

Rational Inequalities

- 1) Replace the inequality symbol with an equal sign and solve the resulting equation, obtaining boundary number(s).
- 2) Set the denominator equal to zero and solve, obtaining another boundary number. (Remember that this number itself cannot be in the solution set since it makes the denominator zero)
- 3) Place the boundary numbers on the number line and test a point in each region to determine which regions satisfy the inequality.
- 4) Write the solution set.

Example: Solve $\frac{x}{x+2} \geq 2$

Step #1

$$\frac{x}{x+2} = 2$$

$$(x+2) \cdot \frac{x}{(x+2)} = 2(x+2)$$

$$x = 2x + 4$$

$$x = -4$$

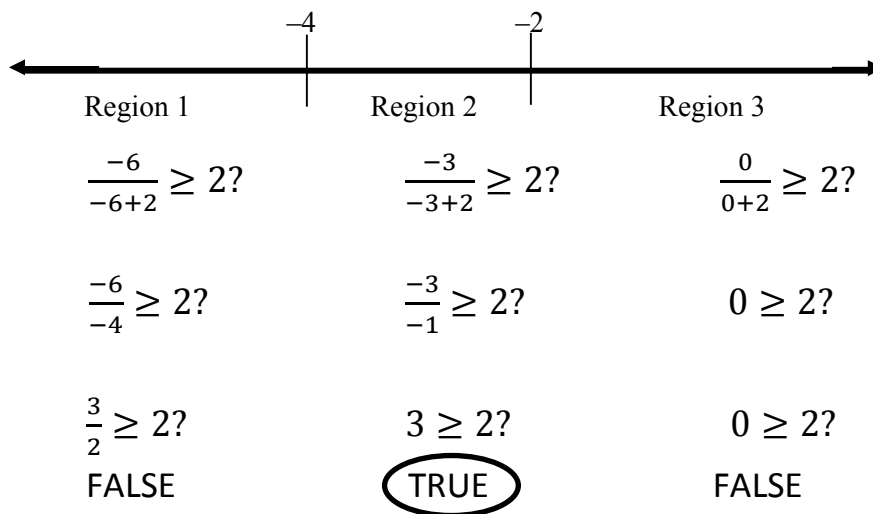
Step #2

$$x + 2 = 0$$

$$x = -2$$

The Boundary Numbers are -4 and -2

3. We pick any number in a region and test to see if that region makes the inequality true. We will test -6 in Region 1, -3 in Region 2, and 0 in Region 3.



5. Solution Set: Numbers in Region 2 (but NOT the endpoint -2 (since we cannot divide by zero))



Interval Notation: $[-4, -2)$

Set Builder Notation: $\{x | -4 \leq x < -2\}$

Rational Inequalities

Alternate Method

- 1) Manipulate the inequality so that we have zero on one side.
- 2) Force the other side of the equation into a single fraction.
- 3) Boundary numbers are found by setting both numerator and denominator to zero.
- 4) Determine the **sign** of the fraction in each region of our number line.
- 5) Graph the “true” intervals on the number line and write the solution set.

Step 1: Get zero on one side

$$\frac{x}{x+2} \geq 2$$

$$\frac{x}{x+2} - 2 \geq 0$$

Step 2: Combine left side into single fraction

$$\frac{x}{x+2} - \frac{2(x+2)}{(x+2)} \geq 0$$

$$\frac{x-2x-4}{(x+2)} \geq 0$$

$$\frac{-x-4}{x+2} \geq 0$$

Step 3: Find the boundary numbers

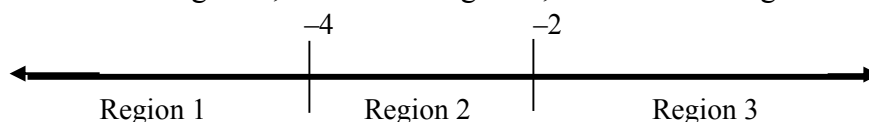
Numerator: $-x - 4 = 0$
 $-x = 4$
 $x = -4$

Denominator: $x + 2 = 0$
 $x = -2$

Draw number line with these 2 numbers marking the boundaries of our testing zones.

4. We pick any number in a region and test to see what the sign of the quotient is in that region.

We will test -100 in Region 1, -3 in Region 2, and 80 in Region 3.



$$\frac{-(-100)-4}{-6+2} = \frac{\text{positive}}{\text{negative}} = \text{negative}$$

FALSE

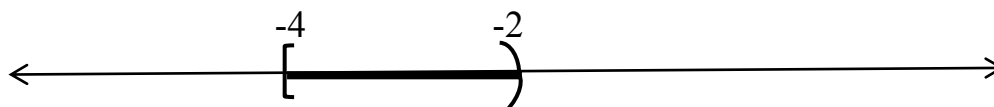
$$\frac{-(-3)-4}{-6+2} = \frac{\text{negative}}{\text{negative}} = \text{positive}$$

TRUE

$$\frac{-(80)-4}{-6+2} = \frac{\text{negative}}{\text{negative}} = \text{positive}$$

FALSE

5. Solution Set: Numbers in Region 2 (but NOT the endpoint -2 (since we cannot divide by zero))



Interval Notation: $[-4, -2)$

Set Builder Notation: $\{x | -4 \leq x < -2\}$