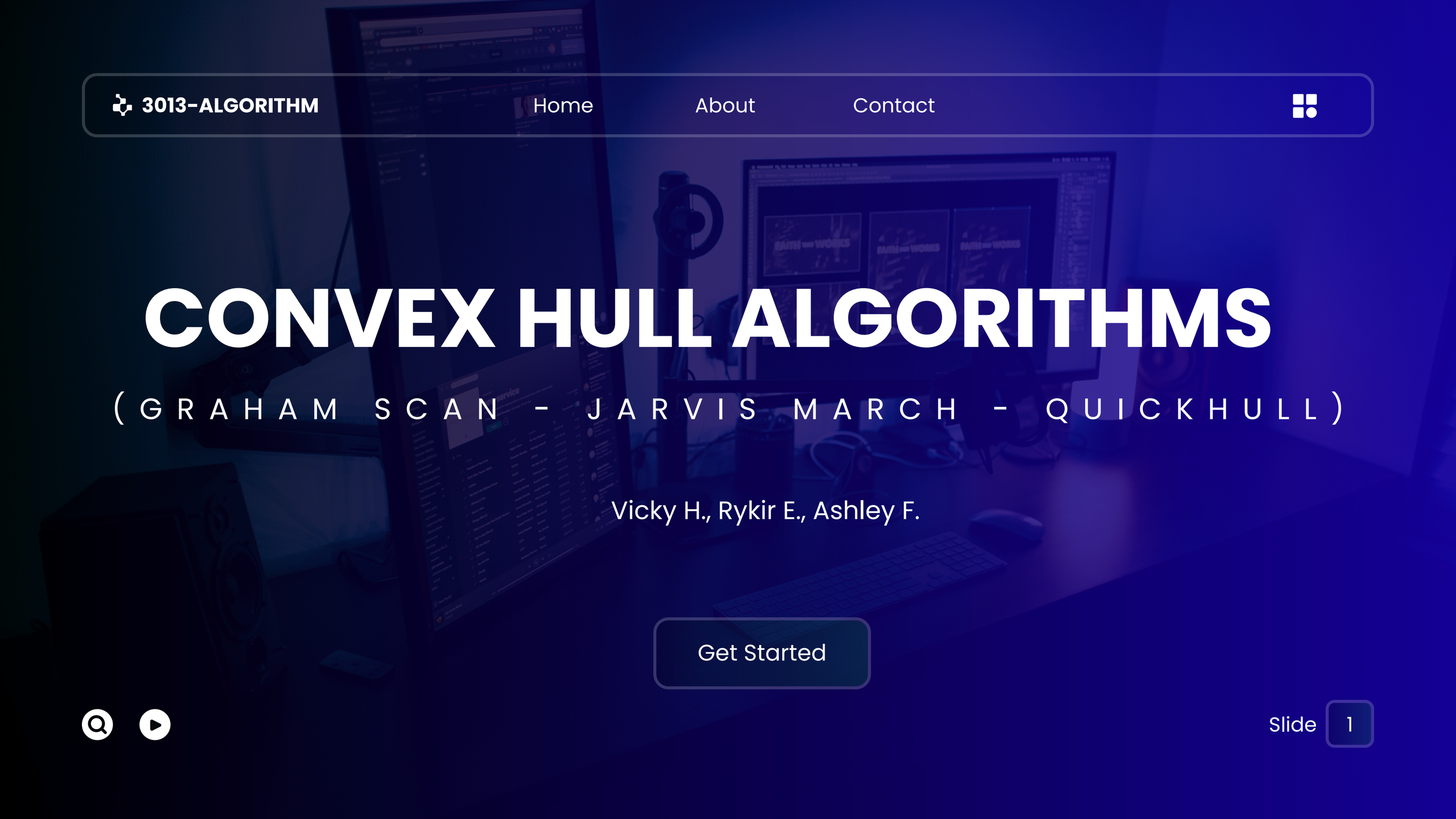

3013-ALGORITHM

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CONVEX HULL ALGORITHMS

(GRAHAM SCAN - JARVIS MARCH - QUICKHULL)

Vicky H., Rykir E., Ashley F.

Get Started

Introduction to Convex Hull

- What is a convex hull?
- Why it matters in computational geometry
- Real-world applications

Convex Hull

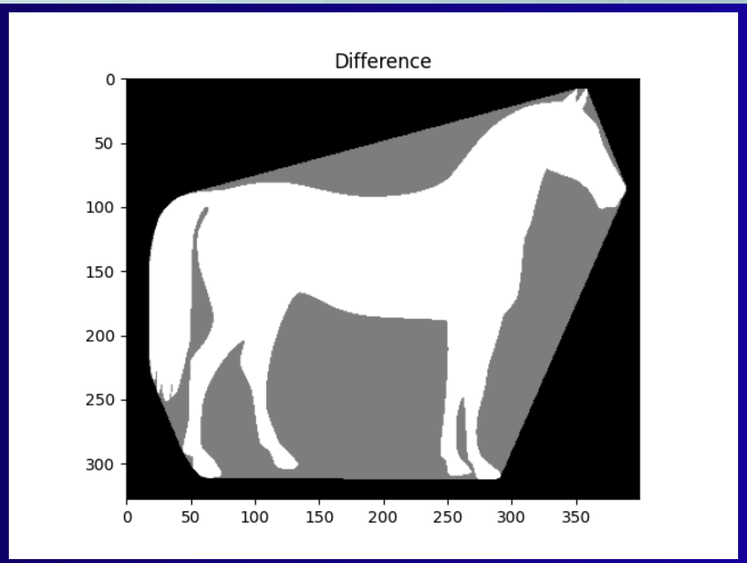
- Convex Shape - Having only interior angles less than 180°
- The smallest set that encloses all vertices in a graph, forming a convex polygon
- Can be expanded to 3D (polyhedron)

Neat Facts

- Convex hulls are unique for each set
- Often analogized to rubber bands stretching around points
- In 2D, Convex Hulls are always polygons.

Why It Matters

- **Simplicity/Efficiency**
  - Shape estimation
  - Reduced rendering intensity
- **Real World Applications**
  - Pattern recognition
  - Image processing
  - Collision detection (physics and games)



Convex Hull Algorithms

Graham Scan

- Published by Ronald Graham 1972
- Time Complexity
  - O(n log n)
  - n = vertices
- Greedy-ish Style

Jarvis March

- Published by Ray Jarvis in 1973
- Time Complexity
  - O(nh)
  - n = vertices
  - h = vertices in hull
- Brute-force-ish Style

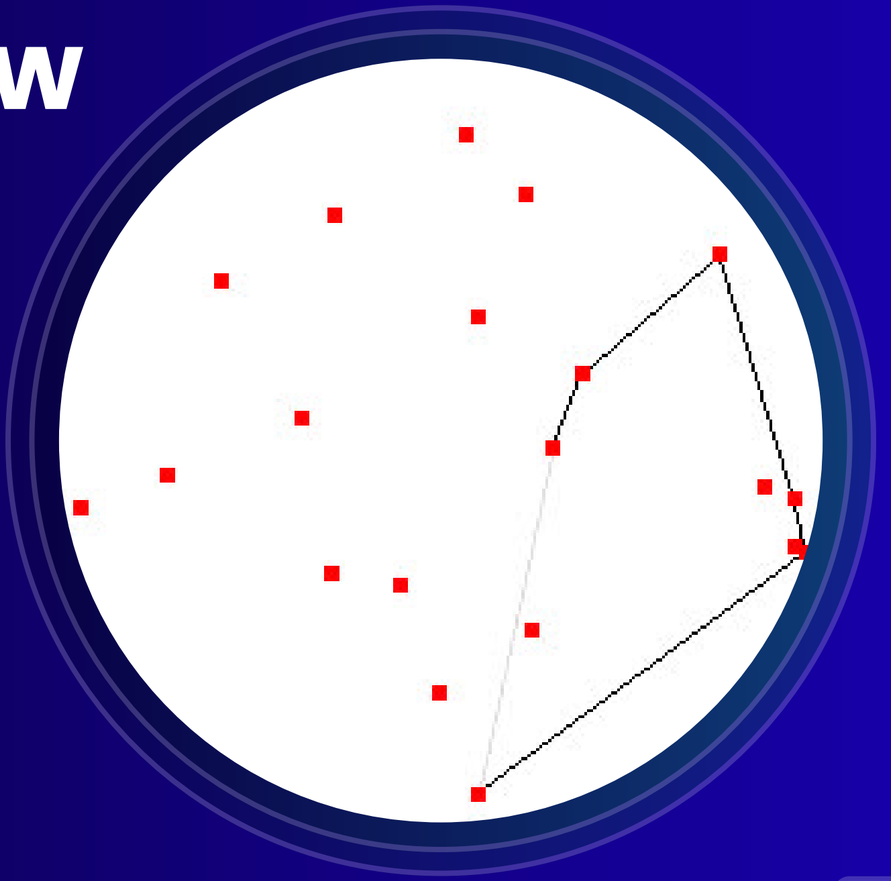
QuickHull

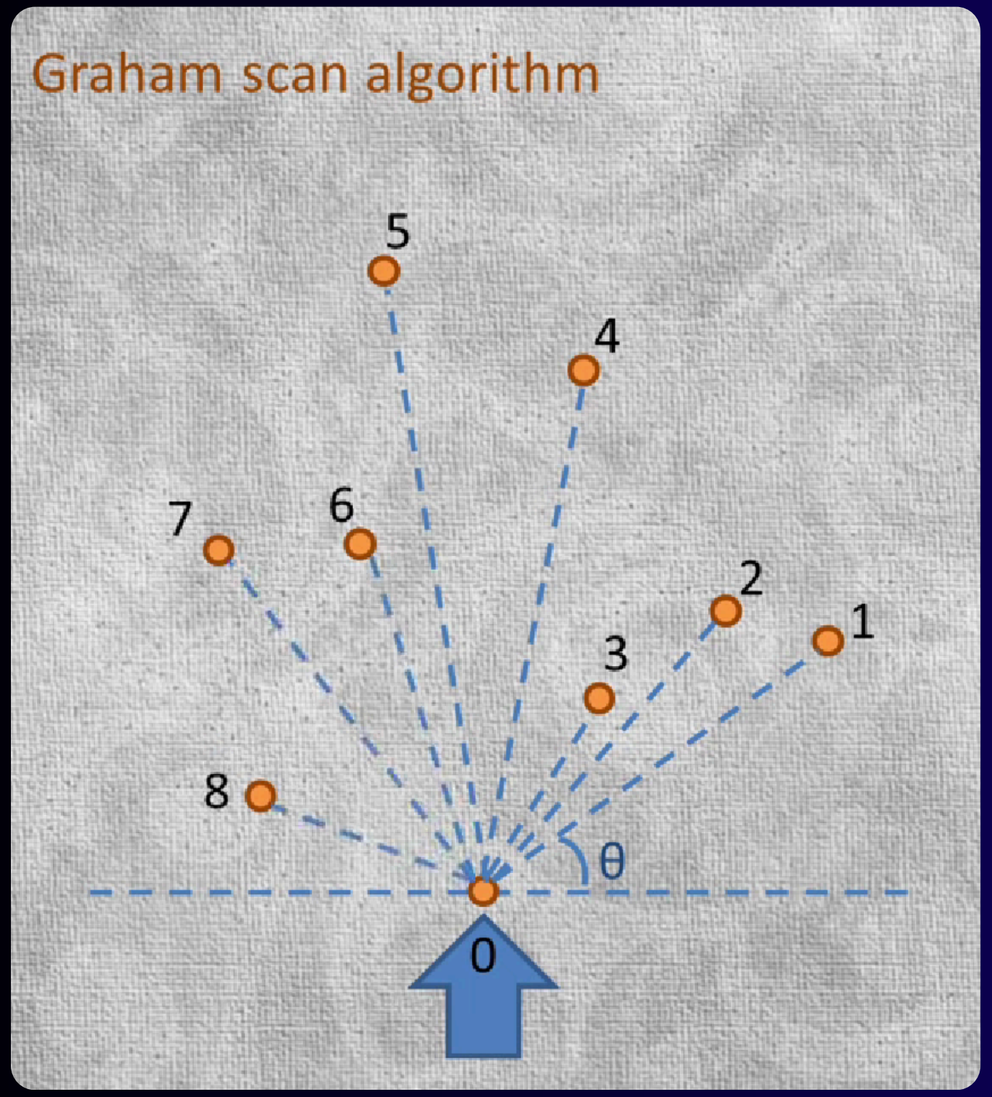
- Published by C. Bradford Barber et. al in 1996
- Time Complexity
  - O(n^2) Worst case
  - O(n log n) Average
  - n = vertices
- Divide and Conquer

Graham Scan – Overview

Steps:

1. Find the lowest point (pivot)
2. Sort points by angle from pivot
3. Use stack to add in-order vertices into the convex hull
4. Keep if the angle is counter-clockwise, remove if clockwise
5. Repeat 3-4 until all points processed



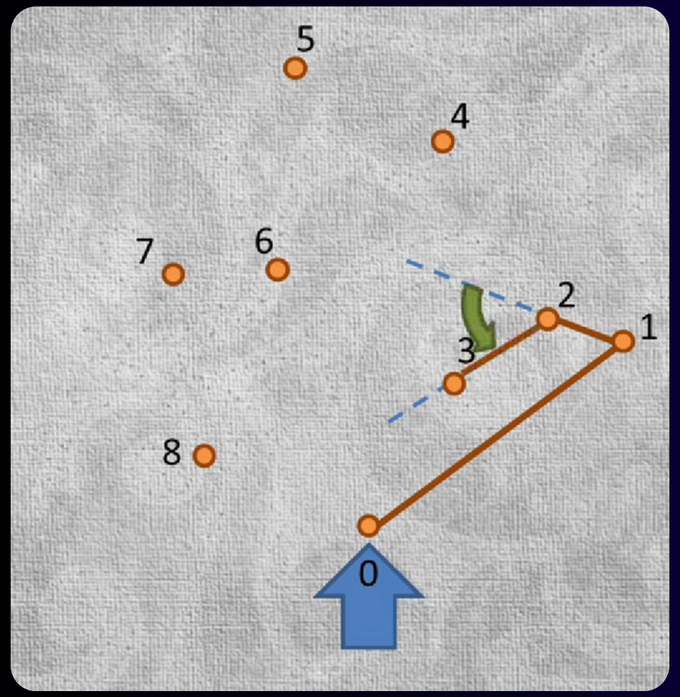


Point Selection and Ordering

Steps 1 and 2:

- Iterate over each point
- Declare the point with the smallest y-value as your pivot
- Order points based on the smallest angle made with the pivot

Point Inclusion and Hull Creation



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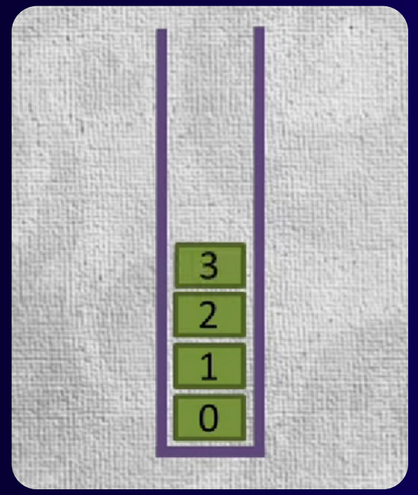
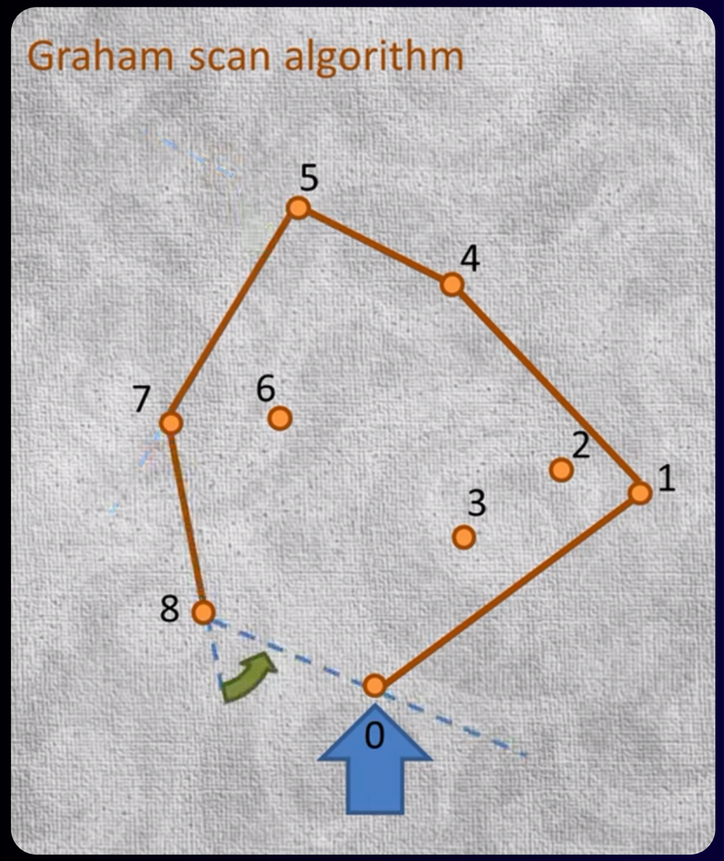


Steps 3 and 4:

- Add in-order point to stack
- Determine if angle to next point creates a clockwise turn (negative angle) or counter-clockwise turn (positive angle)
- if clockwise, remove from stack and go back
- if counter-clockwise, proceed to next point



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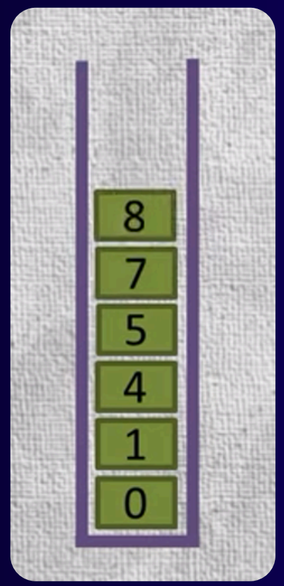
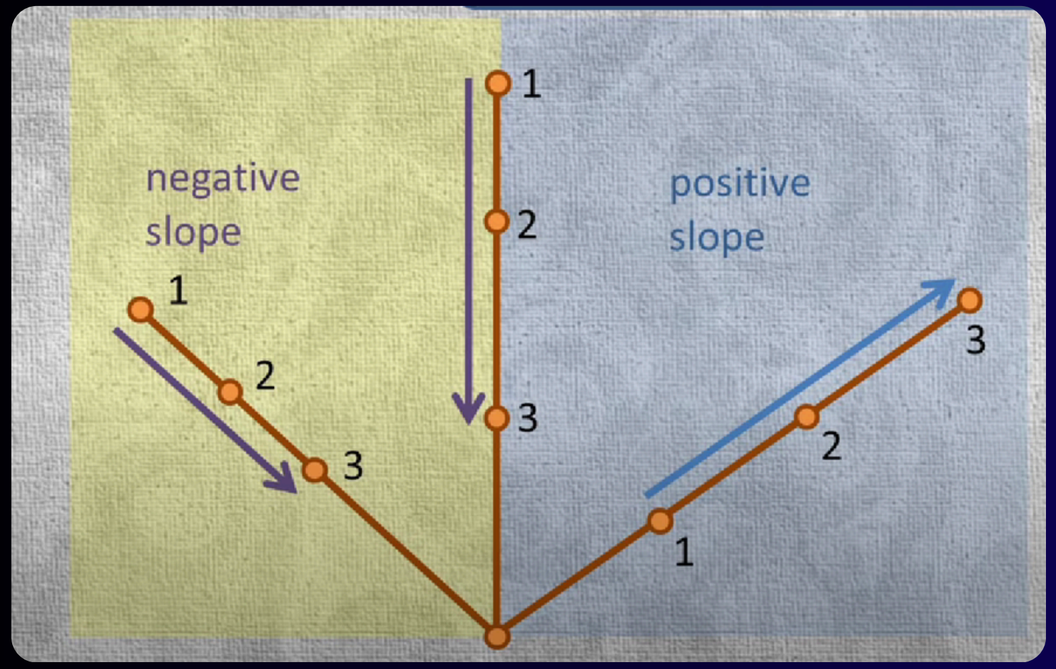
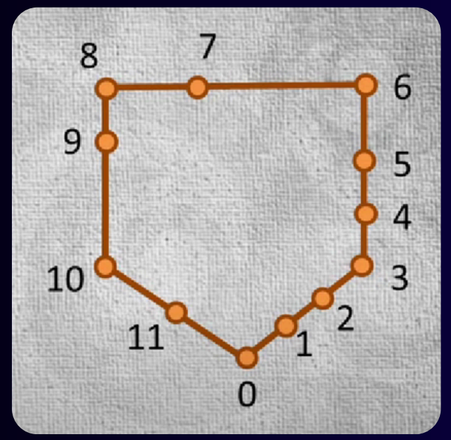


Conclusion

Step 5:

- Once starting point is reached, the algorithm is complete
- The stack contains the points included in the convex hull





Pitfalls & Edge Cases

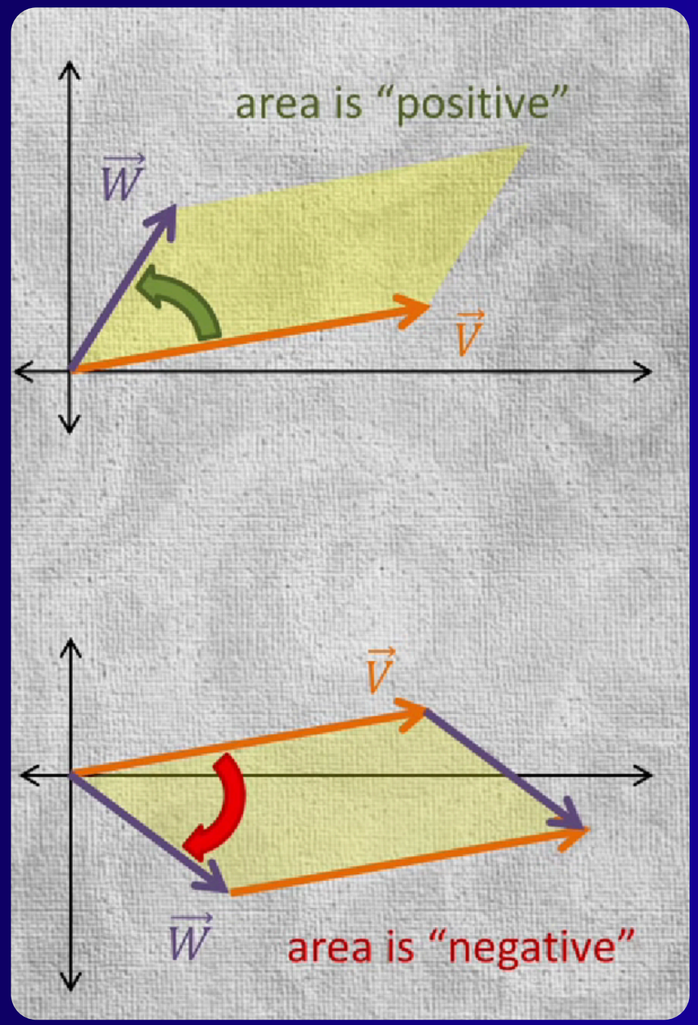
Colinear Points

- Points in a positive slope from start are order higher as distance increases
- Points vertical or in a negative slop from start are ordered lower as distance increases

Implementation:

How do we know if a turn if Clockwise or counter-clockwise?

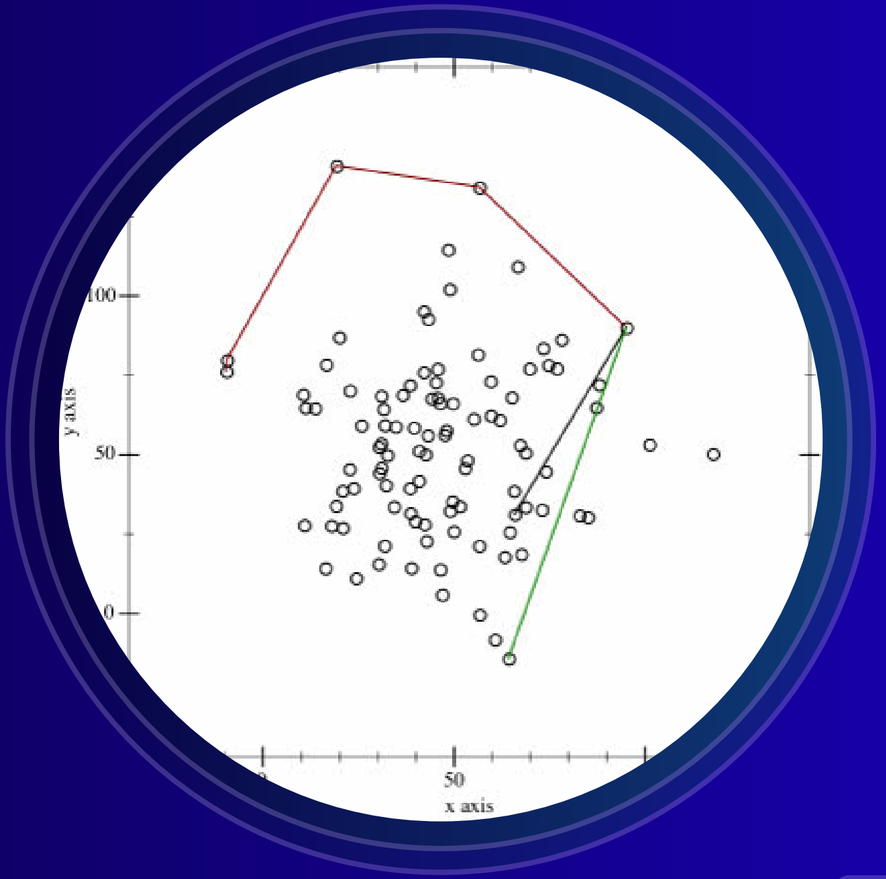
- Calculate Cross product
  - Area is positive if V is on the right of W which forms a clockwise turn.
  - Area is negative if V is on the left of W which forms a counter-clockwise turn.

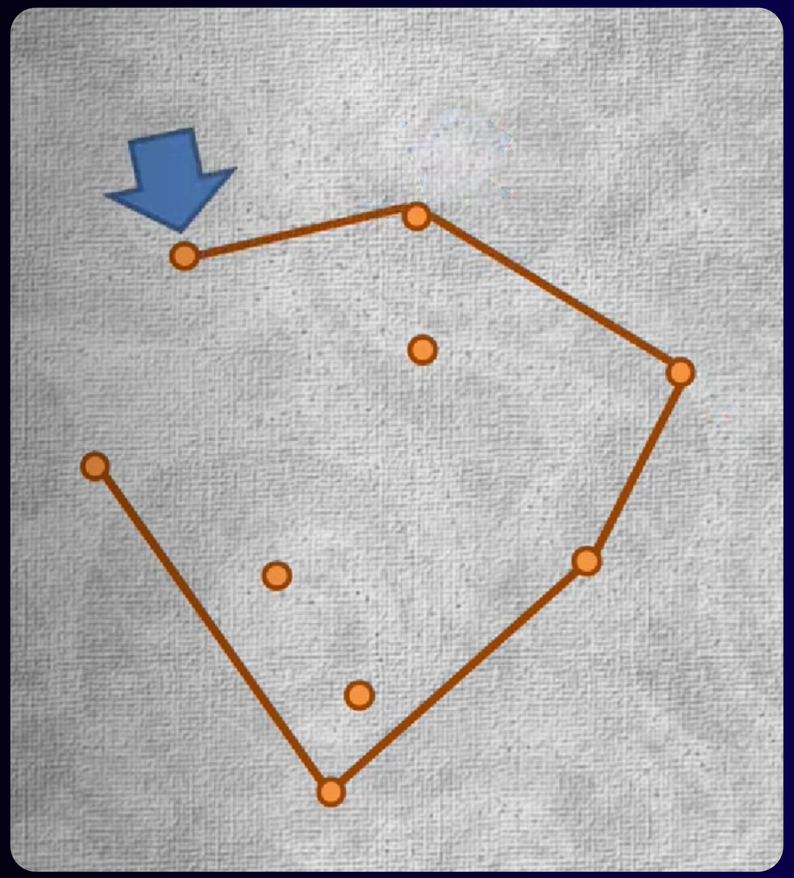


Jarvis March -Overview

Steps:

1. Start with the leftmost point
2. From the current point, include the next point such that the line between these two is “outside” of every other point
3. Repeat until you return to the starting point.

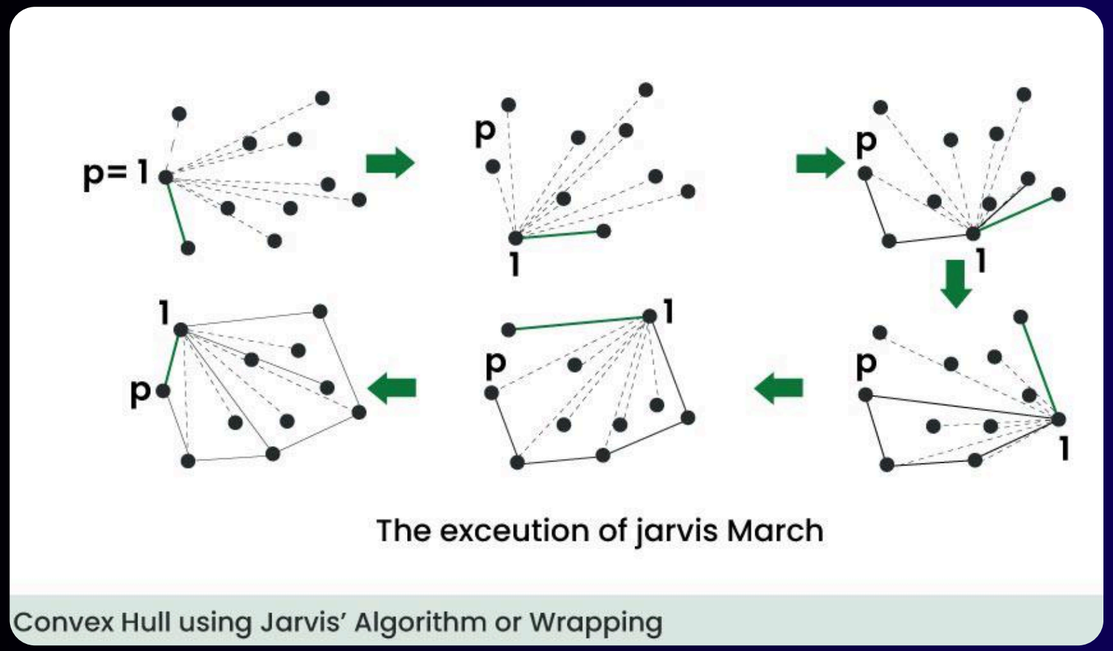




Point Selection

Step 1:

- Iterate over each point
- Declare the point with the smallest x-value as your pivot

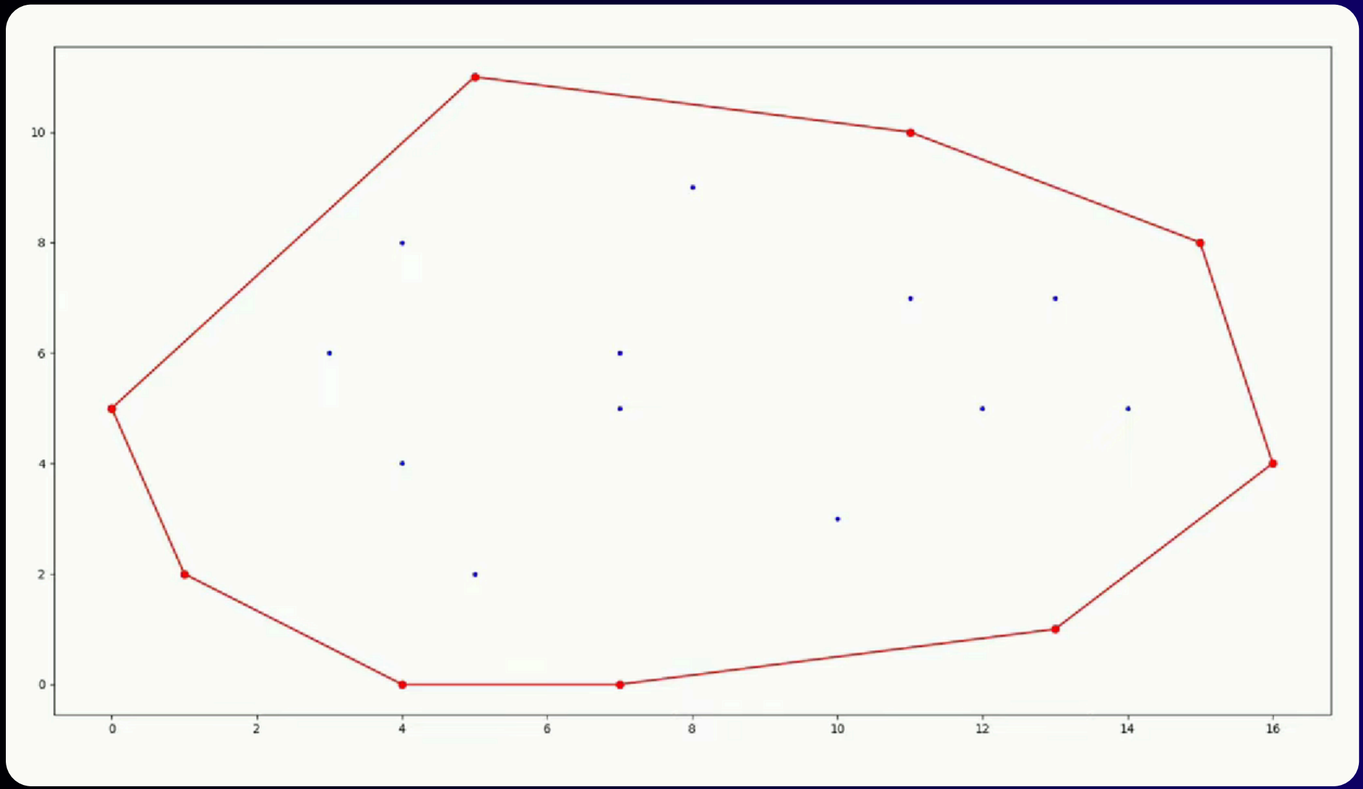


Convex Hull using Jarvis' Algorithm or Wrapping

Main Algorithm

Step 2:

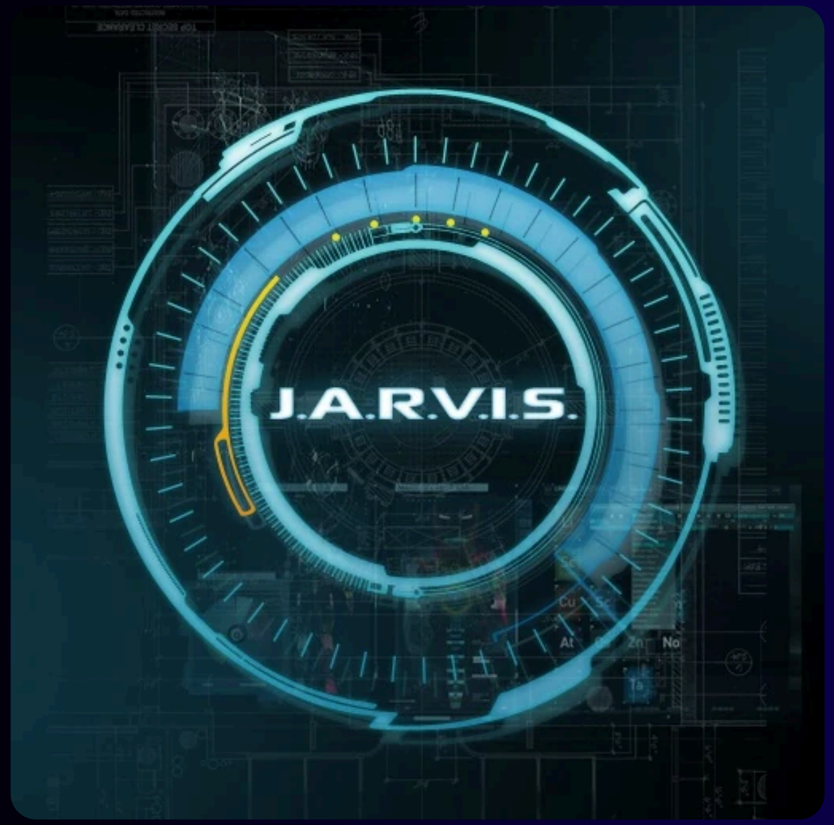
- From the current point, check every other point
- Select the point with the largest angle to connect and insert it into resultant array
- Draw the connecting line
- This line must be outside of all other points



Conclusion

Step 3:

- Once starting point is reached, the algorithm is complete

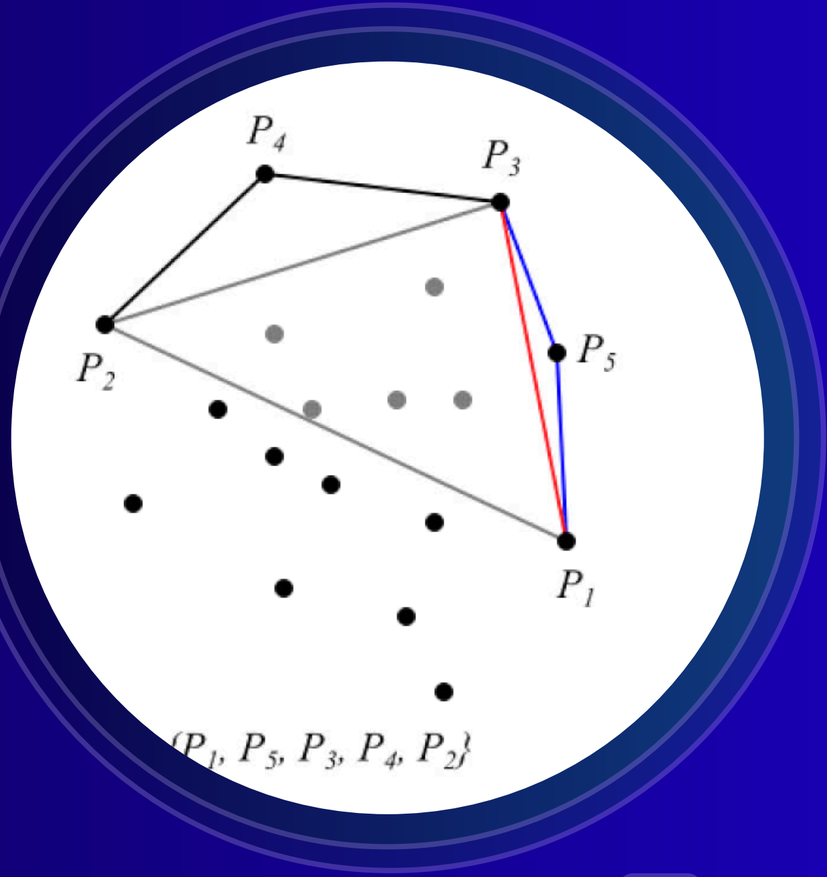


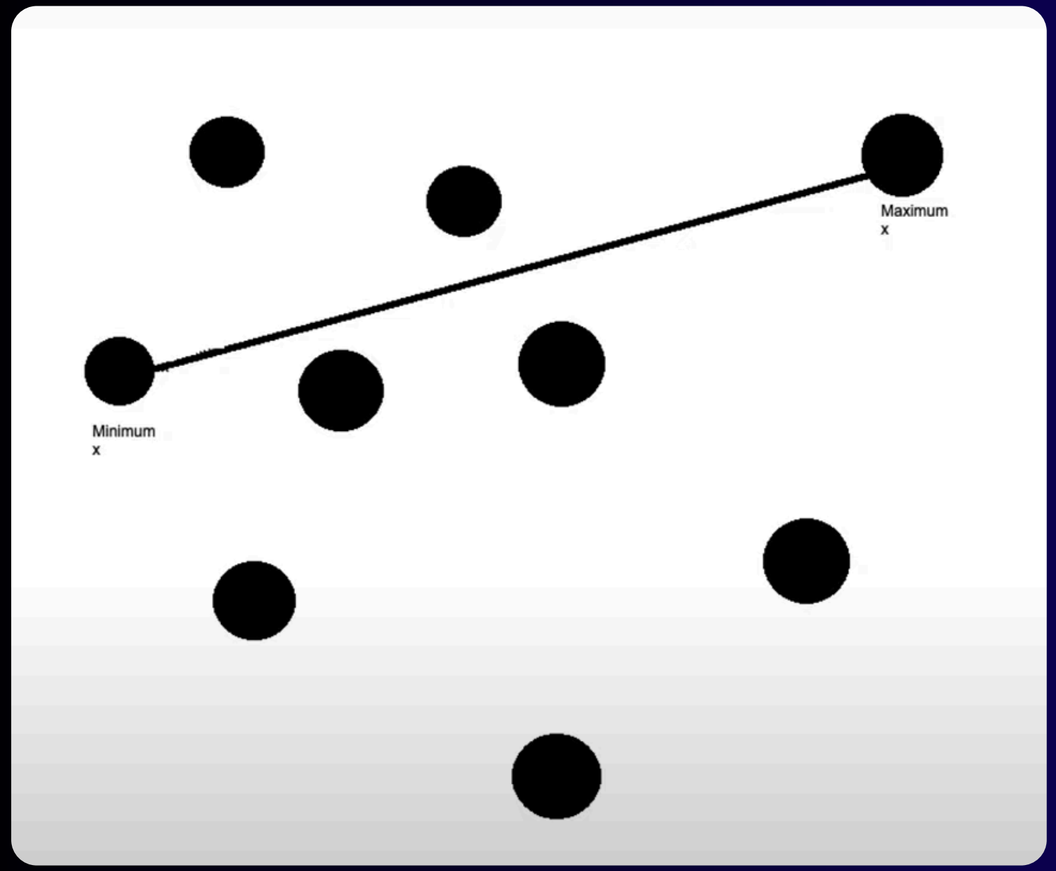
Cons of Jarvis March

1. Inefficient for Large Datasets
2. Slow Compared to Other Algorithms
3. Harder to Optimize
4. Angle Comparison Is Costly
5. Sensitive to Floating Point Errors

Quickhull – Overview

1. Find the point with minimum and maximum x-coordinate.
2. Make a line joining the two points.
3. For a part, find the point with the maximum distance from the line. P, min, and max x-coordinates form a triangle.
4. This step splits the problem into two smaller parts, which are solved one at a time.
5. We treat the lines as new edges, look for points outside the triangle, and repeat.

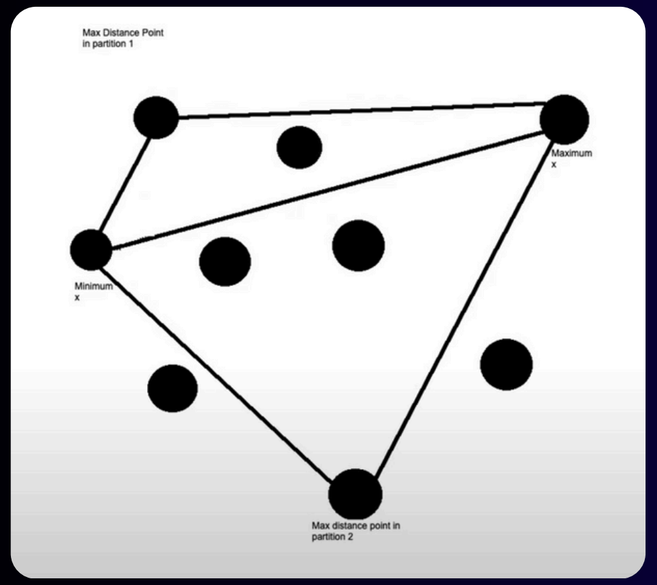


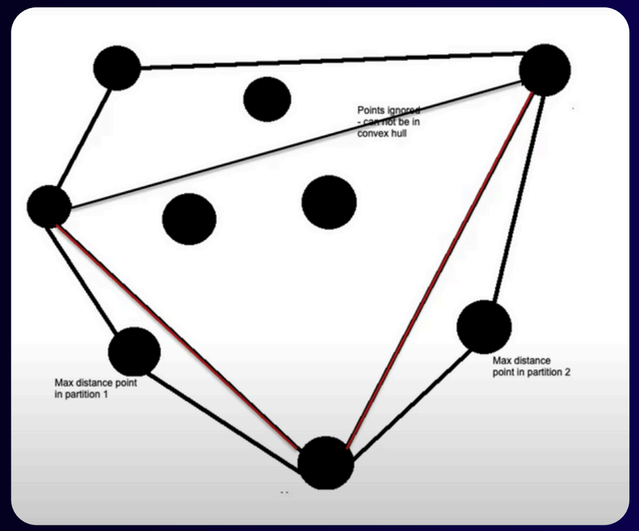


Point Selection

Steps 1 and 2:

- Find the point with minimum x-coordinate and the maximum x-coordinate. (min_x & max_x)
- These two points form a line segment, splitting the set into two subsets: Points above the line and points below the line

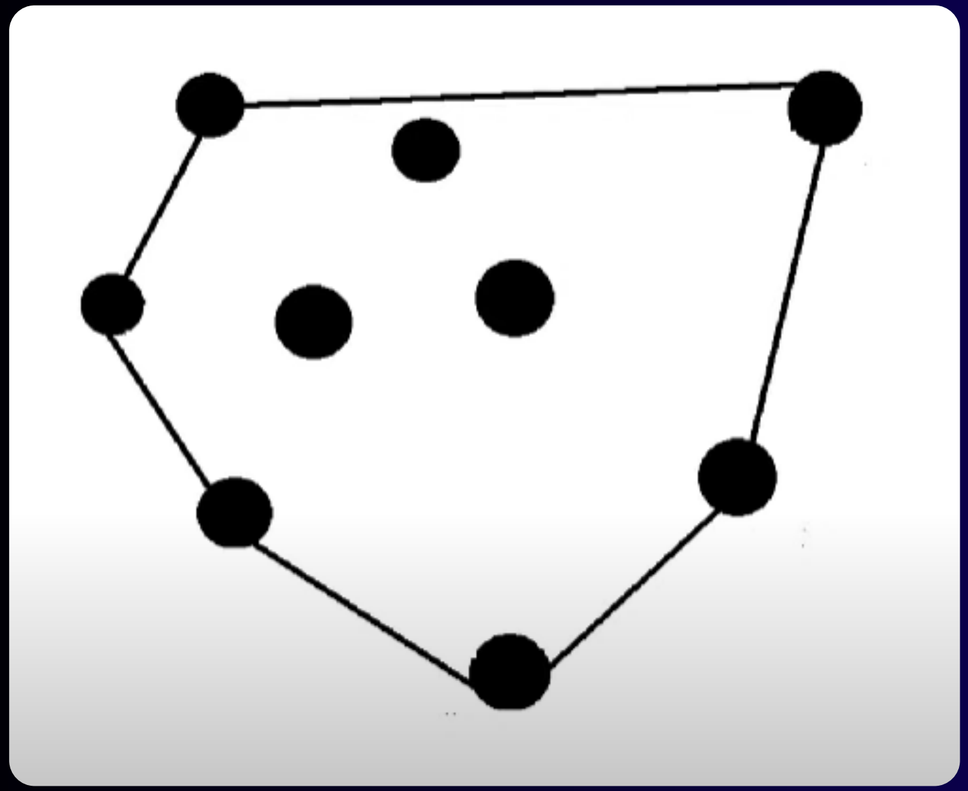




Point Inclusion

Steps 3 and 4:

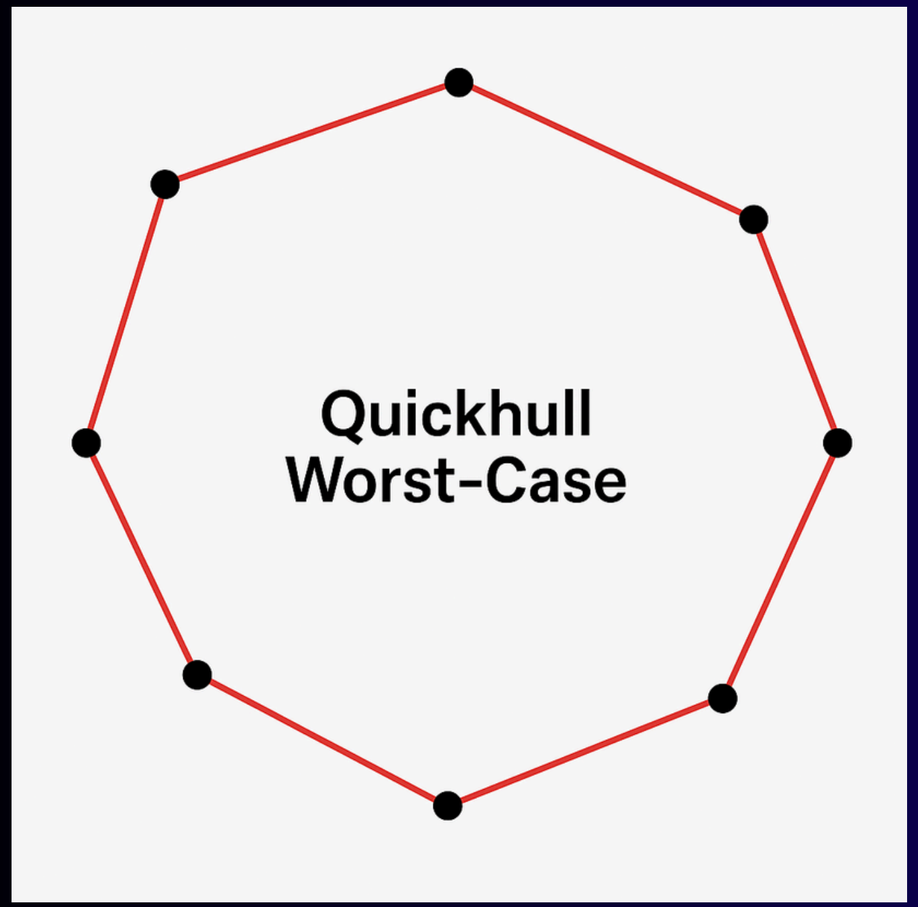
- Recursively find the farthest point from the line to form a triangle.
- Remove points inside this triangle (since they can't be on the hull).
- Recurse on the new edges formed to find more hull points.



Hull Creation

Step 5:

- Continue until no points remain outside the hull edges. The points you've collected form the convex hull.



Worst Case scenario

The worst-case time complexity of Quickhull is $O(n^2)$, and this happens when:

- All or nearly all points lie on the convex hull
- Or, the input points are arranged in such a way that each recursive call removes very few or no points, causing deep recursion and lots of redundant work

Comparison

ALGORITHM	TIME COMPLEXITY	BEST USE CASE
Graham Scan	O(n log n)	Static, sorted data
Jarvis March	O(nh)	Small output size
Quickhull	O(n log n) avg	Large datasets, fast

Our Best Team

Vicky Heredia

Rykir Evans

Ashley Flores

THANK YOU

QUESTIONS?