Reynolds number, assuming that the characteristic length of the flow is the water depth (dimensionless); and W = Lambert function.

However, because of the possible occurrence of overflow errors, the approach in Eq. (37) of the discussed paper with the exponential fast-growing term combined with the Lambert W-function should be used with caution (Sonnad and Goudar 2004; Brkić 2012). Such numerical problems can be solved using the shifted Lambert W-function (Rollmann and Spindler 2015; Biberg 2017; Brkić and Praks 2019), Padé approximants (Praks and Brkić 2018c), or more advanced iterative procedures (Praks and Brkić 2018b).

Transitional Turbulent Regime

The authors put forward the question: "Why is it necessary for the new equation to be valid under all flow regimes?" They provided reasonable explanations. They mentioned the well-known implicit equation by Colebrook (1939), and in light of the new unified flow friction factor formulation valid under all flow regimes, it needs to be emphasized that the Colebrook equation is also actually an attempt to provide smooth transition from the smooth turbulent regime to the rough turbulent regime, here noted as f_{TS} and f_{TR}. The transition process, based on the experiment of Colebrook and White (1937) does not take into account the inflectional form of flow friction curves as suggested by Nikuradse (1950). On the other hand, the suggested unified formulas from the discussed paper and from Brkić and Praks (2018) follow Nikuradse's inflectional form. Brkić and Praks (2018) describe an extension of the Colebrook equation E that is concurrent with the data of Nikuradse (1950) for pipe flow. This extension, adapted for flood flow, is demonstrated in Eq. (5), where $R \sim 4 \cdot R_h$, $\varepsilon/D \sim k_s/(4 \cdot h)$, and $E = 1$; the result is the Colebrook equation in its well-known standard form

$$\left.\begin{aligned} \frac{1}{\sqrt{f}} &= -2 \cdot \log_{10}\left(\frac{2.51}{4 \cdot R_h} \cdot \frac{1}{\sqrt{f}} + \frac{\frac{k_s}{4 \cdot h}}{3.71} \cdot e^E\right) \\ E &= \frac{-31.13}{(4 \cdot R_h) \cdot \left(\frac{k_s}{4 \cdot h}\right)} \cdot \frac{1}{\sqrt{f}} \end{aligned}\right\} \quad (5)$$

where f = Darcy (Moody) flow friction factor (dimensionless); R_h = Reynolds number, assuming that the characteristic length of the flow is the water depth (dimensionless); h = water depth (m); k_s = roughness height (m); and E = Nikuradse's extension to the Colebrook equation (dimensionless).

Roughness height k_s correlates to pipe roughness height ε. The same notation is also used in Eq. (6), where instead of Eq. (11) of the discussed paper, more accurate approximations can be used, starting from the least accurate but the least complex

$$\left.\begin{aligned} \frac{1}{\sqrt{f}} &\approx 0.8686 \cdot \left[s_2 - s_3 + \frac{s_3}{s}\right] \\ \frac{1}{\sqrt{f}} &\approx 0.8686 \cdot \left[s_2 - s_3 + \frac{1.038 \cdot s_3}{0.332 + s}\right] \\ \frac{1}{\sqrt{f}} &\approx 0.8686 \cdot \left[s_2 - s_3 + \frac{1.0119 \cdot s_3}{s} + \frac{s_3 - 2.3849}{s^2}\right] \\ s_1 &\approx \frac{(4 \cdot R_h) \cdot \left(\frac{k_s}{4 \cdot h}\right)}{8.0878} \\ s_2 &\approx \ln(4 \cdot R_h) - 0.7794 \\ s &= s_1 + s_2 \\ s_3 &= \ln(s) \end{aligned}\right\} \quad (6)$$

where f = Darcy (Moody) flow friction factor (dimensionless); R_h = Reynolds number, assuming that the characteristic length of the flow is the water depth (dimensionless); h = water depth (m); k_s = roughness height (m); and s, s_1, s_2, s_3 = auxiliary terms defined in text (dimensionless).

Eq. (6) is based on an approximation of the shifted Lambert W-function (Brkić and Praks 2019).

Swamee (1993) gives also an equation valid for both laminar and turbulent flow (Brkić 2018). On the other hand, the relation given by Swamee and Jain (1976), Eq. (11) of the discussed paper, is valid only for turbulent regime; most of the available approximations are valid only for turbulent regime (Brkić 2011).

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Notation

The following symbols are used in this paper:

f = Darcy (Moody) flow friction factor (dimensionless);
h = water depth (m);
k_s = roughness height, which correlates to pipe roughness height ε (m);
R = Reynolds number (dimensionless);
R_h = Reynolds number R assuming that the characteristic length of the flow is the water depth (dimensionless);
$v, V, s, s_1, s_2, s_3, E$ = auxiliary terms defined in text;
W = Lambert function;
y_1, y_2, and y_3 = switching functions;
α and b = Cheng's power terms; and
ε/D = relative roughness of inner pipe surface (dimensionless).

Subscripts

L = laminar;
TS = turbulent smooth; and
TR = turbulent rough.

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