



# Economic growth and inequality: The role of public investment<sup>☆</sup>

Stephen J. Turnovsky

University of Washington, Seattle, WA 98195, United States



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## ABSTRACT

The relationship between growth and inequality is complex. After discussing some general background issues, motivated by extensive empirical evidence this paper focuses on public investment as a key determinant of the relationship. Two alternative frameworks, each offering sharply contrasting perspectives, are presented. The first employs the “representative consumer theory of distribution” where agent heterogeneity originates with wealth endowments. It yields an equilibrium in which aggregate dynamics drives distributional dynamics. In the second, agent heterogeneity arises from idiosyncratic productivity shocks and generates an equilibrium in which distributional dynamics drive growth. The impact of government investment on growth and inequality are shown to contrast sharply in the two approaches, thus illustrating the complexity of the growth–inequality relationship.

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## 1. Background and overview

Interest in the relationship between the level of economic development, the rate of economic growth, and measures of inequality originated with the seminal paper by Kuznets (1955). In that paper Kuznets argued that the level of a country's development and its degree of income inequality could be described by an inverted-U relationship. The relationship was essentially a statistical one that Kuznets explained in terms of “dual economy dynamics”, associated with the structural transformation from an agricultural to an industrial economy.

Kuznets' proposition has stimulated an extensive literature analyzing the relationship between income inequality and growth and/or economic development. Much of this has taken the form of running regressions of growth rates on measures of inequality, and other control variables, with the results generally being inconclusive. For example, Anand and Kanbur (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1996), and others find that inequality has an adverse effect on growth. Various explanations for this have been offered, including: (i) the political economy consequences of inequality, (ii) the negative impact of inequality on education, and (iii) capital market imperfections and credit constraints.

<sup>☆</sup> This is a revised version of a plenary talk presented to the 2015 Conference of the Society for Computation and Economics, held in Taipei, June 2015. I gratefully acknowledge the sponsorship of the *Journal of Economic Dynamics and Control* for this presentation. My research on economic growth and income inequality has been conducted jointly with various coauthors. I am particularly grateful to Cecilia García-Peñalosa, with whom I began my work in this area some years ago. The two models that are presented in some detail here were developed with Santanu Chatterjee and Yoseph Getachew, respectively. Much of my research on this topic has been supported by the Castor Professorship and more recently by the Van Voorhis Professorship at the University of Washington and that too is gratefully acknowledged.

In contrast, other studies find a positive, or a more ambiguous, relationship; see e.g. Li and Zou (1998), Forbes (2000), and Barro (2000). These explanations include: (i) the relative savings propensities of rich vs. poor, (ii) investment indivisibilities, and (iii) incentives.

From a theoretical standpoint, the diversity of these empirical results is unsurprising. Both the growth rate and income distribution are endogenous outcomes of a larger economic system. Therefore, any relationship between them should reflect the underlying set of forces to which both measures are simultaneously responding. These forces are likely to change over time and vary between economies. As Ehrlich and Kim (2007) have suggested, “association” is a more appropriate characterization of any relationship between growth and inequality, rather than trying to attribute any direct causal link. This means that the growth-inequality relationship can be understood only as a joint equilibrium outcome of a consistently specified general equilibrium growth model.

A fundamental element crucial to the relationship between growth and inequality is the presence of heterogeneity across agents. There are many such sources, the most obvious including the rates of time preference, tastes, endowments, technology, idiosyncratic shocks, progressive tax rates, etc. Under the most general circumstances, to solve for growth and inequality simultaneously is intractable. The interaction between aggregate quantities and their distributions across many diverse individuals is too complex to enable us to advance beyond making a few general qualitative statements about the steady-state equilibrium relationship between per capita output and wealth distribution; see Sorger (2000). In particular, it is infeasible to characterize the dynamic evolution of wealth or income distribution as the economy develops over time. To make progress in this dimension requires some additional structure to be imposed on the system.

Thus, if we assume that the underlying utility function driving individuals' behavior is homogeneous, then for certain important sources of heterogeneity we can exploit the aggregation procedures due to Gorman (1953), which render the problem tractable. In this case, the macroeconomic equilibrium and distribution have a simple recursive structure. First, summing over individuals leads to a macroeconomic equilibrium in which aggregate quantities are determined independently of any distributional aspects. Having derived the aggregate quantities, their distributions across individuals are then determined by how these aggregate quantities influence factor returns. This equilibrium structure has led Caselli and Ventura (2000) to characterize this as a “representative consumer theory of distribution”.<sup>1</sup> In terms of the causality debate relating growth and inequality, this formulation assumes away any causality running from inequality to growth and can address only the reverse, as indeed was Kuznets' original focus.

In constructing any formal economic model, the choice of assumptions one makes involves a tradeoff between realism and tractability. While homogeneity is a strong assumption, in this case the tractability it yields suggests a high payoff, especially in light of the versatility of the questions it enables one to address. Moreover, the assumption of homogeneous utility is routine throughout modern growth theory, and indeed macrodynamics in general. While it includes the widely adopted constant elasticity utility function, it is much more inclusive than that.

Almost all of my research studying the growth-inequality relationship involves the recursive equilibrium structure just described. As the source of heterogeneity it has focused on the initial distribution of endowments of assets across individuals. Usually this has meant endowments of physical capital, although it has also been extended to include human capital and/or ability in considering skills, as well as foreign assets in an international environment.<sup>2</sup>

In my view, dispersion of asset endowments across agents is arguably the most important source of heterogeneity and that is the main reason why I have focused on that aspect. There is certainly much greater diversity among inherited wealth across individuals than can possibility exist between individuals' rates of time preference, which as a practical matter can deviate by only a percentage point or two across agents. Recently, particular prominence to endowments as a source of inequality has been provided by the influential work of Piketty (2013) and Stiglitz (2012).<sup>3</sup>

But, as noted, there are other sources and ways of generating heterogeneity, and these too should be discussed. One of the earliest was heterogeneous rates of time preference. This was first studied by Becker (1980) and shown to lead to the extreme outcome of a degenerate long-run wealth distribution, with the most patient person ultimately owning all the capital. An alternative approach assumes that people are initially identical, with heterogeneity being endogenized through uninsurable idiosyncratic random shocks; see Krusell and Smith (1998), Castañeda et al. (1998), and others. While I shall focus most of my attention on the heterogeneity stemming from endowments, in the latter part of the paper I shall briefly discuss an approach to inequality-growth dynamics which is driven by such idiosyncratic technology shocks.

In parameterizing inequality, one must decide which of the several proposed measures one wishes to consider; see Atkinson (1970). Of these, the Gini coefficient, which is the most prevalent, and the coefficient of variation have the most desirable characteristics, and for our purposes, the latter is particularly convenient. After selecting the appropriate inequality measure, there is the issue of which economic variable is of concern. The most widely analyzed, both theoretically and particularly empirically, is income inequality. Wealth inequality is also important, and indeed in our analysis turns out to be a key driving force generating income inequality. However, because of serious limitations with respect to data availability, empirical studies of wealth inequality are sparse. Also, if one is concerned with more general welfare measures,

<sup>1</sup> As a pedagogic point this means that the representative consumer model can incorporate certain important sources of heterogeneity and does not require that all agents be identical as is typically understood.

<sup>2</sup> Some of this research is summarized in the presentation I made to the NZ Association of Economists, and reported in Turnovsky (2013).

<sup>3</sup> Other papers using this approach include Chatterjee (1994), Maliar and Maliar (2001), and Sorger (2002).

then welfare inequality naturally becomes relevant. In general, these various inequality measures need not move together, thereby giving rise to conflicts between them.<sup>4</sup>

With both aggregate output and inequality being endogenous, their co-movement – albeit it positive or negative—is reflecting some underlying structural change or policy shock. This raises the question of what types of shocks are most pertinent to consider. A natural starting point is to address the distributional consequences of structural changes in the economy, such as productivity increases. Do improvements in technology, which increase the growth rate and enhance development, lead to more or less inequality? That was essentially the motivation underlying the Kuznets curve. A second question relates to the distributional consequences of government policies – particularly fiscal policies—both as exogenous shocks themselves but also as responses to structural change. This involves issues pertaining to the type of government spending (e.g. public investment versus public consumption) and its mode of financing (e.g. taxes on labor income, capital income, or consumption). Moreover, once one introduces distortionary income taxes one has to distinguish between pre-tax and post-tax measures of inequality. These also may move in contrary directions to fiscal policy changes, as we shall demonstrate in [Section 6](#).

In this paper we shall focus exclusively on the role of public investment as a mechanism determining the dynamics of the growth rate and inequality. There are several reasons for doing so. First, the relationship between public investment, economic growth, and the implications for economic inequality is an important issue in its own right, generating extensive empirical research among development economists.<sup>5</sup> Second, this example provides a convenient vehicle for illustrating the “representative consumer” approach to analyzing the growth-inequality relationship already referred to. But, in addition, it has the added advantage that it provides an example for which one can obtain a tractable closed form solution under idiosyncratic productivity shocks. As we shall see, both approaches make dramatically different assumptions, and as a result offer sharply different insights into what is a complex dynamic relationship between growth and inequality.

The remainder of the paper is outlined as follows. [Section 2](#) provides a brief summary of the literature dealing with public investment, economic growth, and inequality. [Section 3](#) sets out an analytical model, based on heterogeneous endowments. [Sections 4](#) and [5](#) discuss its macroeconomic equilibrium and distributional dynamics, while [Sections 6](#) and [7](#) report some numerical simulations. [Sections 8](#) and [9](#) summarize an alternative dynamic model based on idiosyncratic shocks, highlighting the different perspectives that alternative approaches offer. [Section 10](#) concludes with some general comments. In discussing these models, the technical details are minimized; readers interested in pursuing them are referred to the papers from which this material is largely drawn.

## 2. Public investment, growth, inequality: some general observations

Public investment in infrastructure as a source of economic growth has been widely debated in both developing and advanced economies. Several emerging-market countries e.g. India, China, Brazil have undertaken extensive public investment, which has no doubt contributed to their recent high growth rates. In contrast, several European countries have reduced public spending as an austerity measure in response to the financial crisis of 2008. Contemporaneously with these diverse policies, we have witnessed rising income inequality, both in emerging markets and in most OECD countries. This raises the important question of the relationship between public investment directed toward growth enhancement and its consequences for income inequality.

Interest in the theoretical connection between public investment, output, and growth dates back to [Arrow and Kurz \(1970\)](#), who addressed the issue using a one sector neoclassical growth model. Beginning with [Barro \(1990\)](#), an extensive literature has evolved addressing the issue in an endogenous growth framework. Barro’s original model introduced productive government spending as a flow, but many subsequent contributions have introduced it in the form of public capital, which along with private capital can be accumulated and depreciates gradually; see [Futagami et al. \(1993\)](#), [Glomm and Ravikumar \(1994\)](#) and [Turnovsky \(1997\)](#) for early contributions, and more recently [Agénor \(2011\)](#) for an extensive survey of this theoretical literature.<sup>6</sup> There is also an extensive empirical literature that focuses on estimating the productive elasticity of government expenditure in output. The consensus is that infrastructure contributes positively and significantly to output. [Bom and Ligthart \(2014\)](#) provide an exhaustive review of the literature and place the elasticity at between 0.10 and 0.20, far below the original (and controversial) estimate of 0.39 obtained by [Aschauer \(1989\)](#).

By interacting with labor and private capital in the production process, public investment has a direct impact on relative factor returns, and is therefore critical in the evolution of wealth and income distributions. Moreover, because of its diverse nature, public investment is likely to have significant redistributive consequences. Public investment in the form of public transportation is geared toward the less affluent members of the economy and is likely to reduce inequality, whereas public investment directed to enhancing high speed communication is more likely to favor the more affluent having access to these facilities and is likely to exacerbate inequality.

<sup>4</sup> All of these measures are summary measures of inequality. Policymakers may be concerned with the extremes of the wealth or income distributions. As a topical example, the 2012 presidential election campaign in the United States focused much attention on the top 1% of the income distribution. This issue appears to be continuing in the 2016 US presidential campaign.

<sup>5</sup> World Bank and IMF economists in particular focus intensively on this issue; see e.g. [World Bank \(2006\)](#), [Calderón and Servén \(2010, 2014\)](#), [Seneviratne and Sun \(2013\)](#) and [Eden and Kraay \(2014\)](#).

<sup>6</sup> The significance of the stock vs. flow formulation depends upon the issue that one wishes to address; the difference is in the dimension of the dynamics and hence is mainly reflected in the complexity of the transitional path.

The empirical evidence on the relationship between infrastructure investment and inequality is less definitive and more anecdotal. As examples, we may cite the following. [Calderón and Servén \(2004, 2010\)](#), [Fan and Zhang \(2004\)](#), [Ferranti et al. \(2004\)](#), and [Lopez \(2004\)](#) find that public investment has both promoted growth and helped mitigate inequality. In addition, [Brakman et al. \(2002\)](#) find that government spending on infrastructure has increased regional disparities within Europe. On the other hand, [Artadi and Sala-i-Martin \(2003\)](#) suggest excessive public investment has contributed to rising income inequality in Africa. Furthermore, in India, [Banerjee and Somanathan \(2007\)](#) report that access to critical infrastructure services and public goods is positively correlated with social status. [World Bank \(2006\)](#) finds that quality and performance of state-provided infrastructure services worst in India's poorest states.

This diversity of empirical findings highlights the need for a well-specified analytical framework, within which the interaction between public spending, economic growth, and inequality can be systematically addressed. In the remaining sections of this paper we present two alternative models describing the interaction between public investment, growth, and alternative measures of inequality. The first employs the “representative consumer theory of distribution” framework summarized above. The second is based on idiosyncratic shocks. The two approaches differ sharply in terms of their respective frameworks, and as a result they offer sharply contrasting perspectives on the role of public investment in the growth-inequality relationship.

### 3. The “representative consumer” approach

In setting out the representative consumer approach to distribution we focus on two issues pertaining to public spending and its financing:

- (1) The mechanism whereby government spending on public infrastructure and accompanying taxation policies affect the distributions of wealth, and income (pre- and post-tax), in the short run, during transition, and in the long run.
- (2) The dynamics of the growth-inequality relationship along the transitional path.

A key element of the economy is that it experiences a growing stock of a government-provided good (public capital). This interacts with the aggregate stock of private capital to generate composite externalities for both labor productivity in production and the labor-leisure allocation in utility. The government has a range of fiscal instruments available to finance its investment, namely distortionary taxes on capital income, labor income, and consumption, as well as a non-distortionary lump-sum tax (equivalent to government debt). The accumulation of public capital and the spillovers it generates serves both as an engine of sustained growth, and also as a driver of relative returns to capital and labor, with consequences for the evolution of wealth and income inequality.

We compare an increase in the rate of government investment on public capital, financed by the use of the alternative fiscal instruments. But even with the recursive equilibrium structure, the closed-form equilibrium obtained is too complex to study analytically; instead, we must resort to numerical simulations. Nevertheless several interesting and empirically testable hypotheses emerge.

- (1) Government investment leads to a persistent increase in wealth inequality over time, regardless of how it is financed.
- (2) The time paths of both pre- and post-tax income inequality are highly sensitive to the financing policy adopted, and in many cases are characterized by sharp intertemporal tradeoffs. For example, while government investment financed by a lump-sum or consumption tax leads to a short-run *decline* in income inequality, this is completely reversed over time, leading to an *increase* in the long-run dispersion of income. Public expenditure financed by capital or labor income taxes yields sharp differences between pre-tax and post-tax income inequality, both in the short run and over time. Regardless of the financing, both measures of income inequality increase over time.
- (3) The growth-income inequality relationship depends critically on (a) the financing policies for government spending, and (b) the time period under consideration (short run, transition path, or long run). In their sensitivity analysis [Chatterjee and Turnovsky \(2012\)](#) have shown how it also depends on how externalities impinge on allocation decisions. These results underscore the ambiguity in the growth-inequality relationship that is characteristic of the empirical studies, noted.<sup>7</sup>

One of the attractive features of the “representative consumer” model is its versatility. Even though we are focusing on public capital as contributing to some form of physical infrastructure, we can view this as a specific example of the analytical method. It is straightforward to modify this framework so that what we call public investment might in fact be public education, health care, or technology. Essentially, we are offering a blueprint that can be readily modified to address other pertinent aspects pertaining to the link between public expenditure, growth, and inequality.

<sup>7</sup> [Chatterjee and Turnovsky \(2012\)](#) also demonstrate how public investment generates tradeoffs between average welfare and its distribution.

### 3.1. Analytical framework

We consider a closed-economy in which both private and public capital are accumulated, so that the evolution of the economy proceeds gradually along a transitional path.<sup>8</sup>

#### 3.1.1. Firms and technology

Firms are identical and are indexed by  $j$ . The representative firm produces output in accordance with the CES production function:

$$Y_j = A \left[ \alpha (X_p L_j)^{-\rho} + (1 - \alpha) K_j^{-\rho} \right]^{-1/\rho} \quad (1)$$

where  $K_j$  and  $L_j$  represent the individual firm's capital stock and employment of labor, respectively, and  $s \equiv 1/(1 + \rho)$  denotes the elasticity of substitution in production between capital and effective units of labor. Production is influenced by an aggregate composite externality,  $X_p$ , which we take to be a geometric weighted average of the economy's aggregate stocks of private and public capital,  $(K, K_G)$  namely  $X_p = K^\varepsilon K_G^{1-\varepsilon}$ , ( $0 \leq \varepsilon \leq 1$ ). This composite production externality interacts with "raw" labor to create labor efficiency units, which in turn interact with private capital to produce output. The production function has constant returns to scale in both the private factors  $(K_j, L_j)$  and in the accumulating factors  $(K_j, K, K_G)$ , and accordingly, sustains an equilibrium of endogenous growth. The composite externality represents a combination of the role of private capital as knowledge as in [Romer \(1986\)](#), together with public capital as in [Futagami et al. \(1993\)](#) and subsequent authors. Formulating the externality as a composite can be justified by the observation that an economy's infrastructure contributing to labor efficiency typically involves a partnership between the public and private sector; see [Calderón and Servén \(2014\)](#) and [Seneviratne and Sun \(2013\)](#).

All firms face identical competitive production conditions, and hence will choose identical levels of employment of labor and private capital, i.e.,  $K_j = K$  and  $L_j = L$ , for all  $j$ , where  $K$  and  $L$  denote the average economy-wide levels of private capital and labor employment, respectively. Letting  $z \equiv K_G/K$  denote the ratio of the economy-wide stock of public capital to private capital, we can write  $y \equiv Y/K$ , the ratio of economy-wide average output to private capital, in the form:

$$y \equiv y(z, l) = A \left[ 1 - \alpha + \alpha \{ (1 - l) z^{1-\varepsilon} \}^{-\rho} \right]^{-1/\rho} \quad (2)$$

where  $l \equiv 1 - L$  denotes the average allocation of time to leisure in the economy. With both factors being paid their respective private marginal products, the economy-wide returns to capital and labor, determined in competitive factor markets, may be expressed as

$$r = r(z, l) \equiv (1 - \alpha) A^{-\rho} y(z, l)^{1+\rho} \quad (3a)$$

$$w = w(z, l) K; \quad w(z, l) \equiv \alpha A^{-\rho} y(z, l)^{1+\rho} z^{-\rho(1-\varepsilon)} (1 - l)^{-(1+\rho)} \quad (3b)$$

#### 3.1.2. Consumers

There is a continuum of infinitely-lived consumers, indexed by  $i$ , who are similar in all respects except for their initial endowments of private capital,  $K_{i,0}$ .<sup>9</sup> Each consumer is also endowed with one unit of time that can be allocated to either leisure,  $l_i$ , or work,  $L_i = 1 - l_i$ . Consumer  $i$  maximizes utility over an infinite horizon from his flow of consumption,  $C_i$ , and leisure, using the following CES function:

$$U_i = \int_0^\infty \frac{1}{\gamma} [C_i^{-\nu} + \theta (X_U l_i)^{-\nu}]^{-\gamma/\nu} e^{-\beta t} dt \quad (4)$$

where  $q \equiv 1/(1 + \nu)$  denotes the intra-temporal elasticity of substitution between consumption and leisure in the utility function, and  $e \equiv 1/(1 - \gamma)$  represents the inter-temporal elasticity of substitution. Each consumer's utility is also affected by an aggregate composite externality,  $X_U$ , which is a geometric weighted average of the economy's aggregate stocks of public and private capital  $X_U = K^\varphi K_G^{1-\varphi}$ , ( $0 \leq \varphi \leq 1$ ). This composite externality interacts with the time allocated to leisure by consumer  $i$  to generate utility benefits, which in turn are weighted by  $\theta$  in yielding overall utility. Several reasons motivate the preferences specified in (4) (which includes the constant elasticity function as a special case) and are discussed in detail by [Chatterjee and Turnovsky \(2012\)](#).<sup>10</sup>

Each agent chooses  $C_i$ ,  $l_i$ , and his rate of capital accumulation,  $\dot{K}_i$  to maximize (4) subject to the externality, their initial endowment of capital,  $K_{i,0}$ , and the following flow budget constraint

$$\dot{K}_i = (1 - \tau_k) r K_i + (1 - \tau_w) w (1 - l_i) - (1 + \tau_c) C_i - T \quad (5)$$

where  $\tau_k$ ,  $\tau_w$ , and  $\tau_c$  are (constant) tax rates levied on the agent's capital income, labor income, and consumption expenditures, respectively and  $T$  represents a lump-sum tax levied by the government uniformly on all  $i$  individuals. In making

<sup>8</sup> The model discussed here draws heavily on [Chatterjee and Turnovsky \(2012\)](#).

<sup>9</sup> Private capital can be viewed as an amalgam of physical and human capital, as in [Romer \(1986\)](#).

<sup>10</sup> One of the main reasons is that by generalizing the utility function to be of CES form allows consumption and leisure to be Edgeworth substitutes or complements according to whether  $e \gtrless q$ .

these decisions the agent takes the real wage rate and the return on private capital, determined in competitive factor markets, as given, and in addition he also treats all tax and policy variables as given.

Performing the optimization implies standard first-order conditions with respect to  $C_i, l_i$ , and  $K_i$  together with the transversality condition  $\lim_{t \rightarrow \infty} \lambda_i K_i e^{-\rho t} = 0$ , where  $\lambda_i$  is agent  $i$ 's shadow value of capital. Dividing the optimality condition with respect to  $l_i$  by that with respect to  $C_i$  yields

$$\frac{C_i}{l_i} = \Omega(z, l)K \quad (6)$$

where  $\Omega(z, l) \equiv [(1 - \tau_w)\omega(z, l)z^{u(1-\varphi)}/\theta(1 + \tau_c)]^{1/(1+\nu)}$ . From (6) we see that each agent chooses the same ratio of consumption to leisure, which depends on  $\tau_w, \tau_c, z, l$ , and  $K$ .

### 3.1.3. Government

The government provides non-rival and non-excludable public capital, in accordance with

$$K_G = G = gY, 0 < g < 1 \quad (7)$$

where  $G$  is the flow of new public investment, so that  $g$  represents the fraction of aggregate output allocated to public investment by the government.<sup>11</sup> The government finances its investment by tax revenues and maintains a balanced budget at all points of time:

$$G = \tau_k rK + \tau_w w(1 - l) + \tau_c C + T \quad (8)$$

Dividing (8) by  $K$ , we can write this in the form

$$gy(z, l) = \tau_k r(z, l) + \tau_w \omega(z, l)(1 - l) + \tau_c \Omega(z, l)l + \tau y(z, l) \quad (8')$$

where lump-sum tax revenues are expressed as a proportion  $\tau$  ( $0 < \tau < 1$ ) of aggregate output, namely  $T = \tau Y$ . It is clear from (8') that if the tax and expenditure rates,  $\tau_k, \tau_w, \tau_c$ , and  $g$  are maintained constant, then as  $z$  and  $l$  progress along a transitional path the fraction of output levied as lump-sum taxes,  $\tau$ , will continually vary in order for the government budget to remain in balance.<sup>12</sup>

## 4. Macroeconomic equilibrium

In general, the economy-wide *average* of a variable,  $X_i$  is represented by  $(1/N) \sum_i^N X_i \equiv X$ . Because of the homogeneity of the utility function and perfect factor markets, we can show that all individuals choose the same growth rates for consumption and leisure, implying that average consumption,  $C$ , and leisure,  $l$ , will also grow at the same rates; i.e.,

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C} = \frac{\dot{l}_i}{l_i} = \frac{\dot{l}}{l}, \quad \text{for each } i \quad (9)$$

As a result, the system can be aggregated perfectly over agents.<sup>13</sup> Each individual, however, will choose different *levels* of consumption and leisure, depending upon his resources; in particular,

$$l_i = \pi_i l \text{ and } \frac{1}{N} \sum_i^N \pi_i = 1 \quad (10)$$

where *relative* leisure,  $\pi_i$ , chosen by agent  $i$  is constant over time and is determined by (19) below. Summing (6) over all agents yields the aggregate consumption-leisure ratio

$$\frac{C}{l} = \Omega(z, l)K \quad (6')$$

Next, summing (5) and invoking  $T/K = \tau Y/K = \tau y(k, l)$  yields the growth rate of aggregate capital

$$\frac{\dot{K}}{K} = (1 - \tau_k)r + [(1 - \tau_w)\omega(z, l)(1 - l) - (1 + \tau_c)\Omega(z, l)l - \tau y(z, l)] \quad (11)$$

Combining the latter with (8') yields the aggregate goods market clearing condition

$$\frac{\dot{K}}{K} = (1 - g)y(z, l) - \Omega(z, l)l \quad (11')$$

<sup>11</sup> For simplicity, we abstract from depreciation of either form of capital.

<sup>12</sup> With lump-sum taxes our abstraction of government debt involves little loss of generality.

<sup>13</sup> The common growth rate across agents, expressed by (9) is critical in facilitating the aggregation; see Turnovsky and García-Peñalosa (2008).



The transitional dynamics of the aggregate economy are driven by the evolution of the ratio of public to private capital,  $z$ , and leisure,  $l$ :

$$\frac{\dot{z}}{z} = g \frac{y(z, l)}{z} - [(1-g)y(z, l) - \Omega(z, l)l] \quad (12a)$$

$$\frac{\dot{l}}{l} = \frac{H(z, l)}{J(z, l)} \quad (12b)$$

where

$$\begin{aligned} H(z, l) &\equiv (1 - \tau_k)r(z, l) - \beta - (1 - \gamma)\frac{\dot{K}}{K} \\ &\quad + \left\{ \frac{\theta [z^{-(1-\varphi)}\Omega]^v (v + \gamma)(1 - \varphi)}{1 + \theta [z^{-(1-\varphi)}\Omega]^v} - \left[ \frac{(1 - \gamma) + (1 + v)\theta [z^{-(1-\varphi)}\Omega]^v}{1 + \theta [z^{-(1-\varphi)}\Omega]^v} \right] \frac{\Omega_z z}{\Omega} \right\} \dot{z} \\ J(z, l) &\equiv 1 - \gamma + \left[ \frac{(1 - \gamma) + (1 + v)\theta [z^{-(1-\varphi)}\Omega]^v}{1 + \theta [z^{-(1-\varphi)}\Omega]^v} \right] \frac{\Omega_l l}{\Omega} \end{aligned}$$

and  $\dot{K}/K$  is determined by (11').

Eq. (12a) is obtained directly from the definition of  $z$  and asserts that the growth of the aggregate public to private capital ratio equals the differential growth rates of the two components. Eq. (12b) describes the required adjustment in leisure necessary to ensure that the rate of return on consumption equals the changing rate of return on capital, as the productive capacity of the economy evolves through the accumulation of public and private capital. These two relationships are the analogs to the equilibrium relationships in Futagami et al. (1993) and Turnovsky (1997) and are independent of any distributional measures.

#### 4.1. Steady state and aggregate dynamics

Provided the system is stable, the aggregate economy will converge to a balanced growth path characterized by a constant public to private capital ratio,  $\tilde{z}$ , and leisure,  $\tilde{l}$ . Setting  $\dot{z} = \dot{l} = 0$  in (12), the corresponding steady-state conditions can be expressed as:

$$g \frac{y(\tilde{z}, \tilde{l})}{\tilde{z}} = (1 - g)y(\tilde{z}, \tilde{l}) - \Omega(\tilde{z}, \tilde{l})\tilde{l} \quad (13a)$$

$$\frac{(1 - \tau_k)r(\tilde{z}, \tilde{l}) - \beta}{1 - \gamma} = g \frac{y(\tilde{z}, \tilde{l})}{\tilde{z}} \equiv \tilde{\psi} \quad (13b)$$

These two equations determine  $\tilde{z}$  and  $\tilde{l}$ , such that public capital, private capital, and consumption, all grow at the common rate, that we denote by  $\tilde{\psi}$ . Given  $\tilde{z}$  and  $\tilde{l}$ , (6') then determines the steady-state consumption-private capital ratio,  $\tilde{c} = \Omega(\tilde{z}, \tilde{l})\tilde{l}$ . Finally, the transversality condition, together with the optimality condition for private capital accumulation implies  $\tilde{\psi} < \tilde{r}(1 - \tau_k)$ , which combined with (11) in steady state, yields

$$\tilde{c} > \frac{(1 - \tau_w)\omega(\tilde{z}, \tilde{l})(1 - \tilde{l}) - \tau y(\tilde{z}, \tilde{l})}{1 + \tau_c} \quad (14)$$

For the long-run growth rate to be sustainable, consumption expenditure (inclusive of tax) must exceed after-tax labor income (inclusive of lump-sum taxes), so that some (net) capital income is allocated to consumption. This viability condition imposes a restriction on leisure that is necessary to constrain the growth rate and is important in characterizing the distributional dynamics.

To approximate the aggregate transitional dynamics, we linearize (12) around its steady state ( $\tilde{z}$  and  $\tilde{l}$ ), expressing the system in the form

$$\begin{bmatrix} \dot{z} \\ \dot{l} \end{bmatrix} = \begin{bmatrix} a_{11}^{(-)} & a_{12}^{(+)} \\ a_{21}^{(+)} & a_{22}^{(+)} \end{bmatrix} \begin{bmatrix} z - \tilde{z} \\ l - \tilde{l} \end{bmatrix} \quad (15)$$

where  $a_{ij}$  are obtained as appropriate partial derivatives of (12a) and (12b), and the signs in parentheses indicate the likely signs of the corresponding element, implying that the dynamics exhibits saddle-point stability. The stable transition path of the aggregate economy is described by

$$z(t) = \tilde{z} + [z(0) - \tilde{z}]e^{\mu t} \quad (16a)$$

$$l(t) = \tilde{l} + \frac{a_{21}}{(\mu - a_{22})}[z(t) - \tilde{z}] \quad (16b)$$

where  $\mu$  is the stable (negative) eigenvalue corresponding to the linearized system in (15). Chatterjee and Turnovsky (2012) have conducted extensive numerical simulations over plausible ranges of parameter values and in all cases find that the slope of the saddle path is negative, implying that along the transition path, the evolution of leisure is inversely related to that of the public-private capital ratio. Intuitively, an increase in public to private capital raises the productivity of private capital, raising the wage rate, and inducing agents to increase their labor supply and to reduce their leisure. Since both infrastructure and private capital represent stocks that are accumulated gradually, we rule out instantaneous jumps in  $z$ . However, leisure, the consumption-capital ratio, and the various growth rates can respond instantaneously to new information.

## 5. Distributional dynamics

The characterization of the aggregate economy in Section 4 represents the behavior of the *averages* of the key economic variables. The fact that this is independent of any distributional aspects is a consequence of the homogeneity of the utility function and the perfect aggregation that this permits. The next step is to characterize the behavior of a cross-section of agents, and to determine the evolution of that cross-section relative to that of the average. We focus on the distributional dynamics of private capital (wealth), pre-tax, and post-tax income.<sup>14</sup>

### 5.1. Distribution of private capital (wealth)

To derive the dynamics of the relative capital stock of individual  $i$ ,  $k_i \equiv K_i/K$  (the agent's relative wealth) we combine (5) and (11), while taking into account (3), (6), and (6'). To facilitate this, we define:

$$\Delta(z, l) \equiv (1 - \tau_w)\omega(z, l) - \tau_y(\tilde{z}, \tilde{l}); \Gamma(z, l) \equiv [(1 + \tau_c)\Omega(z, l) + (1 - \tau_w)\omega(z, l)] > 0$$

This enables us to express the evolution of relative capital (wealth) in the convenient form

$$\dot{k}_i(t) = -\Gamma(z, l)(l_i - l) + [\Gamma(z, l)l - \Delta(z, l)](k_i(t) - 1) \quad (17)$$

Using this notation, the viability condition (14) can be expressed as  $\Gamma(\tilde{z}, \tilde{l}) > \Delta(\tilde{z}, \tilde{l})$  implying that the dynamic Eq. (17) is locally unstable near the steady state. A key element of a stable (bounded) solution includes the steady state to (17), which implies the positive relationship between the agent's steady-state share of the private capital stock and leisure:

$$\tilde{l}_i - \tilde{l} = \left[ \tilde{l} - \frac{\Delta(\tilde{z}, \tilde{l})}{\Gamma(\tilde{z}, \tilde{l})} \right] (\tilde{k}_i - 1) \quad (18)$$

Thus the transversality condition implies that an individual who in the long run has above-average private capital, given by  $\tilde{k}_i - 1$ , enjoys above average leisure in steady state, i.e.,  $\tilde{l}_i - \tilde{l} > 0$ .<sup>15</sup> Using (10), this equation also yields agent  $i$ 's (constant) allocation of leisure time:

$$\pi_i - 1 = \left( 1 - \frac{\Delta(\tilde{z}, \tilde{l})}{\Gamma(\tilde{z}, \tilde{l})} \right) (\tilde{k}_i - 1) \quad (19)$$

Linearizing (17) around the steady-state levels  $\tilde{z}$ ,  $\tilde{l}$ , and  $\tilde{k}_i$ , while noting (16a), (16b), and (18), implies the following dynamic equation for relative wealth

$$\dot{k}_i = \delta_1(\tilde{z}, \tilde{l})(\tilde{k}_i - 1)[z(t) - \tilde{z}] + \delta_2(\tilde{z}, \tilde{l})[k_i(t) - \tilde{k}_i] \quad (20)$$

where

$$\begin{aligned} \delta_1(\tilde{z}, \tilde{l}) &\equiv \frac{1}{\Gamma(\tilde{z}, \tilde{l})} (\Delta(\tilde{z}, \tilde{l})\Gamma_z(\tilde{z}, \tilde{l}) - \Delta_z(\tilde{z}, \tilde{l})\Gamma(\tilde{z}, \tilde{l})) \\ &\quad + \left( \frac{\Delta(\tilde{z}, \tilde{l})}{\Gamma(\tilde{z}, \tilde{l})} \Gamma_l(\tilde{z}, \tilde{l}) - \Delta_l(\tilde{z}, \tilde{l}) \right) \left( \frac{a_{21}}{\mu - a_{22}} \right) \\ \delta_2(\tilde{z}, \tilde{l}) &\equiv \Gamma(\tilde{z}, \tilde{l})\tilde{l} - \Delta(\tilde{z}, \tilde{l}) > 0 \end{aligned}$$

<sup>14</sup> Chatterjee and Turnovsky (2012) also address the implications for welfare inequality.

<sup>15</sup> This is consistent with various sources of empirical evidence that finds a negative relationship between wealth and relative labor supply; see for example, Holtz-Eakin et al. (1993), Cheng and French (2000), and Algan et al. (2003).



Eq. (20) highlights how the evolution of the economy-wide ratio of public to private capital impacts the evolution of relative wealth, both directly, and indirectly through  $l(t)$ . With  $\delta_2 > 0$ , the bounded solution to (20) is of the form

$$k_i(t) - 1 = (\tilde{k}_i - 1) \left[ 1 + \frac{\delta_1}{\mu - \delta_2} [z(t) - \tilde{z}] \right] = (\tilde{k}_i - 1) \left[ 1 + \frac{\delta_1}{\mu - \delta_2} (z_0 - \tilde{z}) e^{\mu t} \right] \quad (21)$$

Setting  $t=0$  in (21) gives

$$k_i(0) - 1 \equiv k_{i,0} - 1 = (\tilde{k}_i - 1) \left[ 1 + \frac{\delta_1}{\mu - \delta_2} (z_0 - \tilde{z}) \right] \quad (21')$$

Thus the evolution of agent  $i$ 's relative capital stock is determined as follows. First, given the steady-state of the aggregate economy, and his initial endowment,  $k_{i,0}$ , (21') determines the agent's steady-state relative stock of capital,  $(\tilde{k}_i - 1)$ , which together with (21) then yields its entire time path,  $k_i(t)$ , and together with (19) determines the agent's (constant) relative leisure,  $\pi_i$ .

Since all distributional variables are expressed relative to the mean, we can measure their dispersion in canonical form, by using the coefficient of variation. For example, given the linearity of (21) and (21') in terms involving  $k_i$ , we can immediately transform these equations into corresponding relationships for the coefficient of variation for the distribution of capital, which serves as a convenient measure of wealth inequality. Therefore,

$$\sigma_k(t) = \left[ 1 + \frac{\delta_1}{\mu - \delta_2} (z(t) - \tilde{z}) \right] \tilde{\sigma}_k, \quad (22a)$$

where  $\sigma_k(t)$  denotes the coefficient of variation for relative wealth at time  $t$ .<sup>16</sup> Setting  $t = 0$ , the relationship between the initial distribution of wealth and its steady-state distribution is given by

$$\tilde{\sigma}_k = \left[ 1 + \frac{\delta_1}{\mu - \delta_2} (z_0 - \tilde{z}) \right]^{-1} \sigma_{k,0} \quad (22b)$$

Thus (22a) and (22b) completely characterize the evolution of wealth inequality, given its initial distribution and the initial stock of the economy-wide infrastructure to private capital ratio. From (22b), we see that the steady-state distribution of relative wealth is therefore determined by (i) its initial distribution,  $\sigma_{k,0}$ , and (ii) the long-run change in the public-private capital ratio, which depends upon the source or nature of the structural change.<sup>17</sup>

## 5.2. Distribution of pre-tax income

Gross income of individual  $i$  is defined as  $Y_i = r(z, l)K_i + \omega(z, l)(1 - l_i)K$ , while average income is  $Y = [r(z, l) + \omega(z, l)(1 - l)]K$ . Using (18), the relative pre-tax income of agent  $i$ ,  $y_i \equiv Y_i(t)/Y(t)$ , is

$$y_i(t) - 1 = s_k(t)(k_i(t) - 1) - [1 - s_k(t)] \frac{l(t)}{1 - l(t)} \left[ 1 - \frac{\Delta(\tilde{z}, \tilde{l})}{\Gamma(\tilde{z}, \tilde{l})} \right] (\tilde{k}_i - 1) \quad (23)$$

where  $s_k(t) \equiv r(z, l)/y(z, l) = (1 - \alpha)[Ay(z, l)]^{-\rho}$  denotes the equilibrium share of output received by capital. Thus the distribution of pre-tax income can be written as

$$\sigma_y(t) = \zeta(t)\sigma_k(t) \quad (24a)$$

where

$$\zeta(t) \equiv s_k(t) + [1 - s_k(t)] \frac{l(t)}{1 - l(t)} \left[ 1 - \frac{\Delta(\tilde{z}, \tilde{l})}{\Gamma(\tilde{z}, \tilde{l})} \right] \left[ 1 + \frac{\delta_1}{\mu - \delta_2} (z(t) - \tilde{z}) \right]^{-1} \quad (24b)$$

Thus pre-tax income inequality can be shown to be a weighted average of wealth and labor income inequality. While wealth inequality,  $\sigma_k(t)$ , evolves gradually, the initial jump in leisure,  $l(0)$ , which impacts on  $\sigma_y(0)$  through its effect on labor income inequality, means that any structural or policy shock, causes an initial jump in income inequality, after which it too evolves continuously. As a result, short-run pre-tax income inequality,  $\sigma_y(0)$ , may over-shoot its long-run equilibrium,  $\tilde{\sigma}_y$ .

<sup>16</sup> Eq. (22a) provides a summary measure of total wealth inequality. It is straightforward to integrate the distributions across a sub-set of agents, enabling one to focus on the relatively poor or the relatively rich.

<sup>17</sup> The fact that the long-run distribution depends upon the initial distribution reflects a hysteresis property resulting from the "zero root" associated with (9). This turns out to have important implications for wealth and income inequality that are explored in another context by Atolia et al. (2012).

### 5.3. Distribution of post-tax income

A key function of income taxes is their redistributive property, necessitating the distinction between pre-tax and post-tax income equality, the latter being arguably of greater significance. We define after-tax relative income as<sup>18</sup>:

$$y_i^N(t) = \frac{(1 - \tau_k)rK_i + (1 - \tau_w)wK(1 - l_i(t))}{(1 - \tau_k)rK + (1 - \tau_w)wK(1 - l)} \quad (25)$$

Recalling the expression for the before-tax relative income and the definition of  $\zeta$ , we can express after-tax income inequality as

$$\sigma_y^N(t) = \zeta^N(t)\sigma_k(t) \quad (26a)$$

where

$$\zeta^N(t) = \frac{(1 - \tau_w)\zeta(t) + s_k(t)(\tau_w - \tau_k)}{(1 - s_k(t))(1 - \tau_w) + s_k(t)(1 - \tau_k)} = \zeta(t) + (1 - \zeta(t)) \frac{s_k(t)(\tau_w - \tau_k)}{(1 - s_k(t))(1 - \tau_w) + s_k(t)(1 - \tau_k)} \quad (26b)$$

Thus, after-tax income distribution will be more (less) equal than the before-tax income distribution, according to whether  $\tau_k > (<) \tau_w$ . Eqs. (26a) and (26b) also indicate that the income tax rates,  $\tau_k$  and  $\tau_w$ , exert two effects on the after-tax income inequality. First, by influencing  $\zeta(t)$ , they influence gross factor returns, and therefore the before-tax distribution of income. But in addition, they have direct redistributive effects that are captured by the second term on the right hand side of (26b) and influence after-tax income distribution.

One important characteristic of the distributional equilibrium described above is that the effects of any changes on the measures of inequality—whether due to structure or policy—are path-dependent, meaning that their impact on long-run inequality will depend upon the time path followed by the underlying change. This implies that economies that have similar aggregate characteristics, in terms of their respective levels of income, may nevertheless have substantially different distributions of wealth and income if their (common) level of technology was accumulated through different time paths.<sup>19</sup> This observation has potentially important implications, with respect to policies, such as the granting of foreign aid, that are typically implemented over time. It means that the time path over which a fixed amount of foreign aid is granted will permanently influence the distribution of wealth and income in the recipient economy.<sup>20</sup>

## 6. Numerical analysis

As indicated, given the complexity of the model, we analyze it using numerical simulations. These are based on the following parameterization of a benchmark economy.

Benchmark specification of structural parameters	
Preferences	$\gamma = -1.5, \beta = 0.04, \theta = 1.75, v = 0$
Production	$A = 0.6, \alpha = 0.6, \rho = 0$
Externalities	$\varphi = \varepsilon = 0.6$
Fiscal	$g = 0.05 \quad \tau = 0.05$

The choice of parameters have been justified and discussed at some length by Chatterjee and Turnovsky (2012). They are generally standard and non-controversial. Here it suffices to observe that the benchmark corresponds to the standard Cobb–Douglas production function and constant elasticity utility function. The other point to note is that the externality parameters are set at  $\varphi = \varepsilon = 0.6$ . As is discussed elsewhere, this choice of parameters is chosen mainly to ensure that (i) the elasticity of public capital lies in the realistic range reported by Bom and Ligthart (2014) and that (ii) the productive elasticity of aggregate capital does not dominate that of the firm's capital stock, problems that typically arise with the more conventional polar specifications of the externalities.

The benchmark government spending ratio,  $g$ , is assumed to be 5% of GDP, roughly consistent with the rate of public infrastructure spending for most OECD countries. Table 1A summarizes the benchmark equilibrium, in which the ratio of public-private capital is 0.53, output-private capital ratio is around 0.24, leisure is  $\bar{l} = 0.714$ , and the growth rate is 2.29%. These equilibrium values are all plausible, and generally consistent with the available empirical evidence.

<sup>18</sup> Note that this measure ignores the direct distributional impacts of lump-sum transfers, which are arbitrary.

<sup>19</sup> For example, Mexico and Turkey have comparable levels of per capita income GDP (around \$15,000 in 2010) but very different income distributions as measured by their Gini coefficients (around 0.52 and 0.40, respectively).

<sup>20</sup> We do not illustrate this here, but the issue is addressed in detail by Atolia et al. (2012).

Table 1

A Benchmark steady-state equilibrium.

$g=0.05$ $\varepsilon=\varphi=0.6$ (composite externality), $q=s=1$				
Financing policy	$\tilde{z}$	$\tilde{l}$	$\tilde{y}$	$\tilde{\psi}$ (%)
Lump-sum tax-financing $\tau=0.05$	0.531	0.714	0.243	2.29

B Increase in government spending: aggregate and distributional effects benchmark specification  
 $g = 0.05\text{--}0.08$

i. Steady-state aggregate Effects

Policy change		$d\tilde{z}$	$d\tilde{l}$	$d\tilde{\psi}$
Lump-sum tax-financed increase in $g$	$d\tau=0.030$	0.259	−0.01	0.206
Capital income tax-financed increase in $g$	$d\tau_k=0.075$	0.353	−0.006	0.101
Labor income tax-financed increase in $g$	$d\tau_w=0.05$	0.268	0.002	0.168
Consumption tax-financed increase in $g$	$d\tau_c=0.096$	0.265	−0.001	0.179

ii. Distributional effects<sup>a</sup>

Policy change	Wealth inequality		Pre-tax income inequality		Post-tax income inequality	
	Short-run	Long-run	Short-run	Long-run	Short-run	Long-run
Lump-sum tax-financed increase in $g$	0	2.736	−2.602	4.996	−2.602	4.996
Capital income tax-financed increase in $g$	0	3.527	2.707	11.976	−9.174	−0.149
Labor income tax-financed increase in $g$	0	2.805	−8.256	−0.331	−0.110	7.933
Consumption tax-financed increase in $g$	0	2.952	−3.117	4.955	−3.117	4.955

<sup>a</sup> All distributional effects are reported as percentage changes relative to their pre-shock levels:  $[(\sigma_j(t) - \bar{\sigma}_{j,0})/\bar{\sigma}_{j,0}] \times 100$ ,  $j = k, y, u$ .

### 6.1. Increase in government spending on infrastructure

We consider the effect of an unanticipated and permanent increase in the rate of government investment from its benchmark rate of 5–8% of GDP. We compare the responses under the following four financing schemes: (i) lump-sum tax, (ii) capital income tax, (iii) labor income tax, and (iv) consumption tax. In all cases we assume that the economy starts from an initial benchmark equilibrium in which government expenditure is fully financed by lump-sum taxes, and all distortionary tax rates are zero, i.e.  $\tau_c = \tau_w = \tau_k = 0$ , so that  $g_0 = \tau_0 = 0.05$  in Eq. (8'). For the distortionary taxes, we assume that the corresponding tax rate is set such that it fully finances the long-run *change* in government expenditure. Thus starting from  $\tau_c = \tau_w = \tau_k = 0$ , the corresponding required changes in the tax rates (given the underlying tax base) are respectively  $d\tau = 0.03, d\tau_k = 0.075, d\tau_w = 0.05, d\tau_c = 0.096$  [see Table 1B]. This means that during the transition as the tax base is changing, residual lump-sum tax financing must also be employed to ensure that the budget remains balanced at all times.

#### 6.1.1. Aggregate effects

Table 1B(i) shows the effect of an increase in government spending on the steady-state of the aggregate economy. In all cases, the direct stimulus to public investment causes the equilibrium ratio of public to private capital,  $\tilde{z}$ , to increase. Except when spending is financed by a tax on labor income, leisure falls in the long run, as the higher spending raises the marginal product of labor through the composite externality in the production function. In contrast, when  $g$  is financed by a tax on labor income, the time allocated to leisure increases, as the higher tax rate reduces the after-tax return on labor. But in all cases the effects are small. For all forms of financing, the productive benefits of public capital spending and the consequent private capital accumulation ensure that the equilibrium growth rate increases. They dominate any negative tax effects, although in the case of capital income tax-financing with its direct adverse impact on the return to capital, the positive

growth effects are small. Overall, the differential impacts on growth, leisure and the ratio of public to private capital reflect the varying degrees of distortions associated with the different tax rates.<sup>21</sup>

### 6.1.2. Distributional effects

Table 1B(ii) reports the short-run (instantaneous) and long-run effects on wealth, pre-tax, and post-tax income inequality. All these effects are calculated as percentage changes in the coefficient of variation relative to its pre-shock steady-state level.

Row 1 reports the case where the increase in government spending is financed by a lump-sum tax. Being non-distortionary, this policy isolates the pure effect of a government spending increase on the distributional measures. Since the stock of private capital, its initial distribution, and the stock of public capital are initially given, wealth inequality does not change on impact. It does so only gradually, increasing by about 2.7% in the long run. Since the lump-sum tax is non-distortionary, the pre-tax and post-tax distributions of income are identical. In the short run, income inequality declines by 2.6% relative to its pre-shock level. However, over time this decline is reversed, and in the long run income inequality increases by about 5%, thus highlighting how government investment generates a sharp intertemporal trade-off for the distribution of income.

During the transition, the increasing stock of public capital raises the marginal product of private capital and encourages private capital accumulation. Since private capital is unequally distributed in the economy, capital-rich agents experience a larger increase in their income from capital investment than do capital-poor agents. Wealth inequality therefore increases in transition to the long-run. By raising the expected long-run return to capital and labor, the higher government spending has a productivity impact on labor supply, causing the real wage to rise and labor supply to increase. In the short run, since capital-poor agents supply more labor relative to the capital-rich, their higher wage income compresses the dispersion of labor supply, thereby leading to an instantaneous decline in income inequality on impact of the shock. In transition, however, this trend is reversed due to two reinforcing effects. First, the increase in wealth inequality increases the dispersion of income. Second, as the productivity benefits of the gradually accumulating stock of infrastructure are realized along the transition path, average labor supply increases. Since capital-poor agents work more than the capital-rich, their labor incomes increase at a slower rate than those of the capital-rich, due to diminishing returns. This tends to widen the dispersion of labor income over time, consequently increasing income inequality. In the long-run, this leads to an overall increase in income inequality relative to its initial benchmark.

Table 1B(ii) (Rows 2–4) reports the distributional responses to the government spending shock when financed by the three distortionary taxes (capital income, labor income, and consumption). The results here depend on the interaction between two counter-acting effects along the transitional path. On one hand, the higher public spending tends to increase the productivity of both capital and labor, thereby affecting the labor-leisure choice and raising average factor incomes. On the other hand, each distortionary tax permanently reduces the after-tax return on the variable it impinges on, which in turn has a dampening effect on productivity and consequently, the labor-leisure allocation decision. Since the financing instruments are distortionary, the response of pre-tax and post-tax income inequality will now be distinct, except for the consumption tax, as it does not impinge directly on factor incomes.

Long-run wealth inequality increases in all three cases, with the largest increase of 3.5% occurring when the spending is financed by taxing capital income, one effect of which is to reduce the after-tax return on capital and the average capital stock. This, combined with the higher spending on the public good, leads to a large increase in the ratio of public to private capital, which more than offsets the decline in after-tax return on capital. Again, capital-rich agents experience higher long-run returns on capital than the capital poor, and wealth inequality increases. In the case of the labor tax, the same effect now operates through the after-tax return on labor. The effects of the consumption tax are qualitatively similar to that of the lump-sum tax-financing case.

Pre-tax and post-tax income inequality move in opposite directions in response to the capital and labor tax financing policies, while for the consumption tax their dynamics are identical. Capital tax-financing raises pre-tax income inequality both in the short run and the long run, while it has exactly the opposite effect on post-tax income inequality. For spending financed by a labor income tax, pre-tax income inequality falls both in the short run and long run, while after a small initial decline, post-tax inequality increases. The long-run decline in post-tax income inequality under capital tax-financing reflects the redistributive effects of the financing policy, since wealth is the primary source of inequality in this economy. Labor tax-financing policy increases long-run post-tax income inequality by reducing after-tax labor income. Since the capital-poor supply more labor, this increases the dispersion of labor supply which, combined with the higher wealth inequality, increases long-run post-tax income inequality.

## 6.2. The growth-income inequality relationship

Table 2 reports the short-run and long-run relationships between growth and post-tax income inequality resulting from the alternative modes of expenditure financing considered in Section 6.1. Whether this relationship is positive or negative is indicated by the signs in the table. Overall, our findings underscore the ambiguity in the direction of the growth-inequality relationship that is characteristic of recent empirical studies. The results in Table 2 indicate that the qualitative nature of this

<sup>21</sup> We do not discuss the transitional adjustment paths for the aggregate economy, as these are well-known from the public investment-growth literature; see Turnovsky (1997) for an early example.

**Table 2**Increase in government spending: the growth-inequality relationship<sup>a</sup>.

$g=0.05-0.08$				
$q=s=1$				
A Composite externality in utility and production, $\varepsilon=\varphi=0.6$ (Benchmark Case).				
Policy change	Short run change		Long run change	
	Growth	Post-tax income inequality	Growth	Post-tax income inequality
Lump-sum tax-financed increase in $g$	0.129	−2.602	0.206	4.996
Capital income tax-financed increase in $g$	0.044	−9.174	0.101	−0.149
Labor income tax-financed increase in $g$	0.096	−0.110	0.168	7.933
Consumption tax-financed increase in $g$	0.106	−3.117	0.179	4.955

<sup>a</sup> Distributional effects are reported as percentage changes relative to their pre-shock levels:  $[(\sigma_j(t) - \bar{\sigma}_{j,0})/\bar{\sigma}_{j,0}] \times 100$ ,  $j = k, y, u$ .

relationship depends critically on (i) the tax policy used to finance government investment, and (ii) the time horizon, namely short run or long run.

## 7. Sensitivity analysis

The numerical simulations discussed in Section 6 are based on one plausible, but specific, set of parameters. Given the complex nature of the interactions in the model, it is important to examine the robustness of the results to changes in the specification of the key parameters. Chatterjee and Turnovsky (2012) in their detailed analysis of this model perform an extensive sensitivity analysis of the main policy experiments, focusing on three key aspects of the model's structure.

First, they consider the sensitivity to the relative magnitude of the composite externality parameters,  $\varphi$  and  $\varepsilon$ , focusing on two polar cases in detail: (i) where the public good externality affects only the utility function ( $\varphi=0$ ,  $\varepsilon=1$ ), and (ii) where it enters only the production function ( $\varphi=1$ ,  $\varepsilon=0$ ). The basic qualitative results from Table 1 are found to remain robust to these variations in the externality parameters.<sup>22</sup> However, the growth-inequality tradeoff is quite sensitive to the structure of the externality. Second, they consider variations in the intratemporal elasticity of substitution between private capital and effective labor in production,  $s = 1/(1+\rho)$ . They consider three cases (i)  $s=0.4$ , (ii)  $s=0.8$ , and (iii)  $s=1.2$ . Again, the patterns observed in the benchmark experiments remain unchanged, except for a decline in wealth inequality with capital tax-financing when  $s=0.4$ . This is because a large increase in the capital income tax is required to balance the government's budget when the elasticity of substitution in production is small. Finally, they illustrate the robustness of the benchmark results to changes in the intratemporal elasticity of substitution between consumption and leisure in utility,  $q = 1/(1+\nu)$  and again consider the three cases (i)  $q=0.4$ , (ii)  $q=0.8$ , and (iii)  $q=1.2$ . Once again, the patterns observed in the benchmark experiments remain unchanged. The overall conclusion is that the results of this specific model are generally robust to extensive variations in these crucial parameters.

## 8. OLG model with idiosyncratic productivity shocks

We now turn to the second model, which actually has two independent sources of heterogeneity: (i) initial endowments of private capital, (ii) idiosyncratic productivity shocks. Both sources of heterogeneity are lognormally distributed and the production function is CES, having constant returns to scale in public and private capital, thereby generating an equilibrium of ongoing growth. In general, the introduction of idiosyncratic productivity shocks does not readily admit a closed form solution, requiring instead numerical simulations. However, the assumption of lognormality, coupled with the CES production function, enables one to exploit analytical properties of lognormal distributions that facilitate aggregation, and to derive the joint distributional and aggregate dynamics of the CES economy in a very tractable closed form.

Overall the model is a specific one and lacks the versatility of the “representative consumer” model discussed in earlier sections. One important assumption is that there are no credit or insurance markets, the absence of which is a key element generating inequality, (Loury 1981; Bénabou 2000, 2002). By abstracting from borrowing or lending, the analysis is clearly more relevant for developing countries.

As we shall show in Section 9 below, the equilibrium dynamics has a simple recursive structure, but the reverse of that of the previous model! Now it is the dynamics of inequality that drive the growth of the aggregate variables, private capital, public capital, and output. In addition, the impact of public investment on inequality has very different determinants from that in the previous model, the crucial factor now being the elasticity of substitution in production between public and private capital.

Specifically, an increase in public investment will decrease or increase inequality, both in the short run and over time, according to whether the elasticity of substitution between public and private capital,  $\varepsilon$  say, is greater than, or less than,

<sup>22</sup> They also considered all possible comparisons between  $\varepsilon$  and  $\varphi$ , i.e.,  $\varepsilon > \varphi$ ,  $\varepsilon < \varphi$ , and  $\varepsilon = \varphi$ . The qualitative results from Table 1 remain robust.

unity. In the case that  $\varepsilon > 1$  it serves as a substitute for private capital and favors the poor, leading to more equitable distribution of wealth. If  $\varepsilon < 1$ , it is a complement to private capital and favors the rich, thereby exacerbating inequality.

Structurally the equilibrium is closer to that of the early Barro (1990) model, in part because the productive public good is now introduced as a flow, rather than as a stock. But in contrast to Barro (1990), public investment now impacts the equilibrium growth rate through two channels. The first is the direct productivity effect, as in the original Barro model. But now there is an additional indirect channel, through its impact on inequality, which in turn impinges on the growth rate. The presence of the second channel influences the choice of optimal tax, which now depends on the degree of wealth heterogeneity as well as upon  $\varepsilon$ . Overall, this model offers a sharp contrast to the previous one, emphasizing different aspects, particularly factor substitution.

### 8.1. Analytical framework<sup>23</sup>

The model is an overlapping generations model in which individuals live for two periods; households consist of a young and old. Each parent of the initial generation ( $t=0$ ) is endowed with private capital,  $k_0^i$ . They have access to public capital,  $g_0$ , equally available to all. As in the previous model, the initial distribution of wealth is given, and evolves endogenously over time. In the first period, when agents are young, they acquire capital from their parents, while their consumption is included in that of their parents. All decisions are made during the second period of life, when children are adults. Parents earn income by supplying capital and labor to privately-held firms, which produce output by combining these factors with public capital. The government taxes income at a flat rate to finance the public investment good. Agents allocate their after-tax income between consumption and saving, while accumulated capital at the end of the second period is endowed to their offspring. The model is much more stylized than the one developed in earlier sections and should be viewed more as a specific example that focuses on very different aspects.

Household  $i$ 's preferences over the period  $(t, t+1)$  are described by a logarithmic utility function, defined over their consumption, leisure, and the capital they leave as bequests

$$W_t^i \equiv \ln c_t^i + \eta \ln(1 - l_t^i) + \beta \ln k_{t+1}^i \quad (27)$$

They choose  $c_t^i$ ,  $s_t^i$ , and  $l_t^i$  to maximize (27) subject to the constraints

$$c_t^i + s_t^i = (1 - \tau)y_t^i \quad (28a)$$

$$y_t^i = a_s^i \left( (1 - \alpha) (k_t^i)^\rho + \alpha (g_t)^\rho \right)^{\frac{1-\theta}{\rho}} (l_t^i k_t^i)^\theta \quad (28b)$$

$$k_{t+1}^i = s_t^i \quad (28c)$$

Eq. (28a) is the budget constraint, where all income is taxed at the flat rate  $\tau$ . Eq. (28b) describes the production function and is a critical element of the model. It is a two-level production function, in which at the first level private capital and public capital combine in a CES production function, with elasticity of substitution  $\varepsilon = 1/(1 - \rho)$ , which then combine with labor measured in efficiency units to produce final output.<sup>24</sup> This production function is somewhat simpler than that introduced earlier, but that is unimportant. We assume that the two capital goods are cooperative in production, (i.e.  $\partial^2 y^i / \partial k^i \partial g > 0$ ) which implies

$$\rho' \equiv \frac{\rho}{1 - \theta} < 1 \quad \text{i.e. } \varepsilon < \frac{1}{\theta} \quad (28d)$$

thus imposing an upper limit on the elasticity of substitution in production.

In addition, production is subject to an idiosyncratic productivity shock,  $\xi_t^i$ , assumed to be i.i.d. and lognormal with mean one:  $\ln \xi_t^i \sim N(-v^2/2, v^2)$ . The initial distribution of capital is also lognormal:  $\ln k_0^i \sim N(\mu_0, \sigma_0^2)$ . Finally, private capital fully depreciates within the period.

The government budget is always balanced. Public capital also completely depreciates within the period, so that

$$g_{t+1} = \tau \int_i y_t^i di \equiv \tau y_t \quad (29)$$

Household  $i$ 's optimality conditions for saving and labor supply are

$$s_t^i = \chi(1 - \tau)y_t^i \quad (30a)$$

$$l_t^i = \frac{\theta}{\eta(1 - \chi) + \theta} \equiv l \quad \chi \equiv \beta/(1 + \beta) \quad (30b)$$

<sup>23</sup> This model is drawn from Getachew and Turnovsky (2015). Since this model is structurally very different from that discussed in earlier sections, it is impractical to employ common notation to cover both models. As far as the technical details of the two models are concerned each should be considered separately.

<sup>24</sup> The production externality in this case corresponds to the original Romer (1986) form, rather than the more general hybrid externality employed in the earlier model.



implying that all agents allocate the same fraction of their time to labor. From (28), the dynamics of capital accumulation for the  $i$ th individual are

$$k_{t+1}^i = (1-\tau)a'\chi_t^{\varepsilon_i} \left( (1-\alpha) \left( k_t^i \right)^\rho + \alpha (g_t)^\rho \right)^{\frac{1-\theta}{\rho}} (k_t^i)^\theta \quad (31)$$

where

$$a' \equiv a \left( \frac{\theta}{\eta(1-\chi) + \theta} \right)^\theta$$

Under these conditions, summation across agents is straightforward, so that aggregate consumption and private capital stock are respectively

$$c_t = (1-\tau)y_t - s_t \quad (32)$$

$$k_{t+1} = s_t \quad (33)$$

where  $c_t = \int_i c_t^i di$ ,  $s_t = \int_i s_t^i di$ ,  $k_t = \int_i k_t^i di$ . It follows from the above relationships that each agent's wealth is proportional to his income (lagged one period), and the same applies to aggregate wealth and income, so that in contrast to our earlier model wealth and income and their respective distributions across agents essentially coincide.

## 9. Infrastructure, inequality and aggregate capital dynamics

Under the conditions specified above, [Getachew and Turnovsky \(2015\)](#) show that the equilibrium of the aggregate economy is described by the following set of dynamic equations<sup>25</sup>:

### 9.1. Inequality dynamics

$$\sigma_{t+1}^2 = v^2 + \frac{1}{\rho'^2} \ln \left[ 1 + \left( \frac{z_t}{1+z_t} \right)^2 (e^{\rho^2 \sigma_t^2} - 1) \right] \quad (34a)$$

where

$$\phi \equiv \frac{k_t}{g_t} = \chi \left( \frac{1-\tau}{\tau} \right) = \left( \frac{\beta}{1+\beta} \right) \left( \frac{1-\tau}{\tau} \right) \quad (34b)$$

$$z_t \equiv \frac{1-\alpha}{\alpha} \phi^{\rho'} e^{\rho(\rho-1)\sigma_t^2/2} \quad (34c)$$

Eq. (34a) describes the fundamental dynamic equation driving the equilibrium. It relates the inequality of wealth (or income), in this case most conveniently measured by the variance of the lognormal distribution,  $\sigma_t^2$ , to the constant (given) variance of the idiosyncratic shocks,  $v^2$ , plus various terms and parameters relating to the distribution and the ratio of public to private capital, which depend upon the tax rate and the weight assigned to inheritances in utility.

### 9.2. Growth dynamics

$$\begin{aligned} \gamma_{t+1}^k &\equiv \ln g_{t+1} - \ln g_t = \ln(a'\chi^\theta \alpha^{1/\rho'}) + \theta \ln(1-\tau) + (1-\theta) \ln \tau + \frac{1}{\rho'} \ln(1+z_t) \\ &\quad + \frac{1-\rho'}{2\rho'^2} \ln \left[ 1 + \left( \frac{z_t}{1+z_t} \right)^2 (e^{\rho^2 \sigma_t^2} - 1) \right] \end{aligned} \quad (35)$$

Having determined the degree of inequality at time  $t$ ,  $\sigma_{t+1}^2$ , from (34a), (35) then yields the corresponding growth rate of capital,  $\gamma_{t+1}^k$ , during that period. The key point to observe is that in this model inequality determines the growth rate. The causality is therefore sharply reversed from that of the “representative consumer” model, described previously.

<sup>25</sup> This derivation is non-trivial, involving a lot of technical detail exploiting properties of the lognormal distribution. Details are provided by [Getachew and Turnovsky \(2015\)](#).

### 9.3. Steady state

Assuming that the dynamics are stable, the economy converges to the following steady state

$$\left(\frac{\tilde{z}}{1+\tilde{z}}\right)^2 (e^{\rho^2 \tilde{\sigma}^2} - 1) = e^{\rho^2 (\tilde{\sigma}^2 - v^2)} - 1 \quad (36a)$$

$$\tilde{z} \equiv \frac{1-\alpha}{\alpha} \phi^\rho e^{\rho(\rho-1)\tilde{\sigma}^2/2} \quad (36b)$$

$$\begin{aligned} \tilde{\gamma} = & \ln(\alpha' \chi^\theta \alpha^{1/\rho'}) + \theta \ln(1-\tau) \\ & + (1-\theta) \ln \tau + \frac{1}{\rho'} \ln(1+\tilde{z}) + \frac{1-\rho'}{2} (\tilde{\sigma}^2 - v^2) \end{aligned} \quad (36c)$$

Eqs. (36a) and (36b) jointly determine, the long-run inequality,  $\tilde{\sigma}^2$ , together with  $\tilde{z}$ , and having obtained these quantities, (36c) then determines the implied long-run growth rate,  $\tilde{\gamma}$ .

Before drawing out some of the implications of this steady state, it is of interest to relate the long-run distribution of wealth implied by (36), with that derived using other approaches. First, we see from (36a) that long-run inequality depends critically upon the existence of idiosyncratic productivity shocks. In their absence ( $v^2 = 0$ ), the long-run distribution degenerates to  $\sigma^2 = 0$ . This result is analogous to that of Li and Sarte (2004) who show that progressive taxes in conjunction with a common rate of time discount (the assumption being made here) leads to a long-run degenerate wealth distribution. In effect the idiosyncratic shocks present in this model are playing the role of the progressive tax rates in the Li-Sarte (2004) analysis. Li and Sarte also show that the long-run wealth distribution will not degenerate if agents have different rates of time preference, and that can be shown to be the case here too. The equilibrium wealth distribution also agrees with that of Krusell and Smith (1998) and others, who show if agents are initially identical but are subject to idiosyncratic shocks, then this will lead to a long-run non-degenerate wealth distribution.

Finally, we see in the long run, the impact of the initial wealth distribution,  $\sigma_0^2$ , vanishes and has no effect on the long-run wealth distribution of wealth. This is a further contrast with the representative consumer model, where the only source of heterogeneity was the initial endowments of capital and the non-degenerate long-run distributions of wealth and income are directly tied to the initial distribution. The difference is due to the assumption of complete financial markets that these models impose, making the distributional dynamics path-dependent.

### 9.4. Some properties of the equilibrium

#### 9.4.1. Growth-inequality tradeoff

From the equilibrium and its steady state, Getachew and Turnovsky (2015) establish that an increase in the variance of the idiosyncratic productivity shocks will increase inequality and will lead to a lower growth rate, both in the short run and in steady state. An increase in the inequality in initial endowments will increase inequality and reduce the growth rate temporarily, but decline over time and vanish in the long run. In both cases, an increase in inequality is associated with a reduction in the growth rate.

#### 9.4.2. Local stability

To get a sense of the stability characteristics of the fundamental dynamic equation, we can approximate (34a) by

$$\sigma_{t+1}^2 = v^2 + \left(\frac{(1-\theta)\tilde{z}}{1+\tilde{z}}\right)^2 \sigma_t^2 \quad (37)$$

in the neighborhood of steady state. From this equation we see that it is locally stable, although with an overly fast rate of convergence. This reflects the fact that private and public capital depreciate fully each period, suggesting that there is insufficient “sluggishness” in the economy to replicate the empirical evidence on convergence rates.

#### 9.4.3. Effects of public investment on growth and inequality

The key issue pertains to the long-run effects of an increase in government investment on the degree of inequality and the equilibrium growth rate. The relevant expressions are:

$$\frac{\partial \tilde{\sigma}^2}{\partial \tau} = -\frac{(1-\theta)^2}{\rho} \frac{(e^{\rho^2 \tilde{\sigma}^2} - 1)}{(1-D) \left[ 1 + \left( \tilde{z}^2 / (1+\tilde{z})^2 \right) (e^{\rho^2 \tilde{\sigma}^2} - 1) \right] 2 \frac{(\tilde{z})^2}{(1+\tilde{z})^3} \frac{1}{\tau(1-\tau)}} \quad (38a)$$

$$\frac{\partial \tilde{\gamma}}{\partial \tau} = \frac{1}{\tau(1-\tau)} \left[ \frac{(1-\theta)}{(1+\tilde{z})} - \tau \right] + \frac{1}{2} \left[ \left( 1 - \frac{\rho}{1-\theta} \right) - \frac{(1-\theta)(1-\rho)\tilde{z}}{1+\tilde{z}} \right] \frac{\partial \tilde{\sigma}^2}{\partial \tau} \quad (38b)$$

where  $D$  is a constant, lying in the range  $0 < D < 1$ , reflecting stability of the underlying dynamic system, (34). Recalling  $\varepsilon = 1/(1-\rho)$ , from Eq. (38a) we see that the effect of an increase in government investment on inequality depends critically upon the size of the elasticity of substitution between public and private capital,  $\varepsilon$ , in production. An increase in the rate of

public investment will increase (decrease) both short-run and long-run inequality according to whether the elasticity of substitution is less (greater) than one. Thus to determine the qualitative impact of government investment on inequality and growth requires knowledge of  $\varepsilon$  on which empirical evidence is sparse. Getachew and Turnovsky (2015) explore how it is possible to derive estimates of  $\varepsilon$  by utilizing the much more widely available information on the productive elasticity of public capital summarized by Bom and Ligthart (2014), cited earlier. Based on this information, Getachew and Turnovsky conclude that  $\varepsilon < 1$  is empirically the more plausible scenario, although one cannot rule out  $\varepsilon > 1$  as being a realistic possibility in some circumstances.

Eq. (38b) implies that to the extent that more public investment increases inequality this will tend to increase the growth rate both in the short run, and in the long run, although in all cases the net overall effect will also depend critically upon whether current rate of expenditure  $\tau \geq (1-\theta)(1+\tilde{z})^{-1}$ . This has implications for the growth-maximizing rate of public investment. Setting  $\partial \tilde{y} / \partial \tau = 0$ , growth-maximizing rate of public investment,  $\tau^*$ , and corresponding ratio of private to public capital,  $z^*$ , are related by

$$\frac{1}{\tau^*(1-\tau^*)} \left[ \frac{(1-\theta)}{(1+z^*)} - \tau^* \right] + \frac{1}{2} \left[ \left( 1 - \frac{\rho}{1-\theta} \right) - \frac{(1-\theta)(1-\rho)z^*}{1+z^*} \right] \frac{\partial \tilde{z}^2}{\partial \tau} = 0 \quad (39)$$

Thus, to the extent that government investment in infrastructure increases (decreases) inequality, it will set the long-run growth-maximizing rate of public investment at  $\tau^* > (1-\theta)(1+z^*)^{-1}$ , the optimality condition characterizing a riskless economy. The choice of  $\tau^*$  depends upon the presence and degree of inequality in economy. In comparing (39) to a riskless economy, we must take account of the fact that the equilibrium ratio of private to public capital will also be affected.

## 10. Conclusions

The relationship between economic growth and inequality is arguably one of the most important in economics. It is receiving increasing attention in both academic and policy circles as a result of the general increase in inequality that has occurred during recent years in both developed and developing economies. The relationship is also a complex one, both analytically and substantively. Analytically, the complexity arises from the fact that in general both macroeconomic aggregates and the distribution of those aggregates across agents are jointly determined, and in general solving for them simultaneously poses an intractable problem. Substantively, the complexity arises from the multidimensionality of the forces impinging on this relationship. The dynamics of growth and distribution are driven by many diverse factors spanning economic, sociological, demographic, political, and structural origins.

The treatment in this paper of the growth-inequality relationship has focused on the role of public investment as a determinant of the relationship between the accumulation of physical capital and the degree of wealth and income inequality thus generated. As emphasized, this is an important policy issue, both in developed and developing economies. We have studied this from two points of view, illustrating two approaches to the issue, each offering a different perspective.

In the first we have employed a general equilibrium endogenous growth model with heterogeneous agents, where the heterogeneity is due to the initial endowments of private capital (wealth). Two key features of the model include: (i) the homogeneity of the underlying preferences and (ii) all agents have equal unrestricted access to perfect factor markets. Under these conditions, although growth and inequality are joint equilibrium outcomes, they are nevertheless determined recursively; aggregate behavior determines distribution, but not vice versa. While this specification of preferences renders the analysis tractable, the dynamic structure remains sufficiently complex to require the use of numerical simulations.

Using this approach, the results suggest that government spending on public capital will increase wealth inequality over time, irrespective of how it is financed. The mechanism is straightforward. Government investment tends to enhance the productivity of private capital, thereby stimulating its accumulation, and with private capital being more unequally distributed among agents than is labor this tends to increase wealth inequality.

By contrast, the consequences for income inequality are sensitive to how public investment is financed and may be characterized by sharp intertemporal tradeoffs. This is because the short-run response of income inequality is dominated by the initial response of the labor-leisure choice and its impact on factor returns, while over time it is more influenced by the evolution of both wealth and labor income inequality. The behavior of pre-tax and post-tax income inequality contrasts sharply, depending upon whether the expenditure is financed by a tax on labor or capital. This underscores the point that pro-growth policies may not always be pro-poor, with sharply contrasting outcomes for income inequality over time. These results are generally robust to variations in the economy's key structural parameters.

The second model we have presented makes very different assumptions, and emphasizes very different aspects. It assumes the absence of borrowing or lending, with agents being subject to idiosyncratic productivity shocks. Under the specific assumptions of log-normality and CES production function we are able to obtain a closed form solution, but one that is very different in nature from that of the first model. Now the equilibrium dynamics are driven by the volatility of the productivity shocks, while the role of initial endowments gradually disappears over time. The degree of inequality, as determined by the evolution of the productivity shocks, then determines the growth rate, precisely reversing the causality from that of the “representative consumer” model. This framework also implies an unambiguously negative relationship between inequality and growth. Moreover, the impact of government investment on inequality (both wealth and income)

depends upon the degree of factor substitutability embodied in the production function, which plays a relatively minor role in the first model.

The over-riding conclusion of our analysis is that the growth-inequality relationship is a complex one. This is evident from the conflicting empirical evidence. But it is also supported by the more formal analysis discussed here. Whether this relationship is positive or negative depends critically upon the analytical framework one is employing. But even having adopted a particular framework it may depend upon the relative magnitude of externalities, underlying financing policies, the time period of consideration, as well as factor substitutability. The main contribution of the analysis is that to understand the nature of the relationship one needs to embed it within a consistently specified general equilibrium growth model, recognizing that different frameworks offer different perspectives.

## References

- Agénor, P., 2011. *Public Capital, Growth and Welfare: Analytical Foundations for Public Policy*. Princeton University Press, Princeton NJ.
- Alesina, A., Rodrik, D., 1994. Distributive politics and economic growth. *Q. J. Econ.* 109, 465–490.
- Algan, Y., Cheron, A., Hairault, J.-O., Langot, F., 2003. Wealth effect on labor market transitions. *Rev. Econ. Dyn.* 6, 156–178.
- Anand, S., Kanbur, R., 1993. The Kuznets process and the inequality-development relationship. *J. Dev. Econ.* 40, 25–52.
- Arrow, K.J., Kurz, M., 1970. *Public Investment, the Rate of Return and Optimal Fiscal Policy*. Johns Hopkins University Press, Baltimore, MD.
- Artadi, E.V., Sala-i-Martin, X., 2003. The economic tragedy of the XXth century: growth in Africa. NBER Working Paper 9865.
- Aschauer, D.A., 1989. Is public expenditure productive? *J. Monet. Econ.* 23, 177–200.
- Atkinson, A.B., 1970. On the measurement of inequality. *J. Econ. Theory* 2, 244–263.
- Atolia, M., Chatterjee, S., Turnovsky, S.J., 2012. Growth and inequality: dependence on the time path of productivity increases (and other structural changes). *J. Econ. Dyn. Control* 36, 331–348.
- Banerjee, A., Somanathan, R., 2007. The political economy of public goods: some evidence from India. *J. Dev. Econ.* 82, 287–314.
- Barro, R.J., 1990. Government spending in a simple model of endogenous growth. *J. Polit. Econ.* 98, S103–S125.
- Barro, R.J., 2000. Inequality and growth in a panel of countries. *J. Econ. Growth* 5, 5–32.
- Becker, R.A., 1980. On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households. *Q. J. Econ.* 95, 375–382.
- Bénabou, R., 2000. Unequal societies: Income distribution and the social contract. *Am. Econ. Rev.* 90, 96–129.
- Bénabou, R., 2002. Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency? *Econometrica* 70, 481–517.
- Bom, P., Ligthart, J., 2014. What have we learned from three decades of research on the productivity of public capital? *J. Econ. Surv.* 28, 889–916.
- Brakman, S., Garretsen, H., van Marrewijk, C., 2002. Locational competition and agglomeration: the role of government spending. CESifo Working Paper 775.
- Calderón, C., Servén, L., 2004. The effects of infrastructure development on growth and income distribution. World Bank Policy Research Paper no. 3400.
- Calderón, C., Servén, L., 2010. Infrastructure and economic development in Sub-Saharan Africa. *J. Afr. Econ.* 19, 13–87.
- Calderón, C., Servén, L., 2014. Infrastructure, growth, and inequality: an overview. The World Bank, Policy Research Working Paper Series, 7034.
- Caselli, F., Ventura, J., 2000. A representative consumer theory of distribution. *Am. Econ. Rev.* 90, 909–926.
- Castañeda, A., Díaz-Giménez, J., Rios-Rull, V., 1998. Exploring the income distribution business cycle dynamics. *J. Monet. Econ.* 412, 93–130.
- Chatterjee, S., 1994. Transitional dynamics and the distribution of wealth in a neoclassical growth model. *J. Public. Econ.* 54, 97–119.
- Chatterjee, S., Turnovsky, S.J., 2012. Infrastructure and inequality. *Eur. Econ. Rev.* 56, 1730–1745.
- Cheng, I.-H., French, E., 2000. The effect of the run-up in the stock market on labor supply. *Fed. Reserv. Bank Chic. Econ. Perspect.* 24, 48–65.
- Eden, M., Kraay, A., 2014. “Crowding in” and the returns to government investment in low-income countries. The World Bank, Policy Research Working Paper Series 6781.
- Ehrlich, I., Kim, J., 2007. The evolution of income and fertility inequalities over the course of economic development: a human capital perspective. *J. Hum. Cap.* 1, 137–174.
- Fan, S., Zhang, X., 2004. Infrastructure and regional economic development in rural China. *China Econ. Rev.* 15, 203–214.
- Ferranti, D., Perry, G., Ferreira, F., Walton, M., 2004. *Inequality in Latin America: Breaking with History?*. World Bank, Washington, DC.
- Forbes, K., 2000. A reassessment of the relationship between inequality and growth. *Am. Econ. Rev.* 90, 869–887.
- Futagami, K., Morita, Y., Shibata, A., 1993. Dynamic analysis of an endogenous growth model with public capital. *Scand. J. Econ.* 95, 607–625.
- Getachew, Y.Y., Turnovsky, S.J., 2015. Productive government spending and its consequences for the growth-inequality tradeoff. *Res. Econ.*, 69. (forthcoming).
- Glomm, G., Ravikumar, B., 1994. Public investment in infrastructure in a simple growth model. *J. Econ. Dyn. Control.* 18, 1173–1187.
- Gorman, W., 1953. Community preference fields. *Econometrica* 51, 63–80.
- Holtz-Eakin, D., Joulfaian, Rosen, H.S., 1993. The Carnegie conjecture: some empirical evidence. *Q. J. Econ.* 108, 413–435.
- Krusell, P., Smith, A., 1998. Income and wealth heterogeneity in the macroeconomy. *J. Polit. Econ.* 106, 867–896.
- Kuznets, S., 1955. Economic growth and income inequality. *Am. Econ. Rev.* 45, 1–28.
- Li, H.Y., Zou, H.F., 1998. Income inequality is not harmful to growth: theory and evidence. *Rev. Dev. Econ.* 2, 318–334.
- Li, W., Sarte, P.-D., 2004. Progressive taxation and long-run growth. *Am. Econ. Rev.* 94, 1705–1716.
- Lopez, H., 2004. *Macroeconomics and inequality*. The World Bank Research Workshop, Macroeconomic Challenges in Low Income Countries.
- Loury, G.C., 1981. Intergenerational transfers and the distribution of earnings. *Econometrica* 49, 843–867.
- Maliar, L., Maliar, S., 2001. Heterogeneity in capital and skills in a neoclassical stochastic growth model. *J. Econ. Dyn. Control.* 38, 635–654.
- Perotti, R., 1996. Growth, income distribution, and democracy: what the data say. *J. Econ. Growth* 1, 149–187.
- Persson, T., Tabellini, G., 1994. Is inequality harmful for growth? *Am. Econ. Rev.* 84, 600–621.
- Piketty, T., 2013. *Capital in the Twenty-First Century*. Harvard University Press, Cambridge MA.
- Romer, P.M., 1986. Increasing returns and long-run growth. *J. Polit. Econ.* 94, 1002–1037.
- Seneviratne, D., Sun, Y., 2013. Infrastructure and income distribution in ASEAN-5: What are the links? IMF Working Paper 13/41.
- Sorger, G., 2000. Income and wealth distribution in a simple model of growth. *Econ. Theory* 16, 23–42.
- Sorger, G., 2002. On the long-run distribution of capital in the Ramsey model. *J. Econ. Theory* 105, 226–243.
- Stiglitz, J., 2012. *The Price of Inequality*. W.W. Norton, NY.
- Turnovsky, S.J., 1997. Fiscal policy in a growing economy with public capital. *Macroecon. Dyn.* 1, 615–639.
- Turnovsky, S.J., 2013. The relationship between economic growth and inequality. *N. Z. Econ. Papers* 47, 113–139.
- Turnovsky, S.J., García-Peñalosa, C., 2008. Distributional dynamics in a neoclassical growth model: the role of elastic labor supply. *J. Econ. Dyn. Control.* 32, 1399–1431.
- World Bank. Inclusive growth and service delivery: building on India's success. Development Policy Review. 2006.