Assignment-3

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Team 16

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Contents

1 Guassian Mixture Models

1.1 a) GMM

In Gaussian Mixture Models the whole data is represtend as $\boldsymbol{k}$ mixtrure of Gaussians.

$$p(X) = \pi_1 \mathcal{N}(\mu_1, \Sigma_1) + \pi_2 \mathcal{N}(\mu_2, \Sigma_2) + ... + \pi_k \mathcal{N}(\mu_k, \Sigma_k) \tag{1}$$

π_k is the Probability that $\mathbf{X}$ belongs to that gaussian. π_i is also a distrubution

$$\sum_{i=1}^{K} \pi_i = 1 \tag{2}$$

From fact that

$$\int p(X) d\boldsymbol{x} = 1 \tag{3}$$

$$\int p(X) d\boldsymbol{x} = \int \pi_1 \mathcal{N}(x|\mu_1, \Sigma_1) d\boldsymbol{x} + \int \pi_2 \mathcal{N}(x|\mu_2, \Sigma_2) d\boldsymbol{x} \tag{4}$$

Since $\int \mathcal{N}(x|\mu_1, \Sigma_1) d\boldsymbol{x} = 1$

$$p(X) d\boldsymbol{x} = \pi_1(1) + \pi_2(1) \tag{5}$$

Substitue 1 in place of $\int p(X) d\boldsymbol{x}$ using equation 3. $\pi_1 + \pi_2 = 1$

1.2 b) Expectation Maximization Algorithm

1.2.1 Initialization

Apply K-means Algorithm on the given data.

Initialize μ_k , Σ_k , N_k

1.2.2 Expectation Step

$$\gamma(z_{\mathrm{nk}}) = \frac{\pi_{\mathrm{k}} \mathcal{N}(x_{\mathrm{n}}|\mu_{\mathrm{k}}, \Sigma_{\mathrm{k}})}{\sum_{n=1}^{K} \pi_{\mathrm{k}} \mathcal{N}(x_{\mathrm{n}}|\mu_{\mathrm{k}}, \Sigma_{\mathrm{k}})} \tag{6}$$

1.2.3 Maximization Step

$$N_k = \sum_{n=1}^{N} \gamma(z_{nk}) \tag{7}$$

$$\mu_k^{new} = \frac{1}{N_k} \sum_{n=1}^{N} \gamma(z_{nk}) x_n \tag{8}$$

$$\Sigma_k^{new} = \frac{1}{N_k} \sum_{n=1}^{N} \gamma(z_{nk})(x_n - \mu_k^{new})(x_n - \mu_k^{new})^T \tag{9}$$

$$\pi_k^{new} = \frac{N_k}{N} \tag{10}$$

1.2.4 Evaluating the log likelihood

$$\ln p(X|\mu, \Sigma, \pi) = \sum_{n=1}^{N} \ln \sum_{k=1}^{K} \pi_k \mathcal{N}(x_n|\mu_k, \Sigma) \tag{11}$$

Check for convergance , If not converged go to Expectation Step. Check new_log_likelyhood - old_log_likelyhood. If it's less than 0.1 i.e difference is so low , so our algorithm is converged.

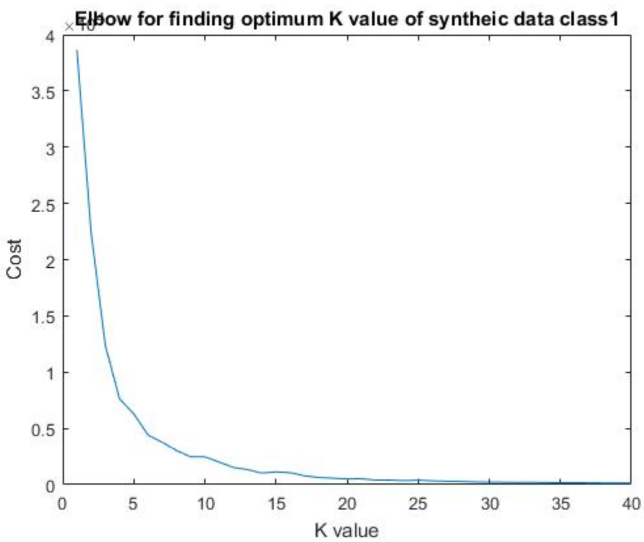
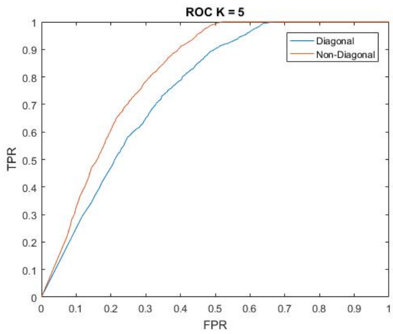


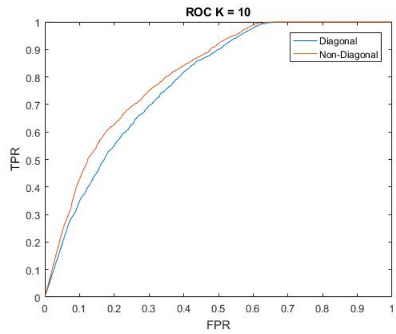
Figure 1: Cost VS K Plot For Deciding K Value

2 Question 1 Real Data

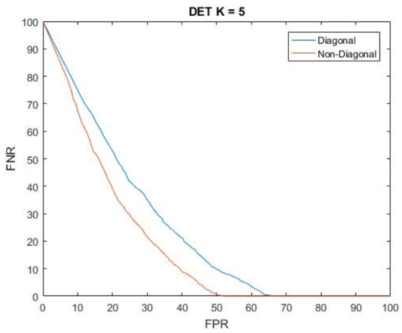
2.1 ROC and DET Plots for both Diagonal and Non Diagonal Covariance Matrices



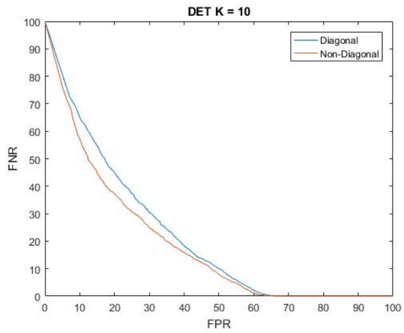
(a) ROC for K=5



(b) ROC for K=10



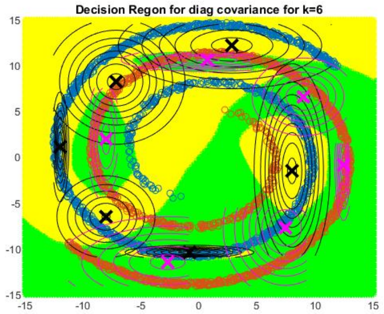
(c) DET for K=5



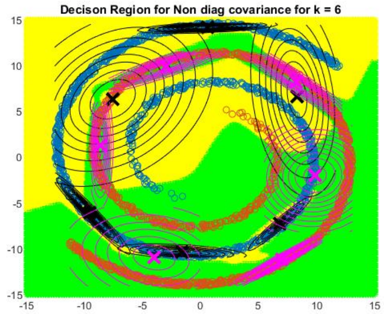
(d) DET for K=10

Figure 2: Plots for Real Data

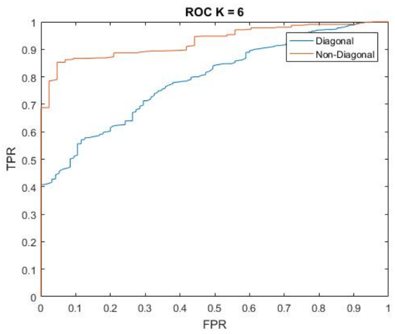
3 Synthetic Data



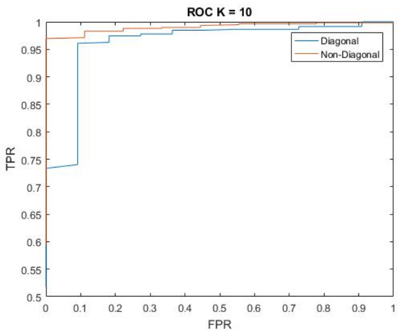
(a) Decision Boundary For Diagonal Covariance K=6



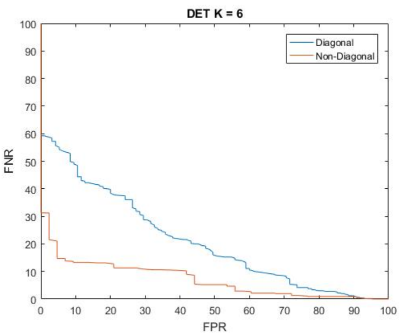
(b) Decision Boundary For Non-Diagonal Covariance k=6



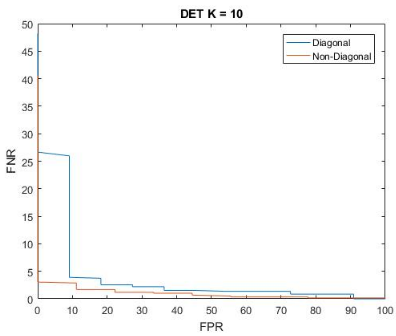
(c) ROC for K=6



(d) ROC for k=10



(e) DET for K=6



(f) DET for K=10

Figure 3: Plots for Synthetic Data

Table 1: Accuracy Values for Different K Values Real Data

K	Diagonal	Non-Diagonal
6	73.7%	76.3%
10	75.9%	78.9%

Table 2: Accuracy Values for Different K Values Synthetic Data

K	Diagonal	Non-Diagonal
6	93.5%	95.7%
10	98.8%	99.2%

3.1 Confusion Matrices

		1	2	3
Output	1	37.7%	7.1%	16.5%
	2	0%	23.7 %	0%
	3	0%	0%	14.9%

Accuracy is 76.3 %

Table 3:Confusion Matrix for Real World Data

		1	2
Output	1	50%	4.3%
	2	0%	45.7%

Accuracy is 95.7% Table 4 Confusion Matrix for Synthetic Data