AMATH 482 - Winter Quarter
Homework 1: Extended Yale Faces B Database – Eigenfaces

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Abstract

This project using single value decomposition to perform image reconstruction from a database of facial images. Two sets of images are used to contrast the difference between cropped and uncropped images. The project concludes that better results are obtained if cropped images are used and significantly better results are produced if the data is grouped by subject.

1 Introduction and Overview

This project aims to investigate how single value decomposition can be used in image processing. Specifically by using a database of images of faces and performing single value decomposition (SVD) then reconstructing these images with a reduced number of values it will show how SVD can be useful in compressing images. Comparing two sets of images, cropped and uncropped will highlight the impact of alignment on the amount of data needed to reconstruct the image. There is also a comparison between combining data from different subjects into one matrix and keeping the data separate.

There is a brief description of the theoretical background behind this method and a discussion of how the process has been implemented here. The results are presented as a series of images and graphs which highlight the comparisons outlined above.

2 Theoretical Background

Singular value decomposition is an important tool used in image processing. It involves breaking a transformation matrix into a serious of matrices that each perform a different task. When vectors are transformed they are subject to a combination of three specific transformations; stretch, compression and, rotation. SVD finds three matrices which each represent one of these transformations.

The transformation can be represented by equation 1 where n represents the rank of matrix $\mathbf{A}$. For a matrix of rank r there will be exactly r values of j that are non-zero, meaning that the rest of the data from matrix $\mathbf{V}$ and $\mathbf{U}$ is redundant. In this project and in most real world data having σ values that are zero is unlikely since the data is full of noise, therefore it is an important task to determine how many σ values are necessary to reconstruct the data and which are small enough to ignore.

$$\mathbf{A}\mathbf{v}_j = \sigma_j \mathbf{u}_j \quad 1 \leq j \leq n. \tag{1}$$

Equation 1 can be shown in matrix form as below and rearranged to achieve the final form of the SVD shown in equation 2.

$$\begin{aligned} \mathbf{AV} &= \mathbf{U\Sigma} \\ \mathbf{A} &= \mathbf{U\Sigma V}^* \end{aligned} \tag{2}$$

In the SVD $\mathbf{U}$ and $\mathbf{V}$ are unitary, hence $\mathbf{V} = \mathbf{V}^*$ and $\mathbf{U} = \mathbf{U}^*$. $\mathbf{\Sigma}$ is diagonal and the elements σ are non-negative and ordered from largest to smallest on the diagonal of the matrix. A further explanation of how the SVD is calculated can be found in section 14.1 of Computational Methods for Data Analysis by Professor J. Nathan Kutz [1].

An important property of single value decomposition is that it is guaranteed to exist for any matrix $\mathbf{A}$[1]. This makes it extremely useful for image processing when data can be very complex.

3 Algorithm Implementation and Development

MATLAB has been used to implement and perform analysis on the images provided by the 'Yale face database'[2]. A brief description of the functions used can be found in Appendix A. Firstly the cropped images were read and the data stored in a matrix with one column per image. Performing a single value decomposition on this matrix produced the three matrices described in the previous section. Before attempting a recomposition of the images a plot of the singular values was produced in order to estimate how many values would be necessary to produce a good reconstruction.

The plot of the single values for the cropped images can be seen in figure 1, from the plot it is clear that some of the single values carry more weight than others, with some of the values being almost 10^10 bigger than the smallest values. This implies that the reconstruction of the image will not be greatly effected by ignoring some of the lower values. The same can be said for the uncropped values in figure 2, but as there are less images and therefore less singular values this indicates that more values may be needed for a better reconstruction.

When all the uncropped images are combined into one matrix it is hard to tell from the first plot, figure 3, which values will be significant, therefore a logarithmic plot of the values has been produced to get a better understanding of the scale of the values, figure 4.
Since visual recognition provides a better indicator of whether the image has been reconstructed correctly, the images are reformed using equation 2 with various values of n to determine how many singular values are needed to reconstruct the images to a level recognizable to the human eye.

This process is repeated for the database of uncropped images. A comparison of the difference between the two sets of images is discussed further below. Another comparison is drawn between performing the analysis on a set of images of one subject and combining the images of several subjects together in one matrix.

4 Computational Results

Figure 5 shows the results of using the SVD decomposition to reconstruct a face using 1-6 singular values. The first image (top left) uses only one value and the final image (bottom right) uses 6 values to do the reconstruction. These images show how the details of the face become sharper as the number of σ values increases. Figure 7 shows the original image, comparing this with the reconstructed images it is clearly recognised to be the same subject however the original image definitely have more definition. It can also been seen that image 1 is less recognisable as the same person in Figure 7 and has more generalised features.

Performing a reconstruction without the first (largest) singular value produces a completely different results as shown in figure 6. These images are far less recognisable as the same person from figure 7, showing the importance of using the larger singular values for reconstruction.

When doing the same process with the uncropped images the reconstruction is of similar quality but requires slightly more singular values to generate an image that is as sharp as that of the cropped images. The example is particularly interesting becaue the subject is wearing glasses, there are no other accessories

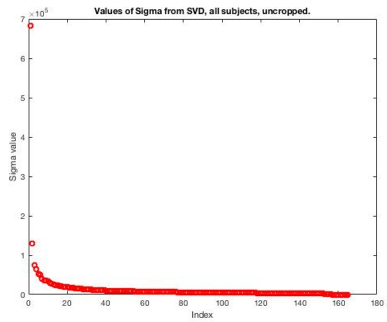

Figure 1: Singular values from matrix Σ of all uncropped images.

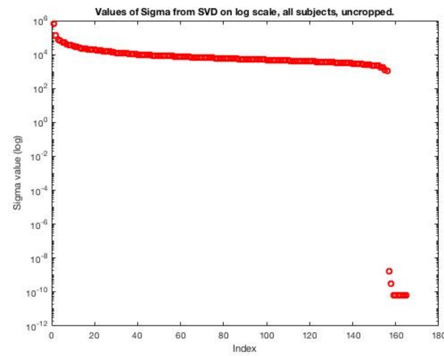

Figure 2: Singular values from matrix Σ of all uncropped images, plotted on a log scale with matlab semilogy function.

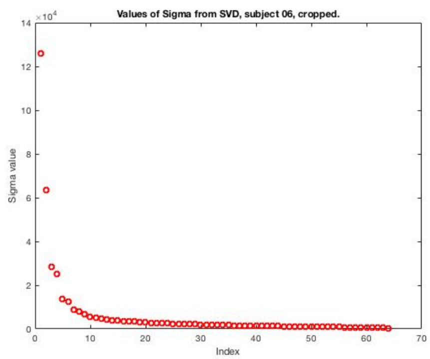

Figure 3: Singular values from matrix Σ for cropped images, subject 06, image 2.

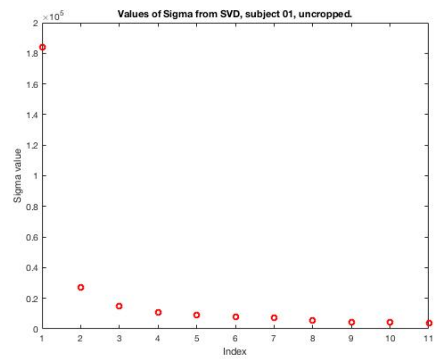

Figure 4: Singular values from matrix Σ for uncropped images, subject 01, image 2.

in any of the other images meaning that the glasses don't appear in the reconstruction. The results can be seen in figure 9.

If we take all the images and put them into one big matrix and then do SVD the reconstruction the results are far less defined. Figure 9 shows what the reconstruction looks like using 6 values of σ and 30 values of σ. Here it can clearly be seen that the result using six singular valuables is not at all recognizable. Using 30 singular values produces an image that is almost recognizable but still no where near as good as when the matrix was only made up of images of a single person.

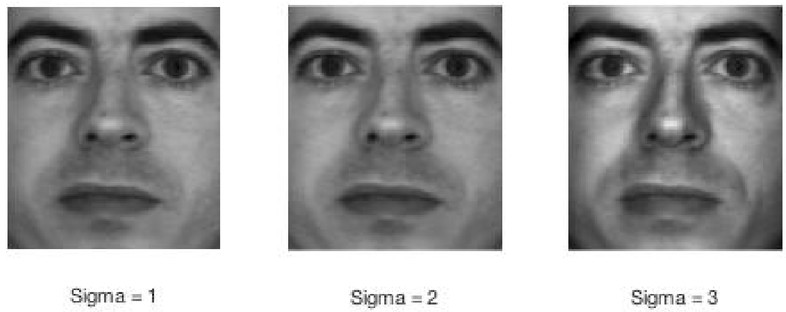

Figure 5: Cropped Yale face reconstruction using SVD. Image 2 of subject 06. Each image uses a different number of Σ values from 1 (top left), to 6 (bottom right) as labeled.

Figure 6: Cropped Yale face reconstruction using SVD. Image 2 of subject 06. Each image uses a different number of Σ values from 2 (top left), to 7 (bottom right) as labeled.

Figure 7: Original image from cropped Yale faces, subject 06, image 2.

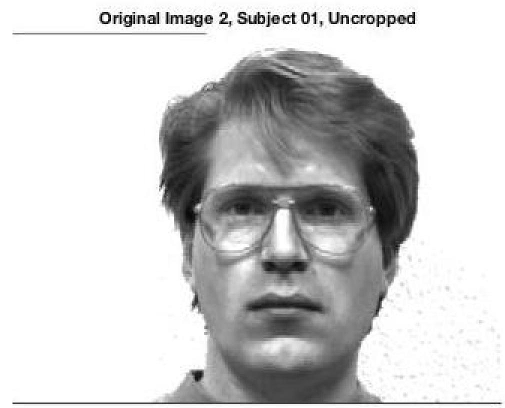

Figure 8: Original image from uncropped Yale faces, subject 01, image 2.

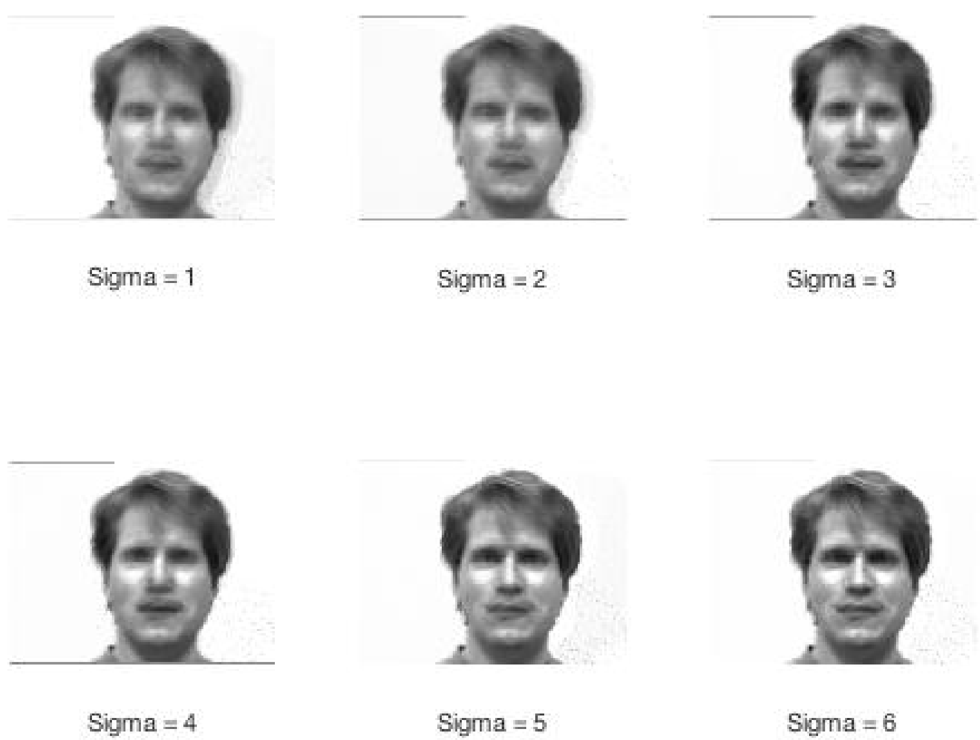

Figure 9: Uncropped Yale face reconstruction using SVD. Image 2 of subject 01. Each image uses a different number of Σ values from 1 (top left), to 6 (bottom right) as labeled.

Figure 10: Reconstruction of subject 01, image 2 from all images SVD using 6 and 50 singular values.

5 Summary and Conclusion

Single value decomposition is a really valuable technique in image processing, this project has shown that it is an effective tool in deconstruction and reconstructing images of faces.

SVD is good because it means the image can be stored in a compressed form, making it possible to store thousands of images on small devices such a smart phones.

An issue with SVD is that it is not always necessary to store the same number of values of σ for all kinds of images. For example if the images are aligned then less values of σ will be required to reconstruct a given image.

References

[1] *Computational Method for Data Analysis* Professor J. Nathan Kutz 2018.

[2] *Yale faces database* Professor David Kriegman, Professor Peter Belhumeur

[3] *mathwork.com*

Appendix

A

- [U,S,V] = svd(A, 'econ') - produces an economy-size decomposition of m-by-n matrix A, S containing the singular values.[3]
- B = reshape(A,sz) - reshapes A using the size vector, sz, to define size(B).[3]
- A = imread(filename) reads the image from the file specified by filename, inferring the format of the file from its contents.[3]

B

MATLAB code for cropped images

```
clear all; close all;
%load images
images = [dir('/Users/vick/Documents/AMATH482/hw1/CroppedYale/YaleB06')];

%read images and store data in a matrix with one column per image
numfiles = length(images);
mydata = cell(1, numfiles);

for k = 3:numfiles
    mydata{k-2} = imread(sprintf(images(k).name));
    data(:,k-2) = reshape(double(mydata{k-2}),[],1);
end

%perform svd
[u,s,v] = svd(data, 'econ');

%plot singular values
s1 = diag(s);
figure
plot(s1,'ro','Linewidth',[2])
title('Values of Sigma from SVD, subject 06, cropped.')
ylabel('Sigma value')
xlabel('Index')
figure
semilogy(s1,'ro','Linewidth',[2])

%%

%show origional image
figure
image = 2;
imshow(uint8(reshape(data(:,image),192,168)))
title('Original image 2, subject 06, cropped.')

%show images with different number of singular values
l = 1;
figure
for j = l+1:l+6
    %reconstruct image using svd
    A = u(:,l+1:j)*s(l+1:j,l+1:j)*v(:,l+1:j)';
```

```
    subplot(2,3,j-l)
    imshow(uint8(reshape(A(:,image),192,168)))
    xlabel(sprintf('Sigma = %d', j))
end
```

MATLAB code for uncropped images, one subject at a time

```
clear all; close all;
%load images
images = [dir('/Users/vick/Documents/AMATH482/hw1/yalefaces_uncropped/yalefaces/subject01.*')];

%read images and store data in a matrix with one column per image
numfiles = length(images)
mydata = cell(1, numfiles);

for k = 1:numfiles
    mydata{k} = imread(sprintf(images(k).name));
    data(:,k) = reshape(double(mydata{k}),[],1);
end

%perform svd
[u,s,v] = svd(data, 'econ');

%plot singular values
s1 = diag(s);
figure
plot(s1,'ro','Linewidth',[2])
title('Values of Sigma from SVD, subject 01, uncropped.')
ylabel('Sigma value')
xlabel('Index')
figure
semilogy(s1,'ro','Linewidth',[2])

%%
%show origional image
figure
image = 2;
imshow(uint8(reshape(data(:,image),243,320)))
title('Original image 2, subject 01, uncropped.')

%show images with different number of singular values
l = 1;
n = 6;
figure
for j = l:l+n-1
    %reconstruct image using svd
    A = u(:,l:j)*s(l:j,l:j)*v(:,l:j)';
    subplot(2,3,j-l+1)
    imshow(uint8(reshape(A(:,image),243,320)))
    xlabel(sprintf('Sigma = %d', j))
end
```

MATLAB code for uncropped images, all subjects

```
clear all; close all;
%load images
images = [dir('/Users/vick/Documents/AMATH482/hw1/yalefaces_uncropped/yalefaces/')];

%read images and store data in a matrix with one column per image
numfiles = length(images);
mydata = cell(1, numfiles);


for k = 4:numfiles
    mydata{k-3} = imread(sprintf(images(k).name));
    data(:,k-3) = reshape(double(mydata{k-3}),[],1);
end

%perform svd
[u,s,v] = svd(data, 'econ');


%plot singular values
s1 = diag(s);
figure
plot(s1,'ro','Linewidth',[2])
title('Values of Sigma from SVD, all subjects, uncropped.')
ylabel('Sigma value')
xlabel('Index')
%plot singular values logarithmic
figure
semilogy(s1,'ro','Linewidth',[2])
title('Values of Sigma from SVD on log scale, all subjects, uncropped.')
ylabel('Sigma value (log)')
xlabel('Index')

%%
%show origional image
figure
image = 2;
imshow(uint8(reshape(data(:,image),243,320)))
title('Original image 2, subject 01, uncropped.')

%show images with different number of singular values
l = 1;
n = 6;
figure
% for j = l:l+n-1
%     reconstruct image using svd
%     A = u(:,l:j)*s(l:j,l:j)*v(:,l:j)';
%     subplot(2,3,j-l+1)
%     imshow(uint8(reshape(A(:,image),243,320)))
%     xlabel(sprintf('Sigma = %d', j))
% end

%reconstruct image using svd
```

```matlab
%show for specific values of sigma
A = u(:,1:6)*s(1:6,1:6)*v(:,1:6)';
subplot(1,2,1)
imshow(uint8(reshape(A(:,image),243,320)))
xlabel('Sigma = 6')
A = u(:,1:50)*s(1:50,1:50)*v(:,1:50)';
subplot(1,2,2)
imshow(uint8(reshape(A(:,image),243,320)))
xlabel('Sigma = 30')
\begin{minted}{matlab}
```