Formulas

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1 Indices

Number of links
number of confluences plus number of barriers.

Number of components
number of connected subgraphs (not number of nodes!!)
(when fully connected should be 1)

Harrary Index (Plavšić 1993)

$$H = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (RD)ij$$
$$(RD)ij = \frac{1}{D_{ij}}, i \neq j \qquad (1)$$

$(RD)ij$ reciprocal distance matrix (replace all matrix elements representing the shortest distances between vertices i and j Dij by their reciprocals)

Between Centrality (Freeman 1977)
Betweenness centrality for each vertex is the number of shortest paths that pass through the vertex

$$BC = \frac{1}{(N-1)(N-2)} \sum_{s \neq v \neq t} \frac{\sigma_{s,t}(v)}{\sigma_{s,t}} \qquad (2)$$

$\sigma_{s,t}$ is the number of shortest paths between node s (source) and t (mouth)
$\sigma_{s,t}(v)$ is the number of shortest paths between node s (source) and t (mouth) that pass through node v

Percolation Centrality
Percolation centrality for a given node, at a given time, is the proportion of ‘percolated paths’ that go through that node. A ‘percolated path’ is a shortest

path between a pair of nodes, where the source node is percolated (e.g., infected). The target node can be percolated or non-percolated, or in a partially percolated state.

$$PC^t(v) = \frac{1}{(N-2)} \sum_{s \neq v \neq r} \frac{\sigma_{s,r}(v)}{\sigma_{s,r}} \frac{x_s^t}{(\sum x_i^t) - x_v^t} \tag{3}$$

$\sigma_{s,t}$ is the number of shortest paths between node s (source) and t (mouth)
$\sigma_{s,t}(v)$ is the number of shortest paths between node s (source) and t (mouth) that pass through node v

Closeness Centrality

$$CC(v) = \frac{1}{\sum_{i \neq v} d_g(v,i)} \tag{4}$$

$d_g(v,i)$ is the shortest path (geodesic) distance between nodes v and i

Eigenvector Centrality
centrality scores of nodes are given by the matrix X and the adjacency matrix of the network is A.
Algorithm can be found here: `https://neo4j.com/docs/graph-data-science/current/algorithms/eigenvector-centrality/`
Freeman Centrality
ask Goncalo

Eccentricity
ask Goncalo

Catchment Area-based Fragmentation Index (Jumani 2022)

$$CAFI = \sum_{i=1}^{n} \frac{a_i}{A} * 100$$
$$CAFI = \sum_{i=1}^{n} \frac{a_i c_i}{A} * 100 \tag{5}$$

a_i total catchment area of dam i
A catchment area of the entire river network

Catchment Area- and Rainfall-based Fragmentation Index

$$CARFI = \sum_{i=1}^{n} \frac{a_i r_i}{AR} * 100 \tag{6}$$

a_i total catchment area of dam i
A catchment area of the entire river network
$r_i$is the average annual rainfall intensity in a_i
R is the average annual rainfall intensity in the entire catchment area

Dentric Connectivity Index (Cote 2009)
(weighted average connectivity value of stream section pairs)
potadromous

$$DCI_P = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \frac{l_i}{L} \frac{l_j}{L} * 100$$

$$c_{ij} = \prod_{m=1}^{M} p_m^u p_m^d \qquad (7)$$

binary passability

$$DCI_P = \sum_{i=1}^{n} \frac{l_i^2}{L^2} * 100$$

$p_m^u p_m^d$ up- and downstream passabilities
l_i length of section i
L length of drainage network

diadromous

$$DCI_D = \sum_{i=1}^{n} \frac{l_i}{L} \left(\prod_{m=1}^{M} p_m^u p_m^d \right) * 100$$
$$DCI_D = \frac{l_i}{L} * 100 \qquad (8)$$

$DCI_{sectional}$ (Mahlum 2014) no access to supplemental data (ask Goncalo)

Conservation Connectivity Index for potamodromous fish species

$$ci_r == s_r \times BI_r$$
$$CCI_P == \sum_{i=1}^{n} \frac{cs_i}{CS^2} x100$$
$$cs_i == \sum_{r=1}^{n} ci_r \qquad (9)$$
$$CS == \sum_{i=1}^{n} cs_i$$

s_r is the length of segment r
BI_r Biodiversity Index of segment r
ci Conservation Interest of a river segment

Breeding Habitat Connectivity Index(Rodeles 2019)

$$HCIb = \sum_{i=1}^{n} \frac{H_{accessible}}{H}$$
$$H_{accessible} = p_j \times SL_i \qquad (10)$$
$$SL_i = Q_i \times l_i$$

p_j total passability to node j
SL_i Index of suitable length
H Total Habitat
Q_i Index of Habitat Quality (see paper)
l_i length of its segment

River connectivity index (Grill 2014)

$$
\begin{aligned}
RCI_{VOL} &= \sum_{i=1}^{n} \frac{v_i^2}{V^2} \\
RCI_{CLASS} &= \sum_{i=1}^{n} \frac{v_i^2 c_i}{V^2 C} 100 \\
RCI_{RANGE} &= \sum_{i=1}^{n} \frac{m_i^2}{M^2} 100
\end{aligned} \tag{11}
$$

v_i is the total river volume of fragment i
V is the total river volume of the entire river network
c i is the total number of distinct river classes in network fragment i
C is the total number of distinct river classes found in the entire river network.
mi is the sum of migration ranges (in terms of river length) of all migratory fish species in network fragment i
M is the total sum of migration ranges (in terms of river length) of all migratory fish species in the entire river network as calculated by the species range model

Fragmentation Index (Díaz 2019)

$$
\begin{aligned}
FI &= 1 - 1.5^{-\sum_{i=1}^{N} IFI(i)} \\
IFI(i) &= \frac{\sum_{j=1}^{M} L_j S_j}{T}
\end{aligned} \tag{12}
$$

L_j Length upstream
S_j Strahler order upstream
M number of stretches in river network upstream of the barrier
N number of Barrier
T maximum value of $IFI(i)$

Population Connectivity Index (Rodeles 2021)

$$
\begin{aligned}
PCI &= \sum_{i=1}^{n} \sum_{j=1} c_{ij} \frac{l_i}{L} \frac{l_j}{L} \\
c_{ij} &= B_{ij} P D^{d_{ij}} \\
B_{ij} &= \prod_{m=1}^{M} p_m
\end{aligned} \tag{13}
$$

Class coincidence probability (Pascual-Hortal 2006)

$$CCP = \sum_{i=1}^{NC} \left(\frac{c_i}{A_C} \right)^2 \qquad (14)$$

with NC number of component
c_i the total area of each component
A_C total habitat area (all habitat patches)

Landscape coincidence probability

$$LCP = \sum_{i=1}^{NC} \left(\frac{c_i}{A_L} \right)^2 \qquad (15)$$

with NC number of component
c_i the total area of each component
A_L total landscape area

Integral index of connectivity

$$IIC = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{a_i a_j}{1+nl_{ij}}}{A_L^2} \qquad (16)$$

a_i area of each habitat patch
nl_{ij} number of links in the shortest path between patches (topological distance)

Flux
i dont know ask Goncalo
Probability of Connectivity

$$PC = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j p_{ij}*}{A_L^2} \qquad (17)$$
$$p_{ij} = e^{-kd_{ij}}$$

p_{ij} dispersal probability (can be obtained in different ways here just example)
$p_{ij}*$ maximum product probability of paths between i and j
d_{ij} edge-to-edge interpatch distance (km)
k species dependant constant

Total remaining core length (Fuller 2015)

$$TAL = \sum_{i=1}^{n} EDH_i \sum_{i=1}^{n} EHU_i \sum_{i=1}^{n} MH_i \qquad (18)$$
$$TCL = TNL - TAL$$

TAL total affected length
EDH_i downstream edge habitat
EDU_i upstream edge habitat
MH_i matrix habitat created by fragmentation agent i
TNL total network length

Dam Impact Index (2017)
River channel connectivity index (Li 2018)

Index of longitudinal riverine connectivity (Crook 2009)

$$ILRC = PrDC_k \times PrUC_k \qquad (19)$$

$PrDC_k$ probability that larvae reach the estuary from above a given point (cumulative downstream passage)
$PrUC_k$ probability that juveniles migrate past multiple intakes to a given point (cumulative upstream passage)
Habitat connectivity index for upstream passage (McKay 2013)

$$HCIU = \frac{Accessiblehabitat}{Totalhabitat} * 100 \qquad (20)$$

Connecitivity Status (Diebel 2014)

$$
\begin{aligned}
\overline{C} &= \frac{\sum_{j=1}^{n}(S_j \cdot \overline{Q}_j \cdot C_j)}{\sum_{j=1}^{n}(S_j \cdot Q_j)} \\
C_j &= \frac{\sum_{t=1, A_{jt}^B > 0}^{m}\left(\frac{A_{jt}}{A_{jt}^B}\right)}{m_j} \\
A_{jt} &= \sum_{i=1}^{n}(S_i \cdot \theta_{it} \cdot Q_i \cdot P_{ij}^{Art} \cdot D_{ij}) \\
D_{ij} &= \frac{1}{1 + (\frac{d_{ij}}{d_0})^2} \\
A_{jt}^B &= \sum_{i=1}^{n}(S_i \cdot \theta_{it} \cdot Q_{it} \cdot P_{ij}^{Nat} \cdot D_{ij})
\end{aligned}
\tag{21}
$$

$\overline{C}$ Overall connectivity status of a watershed
m total number of habitat types m_j number of habitat types for which there is some baseline availability ($A_{jt}^B > 0$) of habitat type t accessible from segment j (no natural barriers $m_j = m$)
D_{ij} value of an inverse distance weighting function that scales the accessibility of nearby segments toward one and distant segments toward zero
d_{ij} is the distance along the stream network between the centroids of two segments i and j, and d_0 is the distance at which the weight equals 0.5.
A_{jt}^B baseline availability (no artificial barriers) of habitat type t from a focal segment j S_i length of segment i
θ_{it} proportion of habitat type t in segment i
Q_{it} Quality of habitat type t

Barrier free length (Jones 2019)
BFL Stream length between two barriers
Conservation Connectivity Index (Rodeles 2020)
for potamodromous fish species

$$
\begin{aligned}
CCI_P &= \sum_{i=1}^{n} \frac{cs_i^2}{CS^2} * 100 \\
cs_i &= \sum_{r=1}^{R} ci_r \\
CS &= \sum_{i=1}^{n} cs_i \\
ci_r &= s_r \cdot BI_r
\end{aligned} \tag{22}
$$

s_r length
BI_r Biodiversity Index of segment r
R segments of a section
n number of sections in network