

Properties of Real Numbers

Property Example
Commutative

$$a+b = b+a$$

$$7+3 = 3+7$$

$$ab = ba$$

$$3 \cdot 5 = 5 \cdot 3$$

Associative

$$(a+b)+c = a+(b+c)$$

$$(2+4)+7 = 2+(4+7)$$

$$(ab)c = a(bc)$$

$$(3 \cdot 7) \cdot 5 = 3 \cdot (7 \cdot 5)$$

Distributive

$$a(b+c) = ab+ac$$

$$2(3+5) = 2 \cdot 3 + 2 \cdot 5$$

$$(b+c)a = ba+ca = ab+ac$$

$$(3+5)2 = 3 \cdot 2 + 5 \cdot 2$$

Properties of Negatives

Property

$$(-1)a = -a$$

Example

$$(-1) \cdot 5 = -5$$

$$-(-a) = a$$

$$-(-5) = 5$$

$$(-a)b = a(-b) = -(ab)$$

$$(-5) \cdot 7 = 5(-7) = -(5 \cdot 7)$$

$$(-a)(-b) = ab$$

$$(-4)(-3) = 4 \cdot 3$$

$$-(a+b) = -a-b$$

$$-(3+5) = -3-5 = -8$$

$$-(a-b) = -a+b = b-a$$

$$-(5-8) = -5+8 = 8-5$$

Properties of Absolute Value

Property

$$|a| \geq 0$$

Example

$$|-3| = 3 \geq 0$$

$$|a| = |-a|$$

$$|5| = |-5| = 5$$

$$|ab| = |a||b|$$

$$|-2 \cdot 5| = |-2| \cdot |5| = 2 \cdot 5$$

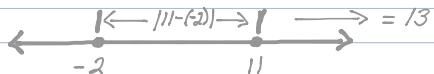
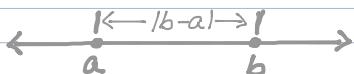
$$\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$$

$$\left| \frac{12}{-3} \right| = \frac{|12|}{|-3|} = \frac{12}{3}$$

Distance between points on the Real Number Line

$$d(a, b) = |b-a|$$

$$\text{Length of a line segment} = |b-a|$$



Distance between points on a graph

distance between $A(x_1, y_1)$ and $B(x_2, y_2)$

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Pythagorean Theorem

if $a^2 + b^2 = c^2$ then

$$d = \sqrt{a^2 + b^2} = c$$

Midpoint formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Exponents

Zero and negative Exponents

if $a \neq 0$ is any real number and n is a positive integer, then

$$a^0 = 1 \quad \text{and} \quad a^{-n} = \frac{1}{a^n}$$

Why is $a^0 = 1 \Rightarrow \frac{a^0}{a^0} = a^{0-0} = a^0 = 1$

Laws of Exponents

$$a^m a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n$$

$$(a/b)^n = \frac{a^n}{b^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$$

$$3^2 \cdot 3^5 = 3^{2+5} = 3^7$$

$$\frac{3^5}{3^2} = 3^{5-2} = 3^3$$

$$(3^2)^5 = 3^{2 \cdot 5} = 3^{10}$$

$$(3 \cdot 4)^2 = 3^2 \cdot 4^2$$

$$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$$

$$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$$

$$\frac{3^{-2}}{4^{-5}} = \frac{4^5}{3^2}$$

Properties of n^{th} Roots

Property

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{\sqrt[n]{a}} = \sqrt[n^2]{a}$$

$$\sqrt[n]{a^n} = a \text{ if } n \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \text{ if } n \text{ is even}$$

Example

$$\sqrt[3]{-8 \cdot 27} = \sqrt[3]{-8} \cdot \sqrt[3]{27} = (-2)(3) = -6$$

$$\sqrt[4]{\frac{16}{81}} = \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{2}{3}$$

$$\sqrt[5]{\sqrt[3]{243}} = \sqrt[5]{243} = 3$$

$$\sqrt[3]{(-5)^3} = -5, \sqrt[5]{2^5} = 2$$

$$\sqrt[4]{(-3)^4} = |-3| = 3$$

Rational Exponents

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{or} \quad a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

Special Product Formulas and Factoring Formulas

$$(A+B)(A-B) = A^2 - B^2 \quad \text{difference of squares}$$

$$(A+B)^2 = A^2 + 2AB + B^2$$

$$(A-B)^2 = A^2 - 2AB + B^2$$

$$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$(A-B)^3 = A^3 - 3A^2B - 3AB^2 - B^3$$

$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$

Formulas and stuff

Point-slope form

Slope intercept form

quadratic formula

$$y - y_1 = m(x - x_1)$$

$$y = mx + b$$

m = slope b = y -intercept

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

distance formula distance between $A(x_1, y_1)$ and $B(x_2, y_2)$

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Complex Numbers $a + bi$

$$\sqrt{-1} = i \Rightarrow i^2 = -1$$

$$i^3 = i^2 i = -1 i = -i$$

$$i^4 = i^2 i^2 = (-1)(-1) = 1$$

$$i^5 = i^2 i^2 i = (-1)(-1)i = i$$

$$i^6 = i^2 i^2 i^2 = (-1)(-1)(-1) = -1$$

Slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{\text{rise}}{\text{run}}$

Remember (order of operation)

Please Excuse My Dear Aunt Sally

P parenthesis

E exponents

M multiplication

D division

A addition

S subtraction

Compass Worksheet A

A1) $x = -1, y = 3$ find $3x^3 - 2xy$

$$\begin{aligned} & 3(-1)^3 - 2(-1)(3) \\ &= 3(-1) + 2(3) \\ &= -3 + 6 \\ &= 3 \end{aligned}$$

$$\begin{aligned} (-1)^3 &= (-1)(-1)(-1) \\ &= 1(-1) \\ &= -1 \end{aligned}$$

A2) The product of three less than x and five more than twice x

three less than $x \Rightarrow x - 3$
 five more than twice $x \Rightarrow 2x + 5$

$$\begin{aligned} (x-3)(2x+5) &= 2x^2 + 5x - 6x - 15 \\ &= 2x^2 - x - 15 \end{aligned}$$

A3) Scores of 83, 78, 77 on 3 of 4 test. What must test 4 be to have an average of 80.

Let x be the unknown test score, then
 $\frac{(83+78+77+x)}{4} = 80$

multiply both sides by 4

$$\frac{4(83+78+77+x)}{4} = 80(4)$$

add the three test scores

$$\begin{array}{r} 238 + x = 320 \\ -238 \quad -238 \\ \hline x = 82 \end{array} \quad \text{subtract 238 from both sides}$$

so the student must make an 82 on the 4th test in order to have an average of 80.

A4) What is the equation of the line that contains the points $(2, 3)$ and $(14, -6)$

Find the slope $\frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-6 - 3}{14 - 2} = \frac{-9}{12} = -\frac{3}{4}$

Using point slope form $y - y_1 = m(x - x_1)$

pt. $(2, 3)$

$$\Rightarrow y - 3 = -\frac{3}{4}(x - 2)$$

$$y - 3 = -\frac{3}{4}x + \frac{6}{4}$$

$$y - 3 = -\frac{3}{4}x + \frac{3}{2}$$

$$y = -\frac{3}{4}x + \frac{3}{2} + 3$$

$$\left. \begin{aligned} y &= -\frac{3}{4}x + \frac{3}{2} + \frac{6}{2} \\ y &= -\frac{3}{4}x + \frac{9}{2} \end{aligned} \right\}$$

Using slope intercept form
 $y = mx + b$ and pt. (2, 3)

$$3 = \frac{-3}{4}(2) + b$$

$$3 = \frac{-3}{2} + b$$

$$\frac{9}{2} = b \quad (3 = \frac{6}{2})$$

$$\frac{9}{2} = b \Rightarrow y = -\frac{3}{4}x + \frac{9}{2}$$

A5) $\frac{x^2 - x - 20}{x^2 - 16}$ Factor the top and the bottom

TOP $x^2 - x - 20$ Look at the factors of 20

1 20

2 10

(4 5) The difference between 4 and 5 is 1 so these are the numbers to use for the factors

$(x+4)(x-5)$ To get $-x$ in the middle the 5 must be neg. and the 4 must be pos.

$$4x - 5x = -x$$

Bottom $x^2 - 16$ is a difference of squares so the factors are $(x+4)(x-4)$

$\frac{(x+4)(x-5)}{(x+4)(x-4)}$ the $(x+4)$ cancels on top and bottom

$$= \frac{x-5}{x-4}$$

A6) First look at the answers. We are not solving for x . We are asked for an equation.

① A rope 36 feet long is cut into 3 pieces,

③ the second piece is four feet longer than the first

④ the last piece is 3 times as long as the second.

② If x represents the length of the first piece

What is the equation?

① $36 = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$ 3 pieces of rope added together equals 36 ft.

② $36 = \underline{x} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$ x represents the first piece

③ $36 = \underline{x} + \underline{x+4} + \underline{\hspace{1cm}}$ the 2nd piece is 4ft. longer than the first.

④ $36 = \underline{x} + \underline{x+4} + \underline{3(x+4)}$ the 3rd piece is 3 times as long as the 2nd.

A7) The product $(x^2+3)(x-1)$ is

$$x^3 \cdot x + x^2 \cdot (-1) + 3(x) + 3(-1)$$
$$x^3 - x^2 + 3x - 3$$

A8) If n is an integer which expression must be an even integer?

integer: whole number
an even number can be divided by 2 so to make any number even multiply by 2

A: $2n+1$ $2n$ is even so $2n+1$ is odd

B: $2n-1$ $2n$ is even so $2n-1$ is odd

C: $n+1$ n can be even or odd so $n+1$ can be even or odd $2+1=3$ $3+1=4$

D: $2n^2$ n^2 can be even or odd but $2n^2$ must be even $2(2)^2=2(4)=8$ $2(3)^2=2(9)=18$

E: n^2 n^2 can be even or odd $2^2=4$ $3^2=9$

A9) $2x^2 + 3x - 5$ where $x = -5$

$$2(-5)^2 + 3(-5) - 5$$
$$= 2(25) - 15 - 5$$
$$= 50 - 20$$
$$= 30$$

A10) Factor $2x^2 - 13x - 24$

Factors of 24

$(2x+3)(x-8)$	$-16+3=-13$	1 2	1 24	$2 \cdot 24 - 1 \cdot 1 = 47$	$1 \cdot 24 - 2 \cdot 1 = 22$
		2 12	2 12	$2 \cdot 12 - 1 \cdot 2 = 22$	$1 \cdot 12 - 2 \cdot 1 = 10$
		3 8	3 8	$2 \cdot 8 - 1 \cdot 3 = 13$	
		4 6			

B1) Keyword is product. multiply $(a+2b)(c-d)$

$$(a+2b)(c-d)$$

$$ac - ad + 2bc - 2bd$$

B2) $3(a+b)(a-b)$ where $a = -2$, $b = 3$

$$3(-2+3)(-2-3)$$

$$3(1)(-5)$$

$$3(-5)$$

$$-15$$

B3) The picture is the graph of which equation

Remember $y = mx + b$
 slope $\frac{\text{rise}}{\text{run}}$ y-intercept

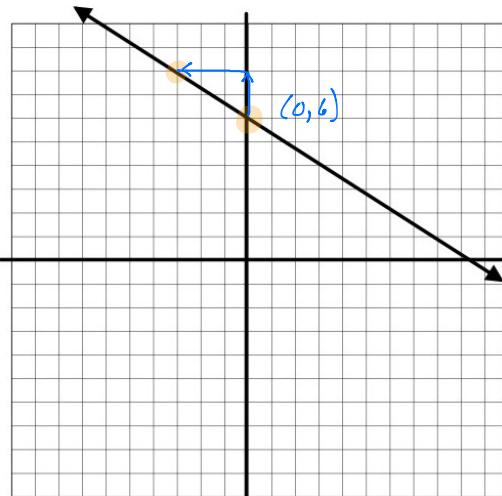
the y-intercept is 6

the slope is up 2 and over -3

$$\frac{\text{rise}}{\text{run}} = \frac{2}{-3} \text{ or } -\frac{2}{3}$$

so the equation is

$$y = -\frac{2}{3}x + 6$$



B4) The solution to $2(x+3) - 3(x+5) = 13$

$$2(x+3) - 3(x+5) = 13$$

$$2x + 6 - 3x - 15 = 13 \quad \text{distribute}$$

$$2x - 3x + 6 - 15 = 13 \quad \text{combine like terms}$$

$$-x - 9 = 13$$

$$\frac{-x - 9 + 9}{(-1)(-x)} = \frac{13 + 9}{(-1)(-1)} \quad \text{add 9 to both sides}$$

$$x = -22$$

B5) D dollars a week plus a commission of 8% on her total sales S

$$D + .08S$$

$$\text{weekly pay} = D + .08S$$

B6) product (multiply)

$$(D-5)(D^3 + 2D^2 - 2D + 3)$$

① multiply D times everyting in the 2nd set of ()

② multiply -5 times everyting in the 2nd set of ()

$$\textcircled{1} \quad D D^3 + 2D^2 D - 2D D + 3D$$

$$= D^4 + 2D^3 - 2D^2 + 3D$$

$$\textcircled{2} \quad -5D^3 + (-5)(2D^2) - (-5)2D + (-5)(3)$$

$$= -5D^3 - 10D^2 + 10D - 15$$

③ line up like terms and add

$$\textcircled{3} \quad D^4 - 3D^3 - 12D^2 + 13D - 15$$

B7) The points on the graph are $(-4, 2)$ and $(3, -4)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(3 - (-4))^2 + (-4 - 2)^2}$$

$$= \sqrt{(3 + 4)^2 + (-6)^2}$$

$$= \sqrt{7^2 + 6^2}$$

$$= \sqrt{49 + 36}$$

$$= \sqrt{85}$$

B8) $\frac{a^{-3}b^2}{a^5b^{-4}}$ Rule for division: For like bases, top exponent minus bottom exponent

$$a^{-3-5} b^{2-(-4)} = a^{-8} b^{2+4} = a^{-8} b^6 = \frac{b^6}{a^8}$$

do not leave a neg. exponent
if exponent on top is neg bring it to the bottom to make it pos.
if bottom is neg bring it to the top to make it pos.

You could also do $\frac{b^2 b^4}{a^5 a^3} = \frac{b^6}{a^8}$

make all exponents pos. first.

B9) $(a^3 b^2 c)^2$ Rule: for multiplication and division
the exponent outside the () is multiplied by the exponents inside the ()

$$a^{3(2)} b^{2(2)} c^{1(2)} = a^6 b^4 c^2$$

B10) $3(2x+5) - 4(x-2) = 3(2x+2) + 1$

$$\begin{aligned} 3(2x) + 3(5) - 4(x) + (-4)(-2) &= 3(2x) + 3(2) + 1 \\ 6x + 15 - 4x + 8 &= 6x + 6 + 1 \\ 6x - 4x + 15 + 8 &= 6x + 7 \\ 2x + 23 &= 6x + 7 \\ -2x - 7 &= -2x - 7 \\ \frac{16}{4} &= \frac{4x}{4} \end{aligned}$$

$$4 = x$$

C1) $4x^3 - 2xy$ where $x = -2$ and $y = -3$

$$\begin{aligned} 4(-2)^3 - 2(-2)(-3) &= (-2)(-2)(-2) \\ &= 4(-2) \\ &= -8 \\ &= -4y \end{aligned}$$

C2) the product of four more than twice x and six less than x
multiply $2x+4$ and $x-6$
separate the two factors

$$(2x+4)(x-6)$$

$$2x(x) - 2x(6) + 4(x) - 4(6)$$

$$2x^2 - 12x + 4x - 24$$

$$2x^2 - 8x - 24$$

Combine like terms

$$-1x + 4 = -8x$$

C3) A student earns scores of 85, 76, and 78 on three of four test
What must the student score on the fourth test to have an average of 80?

The fourth test is the unknown so replace it with a variable. I'll use x .

$$\frac{85+76+78+x}{4} = 80 \text{ solve for } x$$

$$4\left(\frac{239+x}{4}\right) = 80(4) \quad \text{multiply both sides by 4}$$

$$239+x = 320 \quad \text{subtract 239 from both sides}$$

$$\underline{-239} \quad \underline{-239}$$

$$x = 81 \quad \text{so the student must score an 81 to have an 80 average.}$$

C4) the equation of the line through the points $(2, 3)$ and $(4, 6)$

① Find the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$\begin{matrix} x_1, y_1 & x_2, y_2 \\ (2, 3) & (4, 6) \end{matrix}$$

$$m = \frac{6-3}{4-2} = \frac{3}{2}$$

② Use point slope form ($y - y_1 = m(x - x_1)$) to solve for the equation of the line using pt. $(2, 3)$

$$y - 3 = \frac{3}{2}(x - 2) \quad \text{solve for } y$$

$$y - 3 = \frac{3}{2}x - 2\left(\frac{3}{2}\right)$$

$$\underline{y - 3 + 3} = \frac{3}{2}x \quad \underline{-2\left(\frac{3}{2}\right) + 3} \quad \text{add 3 to both sides}$$

$y = \frac{3}{2}x + 3$ is the equation of the line through the two points given

C5) $\frac{x^2 - x - 30}{x^2 - 36}$ factor the top and bottom

$$x^2 - x - 30 \Rightarrow \text{Factors of } 30 \text{ when subtracted gives us } -1$$

$$(x+5)(x-6)$$

$x^2 - 36$ is a difference of squares

$$\text{then } \frac{(x+5)(x-6)}{(x+6)(x-6)} = \frac{x+5}{x+6}$$

C6) Each circle has a diameter of 6 and radius $r = \frac{d}{2} = \frac{6}{2} = 3$

so counting down the y-axis

$$y = (-6) + (-6) + (-3) = -15$$

Counting over the x-axis

$$x = 3$$

so $B = (3, -15)$

C7) $(x-3)(x^2 + 3x + 9)$

$$\begin{aligned} x(x^2) + x(3x) + x(9) \\ - 3(x^2) + (-3)(3x) + (-3)(9) \end{aligned} \Rightarrow \begin{aligned} &x^3 + 3x^2 + 9x \\ &- 3x^2 - 9x - 27 \\ &x^3 + 0 + 0 - 27 \end{aligned} \quad \begin{array}{l} \text{line up like terms} \\ \text{and add} \end{array}$$

$$= x^3 - 27$$

C8) If $A^\circ B = 2A + 3B$, find $4^\circ 5$

This just means to replace A with 4 and B with 5

$$2(4) + 3(5)$$

$$= 8 + 15$$

$$= 23$$

C9) The perimeter, P of $\triangle ABC$

$$P = \overline{AB} + \overline{AC} + \overline{BC}$$

We need the distance of each line

$$\begin{aligned} \overline{AB} &= \sqrt{(4 - (-4))^2 + (2 - 4)^2} \\ &= \sqrt{8^2 + (-6)^2} \\ &= \sqrt{64 + 36} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

$$\begin{aligned} \overline{AC} &= |4 - (-2)| = 6 \\ \overline{BC} &= |-4 - 4| = 8 \end{aligned}$$

points are $A = (4, -2)$

$$B = (-4, 4)$$

$$C = (4, 4)$$

distance formula is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{aligned} \overline{AB} &= 10 \\ \overline{AC} &= 6 \\ \overline{BC} &= 8 \\ P &= 24 \end{aligned}$$

C10) $64^{\frac{3}{2}}$ the exponent is power over root power/root

$$(\sqrt[3]{64})^2 = 4^2 = 16$$

D1) $2x^2y - 3xy$ where $x=-1$ and $y=-2$

$$2(-1)^2(-2) - 3(-1)(-2)$$

$$2(1)(-2) - 3(2)$$

$$2(-2) - 6$$

$$-4 - 6$$

$$-10$$

D2) solutions of (means solve for x) $x^2 - 2x - 48 = 0$

$$x^2 - 2x - 48 = 0 \quad \text{Factor}$$

$$(x+6)(x-8) = 0$$

$$x+6=0 \quad x-8=0 \quad \text{set each factor} = 0$$

$$x = -6 \quad x = 8$$

$$\begin{array}{r} 48 \\ 1 \end{array}$$

$$\begin{array}{r} 24 \\ 2 \end{array}$$

$$\begin{array}{r} 16 \\ 3 \end{array}$$

$$\begin{array}{r} 12 \\ 4 \end{array}$$

$$\begin{array}{r} 8 \\ 6 \end{array}$$

D3) the sum of the solutions of $x^2 - 2x - 48 = 0$
add the solutions together

$$-6 + 8 = 2$$

D4) the sum of the solutions of $x^2 + 3x = 28$

$$x^2 + 3x = 28$$

$$-28 - 28$$

$$x^2 + 3x - 28 = 0$$

$$(x-4)(x+7) = 0$$

$$x-4=0 \quad x+7=0$$

$$x = 4 \quad x = -7$$

$$\begin{array}{r} 28 \\ 1 \end{array}$$

$$\begin{array}{r} 28 \\ 128 \end{array}$$

$$\begin{array}{r} 14 \\ 2 \end{array}$$

$$\begin{array}{r} 14 \\ 47 \end{array}$$

$$4 - 7 = -3$$

D5) sum of solutions $2x^2 - x = 15$

$$2x^2 - x - 15 = 0$$

$$(2x+5)(x-3)$$

$$2x+5=0 \quad x-3=0$$

$$2x = -5 \quad x = 3$$

$$x = \frac{-5}{2}$$

$$\begin{array}{r} 2 \\ 12 \\ \hline 15 \end{array}$$

$$\begin{array}{r} 15 \\ 35 \\ \hline 35 \end{array}$$

$$3 - \frac{5}{2} = \frac{6}{2} - \frac{5}{2} = \frac{1}{2}$$

D6) sum of solutions, $x^2 - x = 6$

$$x^2 - x - 6 = 0$$

$$(x+2)(x-3)$$

$$x = -2 \quad x = 3$$

$$3 - 2 = 1$$

D7) Solutions, $x^2 - 5x = -6$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3)$$

$$x = 2 \quad x = 3$$

$$D8) \frac{x^2 - 5x + 6}{x-2} = \frac{(x-2)(x-3)}{x-2}$$

D9) $x^2 + 11x + K = 0$ where $x = -4$ solve for K

$$(-4)^2 + 11(-4) + K = 0$$

$$16 - 44 + K = 0$$

$$-28 + K = 0$$

$$K = 28$$

D10) solutions, $x^2 - 10x + 24 = 0$

$$x^2 - 10x + 24 = 0$$

$$(x-4)(x-6)$$

$$x = 4 \quad x = 6$$

$$E1) \frac{x^2 + 2x - 24}{x+2} \text{ where } x = -5$$

$$\frac{(-5)^2 + 2(-5) - 24}{-5+2} = \frac{25 - 10 - 24}{-3} = \frac{-9}{-3} = 3$$

$$E2) \text{ Simplify } \frac{\sqrt{50}}{3} + \frac{5\sqrt{3}}{6}$$

$$\frac{\sqrt{50}}{3} + \frac{5\sqrt{3}}{6} = \frac{\sqrt{25 \cdot 2}}{3} + \frac{5\sqrt{3}}{6} = \frac{5\sqrt{2}}{3} + \frac{5\sqrt{3}}{6} = \frac{6(5\sqrt{2})}{6} + \frac{5\sqrt{3}}{6} = \frac{10\sqrt{2} + 5\sqrt{3}}{6}$$

E3) distance between pt. A and pt. B $A = (-3, 3)$ $B = (4, -1)$

$$= \sqrt{(4 - (-3))^2 + (-1 - 3)^2}$$

$$= \sqrt{7^2 + 4^2}$$

$$= \sqrt{49 + 16}$$

$$= \sqrt{65}$$

E4) distance between $(-5, 2)$ and $(1, -6)$

$$\begin{aligned}d &= \sqrt{(1 - (-5))^2 + (-6 - 2)^2} \\&= \sqrt{6^2 + 8^2} \\&= \sqrt{36 + 64} \\&= \sqrt{100} \\&= 10\end{aligned}$$

$$E5) 27^{-2/3} = \frac{1}{27^{2/3}} = \frac{1}{(\sqrt[3]{27})^2} = \frac{1}{3^2} = \frac{1}{9}$$

E6) equation of a parabola opening down with vertex $(-2, 4)$
 $y = (x - h)^2 + k$ where (h, k) is the vertex
 $-(x + 2)^2 + 4$

E7) The solution falls between what two integers

$$\begin{aligned}2x + 8 &= 7x + 20 \\-2x - 20 &\quad -2x - 20 \\-12 &= 5x \\5 &\quad 5 \\x = -\frac{12}{5} &\approx -2.4 \quad -3 < -2.4 < -2\end{aligned}$$

E8) solution of the system of equations

$$\begin{aligned}3x + 4y &= a \\2x - 4y &= 14a \\5x + 0 &= 15a \\5 &\quad 5 \\x = 3a &\quad 3(3a) + 4y = a \\9a + 4y &= a \\-9a &\quad -9a \\4y &= -8a \\4 &\quad 4 \\y &= -2a \\(3a, -2a)\end{aligned}$$

E9) What is the slope of the line

$$\begin{aligned}2x - 3y &= 12 \\-2x &\quad -2x \\-3y &= -2x + 12 \\-3 &\quad -3 \quad -3\end{aligned}$$

$$y = \frac{2}{3}x - 4$$

$$m = \frac{2}{3}$$

$$E10) (x+2y)^2 - 4y(x+y)$$

$$\begin{aligned} &= (x+2y)(x+2y) - 4y(x+y) \\ &= x^2 + 2xy + 2xy + 4y^2 - 4xy - 4y^2 \\ &= x^2 + 4xy + 4y^2 - 4xy - 4y^2 \\ &= x^2 + 4xy - 4xy + 4y^2 - 4y^2 \\ &= x^2 \end{aligned}$$

$$F1) \begin{array}{|c|c|c|c|} \hline x & 0 & 2 & -2 \\ \hline y & 5 & 1 & 9 \\ \hline \end{array} \quad (0, 5) \quad (2, 1) \quad (-2, 9)$$

\uparrow
y-intercept

$$m = \frac{1-5}{2-0} = \frac{-4}{2} = -2$$

$$y = -2x + 5$$

$$F2) \frac{20}{\sqrt{x^2+7}} = 5 \quad x^2 = ?$$

$$\sqrt{x^2+7} \frac{(20)}{\sqrt{x^2+7}} = 5 \sqrt{x^2+7}$$

$$\frac{20}{5} = \frac{5\sqrt{x^2+7}}{5}$$

$$4 = \sqrt{x^2+7}$$

$$4^2 = (\sqrt{x^2+7})^2$$

$$16 = x^2 + 7$$

$$-7 \quad -7$$

$$9 = x^2$$

$$F3) \frac{\sqrt{48}}{3} + \frac{5\sqrt{5}}{6} = \frac{2\sqrt{48} + 5\sqrt{5}}{6} = \frac{2\sqrt{16 \cdot 3}}{6} + \frac{5\sqrt{5}}{6} = \frac{8\sqrt{3} + 5\sqrt{5}}{6}$$

F4) What is the slope of $3x + 2y = 6$

$$\begin{array}{rcl} 3x + 2y = 6 \\ -3x \quad -3x \end{array}$$

$$\frac{2y}{2} = \frac{-3x + 6}{2}$$

$$y = -\frac{3}{2}x + 3 \quad m = -\frac{3}{2}$$

F5) line perpendicular to $y = -2x + 5$
 perpendicular slope is opposite sign and the reciprocal

$$m = \frac{1}{2} \quad y = \frac{1}{2}x + 5 \text{ is } \perp \text{ to } y = -2x + 5$$

F6) distance $(3\sqrt{5}, 0)$ and $(6\sqrt{5}, 4)$

$$\begin{aligned} d &= \sqrt{(6\sqrt{5} - 3\sqrt{5})^2 + (4 - 0)^2} \\ &= \sqrt{(3\sqrt{5})^2 + 4^2} \\ &= \sqrt{45 + 16} \\ &= \sqrt{61} \end{aligned}$$

F7) system of equations, solution

$$\begin{aligned} 2x - 3y &= 4 \quad \text{and } y = x \\ \text{use substitution} \\ 2x - 3x &= 4 \quad y = -4 \quad (-4, -4) \\ -x &= 4 \\ x &= -4 \end{aligned}$$

F8) $(a+b) = 3a - 2b$ and $(6 \times x) = 8$ then $x = ?$

$$\begin{array}{r} 3(6) - 2x = 8 \\ 18 - 2x = 8 \\ -18 \quad -18 \\ \hline -2x = -10 \\ \hline -2 \quad -2 \\ x = 5 \end{array}$$

F9) $12x^2 + 11x - 36$ is the product of $(3x-4)$ and what?

$$\begin{array}{r} 12x^2 + 11x - 36 \\ (3x-4)(4x+9) \end{array} \quad \begin{array}{r} 12 \quad 36 \\ \cancel{3} \cancel{4} \quad \cancel{4} \cancel{9} \end{array}$$

$$F10) 4\sqrt{\frac{16}{81}} = \frac{\sqrt{16}}{\sqrt{81}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

$$G1) \begin{array}{|c|c|c|c|} \hline x & -3 & 0 & 3 \\ \hline y & -6 & -4 & -2 \\ \hline \end{array} \quad (-3, -6) \quad (0, -4) \quad (3, -2)$$

$$m = \frac{-4 - (-6)}{0 - (-3)} = \frac{-4 + 6}{3} = \frac{2}{3} \quad y = \frac{2}{3}x - 4$$

$$\begin{aligned} 3y &= 2x - 12 \\ -2x + 3y &= -12 \\ 2x - 3y &= 12 \end{aligned}$$

$$G2) \frac{18}{\sqrt{x^2+4}} = 6 \quad x^2 = ?$$

$$\frac{\sqrt{x^2+4} \cdot (18)}{\sqrt{x^2+4}} = 6\sqrt{x^2+4}$$

$$\frac{18}{6} = \frac{6\sqrt{x^2+4}}{6}$$

$$3 = \sqrt{x^2+4}$$

$$3^2 = (\sqrt{x^2+4})^2$$

$$9 = x^2 + 4$$

$$-4 \quad -4$$

$$5 = x^2$$

$$G3) \frac{\sqrt{18}}{2} + \frac{\sqrt{32}}{3} = \frac{3\sqrt{18}}{6} + \frac{2\sqrt{32}}{6} = \frac{3\sqrt{9 \cdot 2}}{6} + \frac{2\sqrt{16 \cdot 2}}{6} = \frac{3 \cdot 3\sqrt{2}}{6} + \frac{4 \cdot 2\sqrt{2}}{6} = \frac{9\sqrt{2} + 8\sqrt{2}}{6} = \frac{17\sqrt{2}}{6}$$

$$G4) \frac{\frac{5a}{b}}{\frac{2a}{a-b}} = \frac{5a}{b} \cdot \frac{a-b}{2a} = \frac{5(a-b)}{2b}$$

G5) a line parallel to $y = -\frac{1}{2}x + 5$

$y = -\frac{1}{2}x + 5$ parallel lines have the same slope but different y-intercepts

$$y = -\frac{1}{2}x + 7$$

G6) distance between $(3\sqrt{3}, -1)$ and $(6\sqrt{3}, 2)$

$$\begin{aligned} d &= \sqrt{(6\sqrt{3} - 3\sqrt{3})^2 + (2 - (-1))^2} \\ &= \sqrt{(3\sqrt{3})^2 + 3^2} \\ &= \sqrt{27 + 9} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

G7) solution of the system of equations

$$x + 2y = 7a$$

$$3x - 2y = 5a$$

$$\frac{4x}{4} + 0 = \frac{12a}{4}$$

$$x = 3a$$

$$(3a, 2a)$$

$$3a + 2y = 7a$$

$$\underline{-3a} \quad \underline{-3a}$$

$$\frac{2y}{2} = \frac{4a}{2}$$

$$y = 2a$$

$$G8) (x-3)^2 + 3(2x-3) =$$

$$\begin{aligned} & (x-3)(x-3) + 3(2x-3) \\ &= x^2 - 3x - 3x + 9 + 6x - 9 \\ &= x^2 - 6x + 9 + 6x - 9 \\ &= x^2 - 6x + 6x + 9 - 9 \\ &= x^2 \end{aligned}$$

$$G9) 8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4}$$

$$G10) \frac{x^2 - kx + 24}{x-12} = x-2 \quad \text{solve for } k$$

$$\frac{(x-12)(x^2 - kx + 24)}{x-12} = (x-2)(x-12)$$

$$\begin{aligned} x^2 - kx + 24 &= x^2 - 12x - 2x + 24 \\ -x^2 &\quad -24 \quad -x^2 \quad -24 \\ -\frac{kx}{x} &= -\frac{14x}{x} \\ -k &= -14 \\ k &= 14 \end{aligned}$$