

3a Base case: $()$ is balanced

Inductive case: if b is balanced, (b) , $()b$, and $b()$ are balanced.

b Assume L is regular.

Let w be n '('s followed by n ')'s

$\Rightarrow |w| \geq n \Rightarrow w = xy^kz$ s.t. 1) $y \neq \epsilon$, 2) $|xy| \leq n, \forall k \geq 0, xy^kz \in L$

by (2), y must be i '('s, where $i \geq 1$ (by (1))

However, then xy^2z would have more '('s than ')'s

\Rightarrow not balanced \Rightarrow proof by contradiction.

4. regex of length m , input string length n

Time complexity:

A state has at maximum 2 paths to transition to the next state and the number of states at a time is $\leq \frac{\text{total states}}{\text{the number of}}$. You need to do this for each of the characters of the string, therefore the time complexity is $O(mn)$.

Space complexity:

You only need $O(n)$ to store the input string, and then as each operand has two outgoing links, storing would require $O(2^n + n)$