# The Carnegie RR Lyrae Program: The Mid-Infrared RR Lyrae Period-Luminosity-Metallicity Relations in $\omega$ Cen

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Accepted XXX. Received YYY; in original form ZZZ

#### ABSTRACT

[abstract]

Key words: keyword2 - keyword3

#### 1 INTRODUCTION

The Carnegie RR Lyrae Program (CRRP) is a Warm Spitzer program (Freedman et al. 2012a, PID 90002) with the aim of calibrating the mid-infrared (mid-IR) RR Lyrae period-luminosity (PL) relation. Similar to the Carnegie Hubble Program (CHP) (Freedman et al. 2011), which used mid-IR observations of Cepheids to measure the Hubble constant ( $H_0$  Freedman et al. 2012b), the results of the CRRP will be used to provide an independent, population II calibration of the zero-point of the extragalactic distance scale, and hence an independent measurement of  $H_0$ .

The results of Riess et al. (2011) and Freedman et al. (2012b), both of which use Cepheids and type Ia supernovae (SNe), agree very well at  $74.4 \pm 2.5$  km s<sup>-1</sup> Mpc<sup>-1</sup> and  $74.3 \pm 2.6$  km s<sup>-1</sup> Mpc<sup>-1</sup>, respectivly. However, when we consider the latest results from Planck, who find  $67.48 \pm 0.98$  km s<sup>-1</sup> Mpc<sup>-1</sup> (Planck Collaboration et al. 2015), there is tension. The Planck study derives its measurement from a model of the cosmic microwave background (CMB), so is completely independent of the Riess et al. and Freedman et al. results.

There have been several recent works that have investigated possible sources of uncertainty in the distance ladder that may contribute to the discrepancy between  $H_0$  measurements. For example, Rigault et al. (2015) examine the differences in star formation rates in type Ia SNe host galaxies. They find that the intrinsic brightness of a SNe Ia may be affected by the local host environment; i.e. whether the SN occurs in a locally star forming or locally passive environment. Efstathiou (2014) reanalysed the Cepheid data from Riess et al. (2011), and found that the outlier rejection

criteria in their analysis of NGC4258 lowered the resultant value of  $H_0$  to  $70.6 \pm 3.3$  km s<sup>-1</sup> Mpc<sup>-1</sup>, bringing it into closer agreement with the value from *Planck*.

The CRRP assesses a systematic that was unreachable in the original CHP – the intrinsic accuracy of the zero-point of the mid-IR Cepheid standard candle distance scale. With only one "test candle" it was impossible to make any assessment of this accuracy. However, with two standard candles with similar precision we can make meaningful comparisons and assess their systematic accuracy.

RR Lyrae variables (hereafter RRL) are intrinsically fainter than Cepheids, and in the optical follow a much shallower, even horizontal, PL relation (Catelan et al. 2004). Determining an accurate distance to an RRL in the V band requires knowledge of its metallicity. However, Longmore et al. (1986) showed that the true power of RRL as distance indicators lies in the IR passbands. Recently, everal groups have been studying the populations of RRL in globular clusters and nearby dwarf spheroidal galaxies in the infrared (e.g. Garofalo et al. 2013; Ordoñez et al. 2014; Cusano et al. 2015; Kains et al. 2015, and references therein). Several Galactic RRL now have high precision geometric distances measured as part of the HST-Fine Guidance Sensor (HST-FGS) parallax program (Benedict et al. 2011), and can therefore be used to calibrate the absolute zero-point of the RRL PL relation.

Distance measurements made in the mid-IR benefit from reduced extinction effects, where  $A_{[3.6]}$  and  $A_{[4.5]}$  are 16 to 20 times lower than  $A_V$  (Cardelli et al. 1989; Indebetouw et al. 2005). Additionally, the precision of distances obtained from the RRL PL relation is increased. At the wavelengths observed by Warm Spitzer (3.6 and 4.5  $\mu$ m) we do not see photospheric effects, but only the effects of temperature driving the pulsation; essentially, the mid-infrared light

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curve is tracing the change in radius of the star through the pulsation cycle. A by-product of this effect is that the intrinsic width of the RRL PL relation is also minimised in the mid-IR. The PL relation for pulsational variables can be thought of as a two-dimensional projection of the threedimensional period-luminosity-colour relation (see figure 3 of Madore & Freedman (1991) for a graphical representation). As the effects of temperature are reduced, the colourwidth of the PL decreases in the mid-IR. As one moves from the optical to the mid-IR, the slope of the PL relation steepens and its dispersion dramatically decreases; the slope should asymptotically approach the predicted slope of the period-radius relation, resulting in a slope between -2.4and -2.8, confirmed empirically by Madore et al. (2013). Through this decrease in dispersion we have found that the intrinsic width of the mid-IR PL for RRL is in fact smaller than for Cepheids – less than 0.05 mag compared to 0.10 mag (Neeley et al. 2015). This translates to an uncertainty on an individual RRL of below 2%, compared to 4% for Cepheids. Thus, for nearby systems RRL are the most precise standard candles.

In this work we focus on the effects of metallicity on the RRL PL relation in the mid-infrared. Several Galactic Globular Clusters are being observed as part of CRRP, but  $\omega$  Cen is unique in that it exhibits a measurable spread in metallicity (Freeman & Rodgers 1975; Villanova et al. 2007, 2014).

There are very few metallic or molecular transition lines in the mid-IR at typical RRL temperatures, so the effects of metallicity on luminosity should be minimised. However,  $\omega$  Cen provides the ideal environment for a direct empirical test. Such an effect is not out of the realm of possibility; for example, the strength of the CO band head at 4.5  $\mu$ m has been found to have a significant effect on Cepheid colours (Marengo et al. 2010), and has such prevented the IRAC 4.5  $\mu$ m Cepheid observations from being used for distance measurements in the CHP (Scowcroft et al. 2011; Monson et al. 2012; Scowcroft et al. 2016). Our concern in this program is systematic precision, and therefore we must test for any potential effects directly.

The paper is set out as follows: Section 2 details the observations and data reduction. Section 3 presents the photometry of the  $\omega$  Cen RRL. Section 4 describes the mid-IR PL relations and Section 5 discusses the application of these to a distance measurement of  $\omega$  Cen. Section 6 examines the effects of metallicity on RRL magnitudes and distance estimates. Section 7 discusses the implications of this, and other systematic effects we consider in this work. In Section 8 we present our conclusions.

# 2 OBSERVATIONS & DATA REDUCTION

This work combines mid-IR observations from the Warm Spitzer mission (Freedman et al. 2012a, PID 90002) with supporting near-IR observations from the FourStar instrument on the Baade-Magellan telescope at Las Campanas Observatory (Persson et al. 2013). Figure 1 shows a  $K_s$  FourStar image with the Spitzer fields outlined, the positions of known RRab plotted as circles, and the positions of known RRc as triangles.

#### 2.1 Warm Spitzer Data

The Warm Spitzer observations for this work were taken as part of the CRRP. Three fields in  $\omega$  Cen were chosen; their positions and the positions of known  $\omega$  Cen RRLs are shown in Figure 1. To obtain optimal RRL light curves we observed each field 12 times over approximately 16 hours, roughly corresponding to the period of the longest period RRL we expected in the field. The observations of all three fields were taken on 2013 May 10 and 2013 May 11. Each field was observed using the Spitzer InfraRed Array Camera (IRAC) (Fazio et al. 2004) with a 30s frame time with a medium scale, and gaussian 5-point dither pattern to mitigate any image artefacts. Images were collected in both the 3.6 and 4.5  $\mu m$  channels. The elongated field shapes come from the design of IRAC; while the [3.6] channel is collecting on-target data, the [4.5] channel collects off target data "for free", and vice versa. We chose to include these off-target fields to maximise the number of RRL in our final sample and to increase the legacy value of our data set to the community.

The science images were created using MOPEX (Makovoz et al. 2006), first running overlap correction on the corrected basic calibrated data frames (cBCDs) then mosaicking them at 0.6 arcsec pixel scale using the drizzle algorithm. Mosaicked location-correction images were created at the same time.

PSF photometry was performed using DAOPHOT and Allframe (Stetson 1987, 1994). The PSF model was created for each field/filter combination using the first epoch data. This was then applied to each other epoch. As the observations were taken temporally close together the effects of telescope rotation between epochs on the mosaicked PSF were minimal, so making a single good PSF model for each field/filter combination was much more efficient than creating one for every epoch.

Master star lists for Allframe were created for each filter/field combination using a median mosaicked image created by MOPEX. We did not use the same single master star list for both filters as only a small proportion (1/3) of the [3.6] and [4.5] fields overlap each other. Instead we performed separate Allframe reductions for each filter, and combined the results after the fact using DAOMATCH and DAOMASTER. Our mid-IR photometry is calibrated to the standard system set by Reach et al. (2005).

#### 2.2 FourStar Data

 $J,\,H$  and  $K_s$  data were taken with the FourStar instrument on the Baade-Magellan telescope at Las Campanas Observatory (Persson et al. 2013) on the nights of 2013 June 25, 2013 June 27, and 2013 June 28. Four epochs were obtained each night in each filter for a total of 12 epochs. A mosaic of  $5\times 3$  slightly overlapping pointings (tiles), each with FourStar's native  $10.9\times 10.9$  arcminute field of view, covered a  $50\times 30$  arcminute field of view centered on  $\omega$  Cen. Each tile consists of a 5 point dither pattern with a 5.8 second exposure time. Stacked mosaics of the entire field were made as well as individual tiles using a customized pipeline for FourStar data. The purpose of the individual tiles is to provide photometry with better time resolution than the large mosaic.

PSF photometry of the tiles was performed using

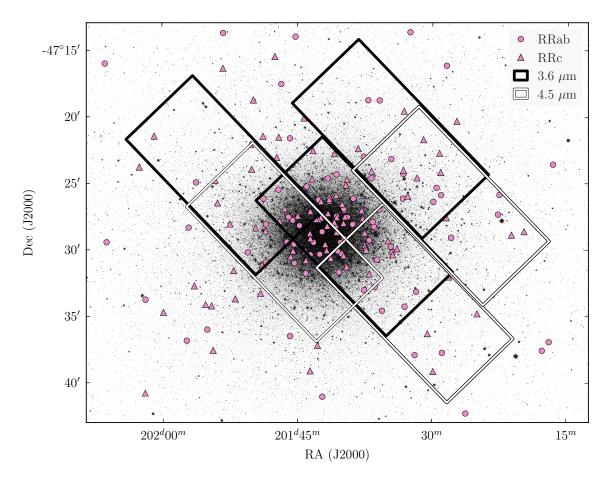


Figure 1. A  $K_s$ -band image of  $\omega$  Cen from the FourStar camera, overlaid with a catalog of RRL from Kaluzny et al. (2004) and footprints of the *Spitzer* IRAC fields. Here the circular points are RRab's and the triangular points are RRc's; we adopt this convention throughout the paper. The three black rectangle outlines are the IRAC field of view for each pointing in the 3.6  $\mu$ m channel, and the white rectangle outlines are the same for 4.5  $\mu$ m.

DAOPHOT and ALLFRAME (Stetson 1987, 1994). A PSF model was created for each epoch/tile/filter combination. A master star list for ALLFRAME was created from the final  $K_s$  mosaic and the multi-wavelength/epoch results were combined using DAOMATCH and DAOMASTER. Our final photometry is calibrated to the 2MASS standard system (Skrutskie et al. 2006).

#### 2.3 Crowding

The primary limiting factor in the photometric precision is crowding. To assess crowding for individual RRLs, we compared the Spitzer images to the FourStar  $K_s$ -band image. The 0.159 arcsec/pixel resolution of the  $K_s$  band image compared to the 0.6 arcsec/pixel resolution of the IRAC images enabled us to more accurately determine which stars were significantly contaminated. 77 RRLs out of the original catalog of 192 (Kaluzny et al. 2004) were rejected due to crowding, another 13 were outside the mosaic field of view, and four more were found to be unusable other reasons.

#### 3 RESULTS

Our final photometry catalog, including magnitudes and uncertainties for  $JHK_s$ , [3.6], and [4.5], is presented in Table A1. Note: this will not be an appendix in the final draft, we're just putting it there for now because it's huge. The average magnitudes presented in Table A1 are flux averages, and the photometric uncertainties of the time series data are the error on the mean. GLOESS fitting Persson et al. (2004) was also attempted on the light curves as done in previous CHP works (e.g. Scowcroft et al. 2011; Monson et al. 2012; Rich et al. 2014; Scowcroft et al. 2016), but the phase coverage is sufficiently extensive that it did not significantly improve results.

Our full, uncrowded RRL sample consists of 96 stars in J and H, 98 in  $K_s$ , 38 in [3.6], and 42 in [4.5]. For the PL fitting, detailed in the next section, we use only the stars for which we have photometry in all five bandpasses, ensuring that the same range of periods and metallicities are sampled for each wavelength. This reduces the possibility of biases being introduced by inconsistent RRL samples to the

**Table 1.** Empirical mid-IR RRL period-luminosity relation coefficients for  $\omega$  Cen (Neeley et al. 2015), for relations of the form  $M=a+b\times(\log(P)+P_0)$  with intrinsic dispersion  $\sigma$ .

| Band  | Mode        | a               | b               | $P_0$            | σ                |
|-------|-------------|-----------------|-----------------|------------------|------------------|
| [3.6] | RRab<br>RRc | -0.558 $-0.192$ | -2.370 $-2.658$ | $0.260 \\ 0.550$ | $0.040 \\ 0.079$ |
| [4.5] | RRab<br>RRc | -0.593 $-0.240$ | -2.355 $-2.979$ | $0.260 \\ 0.550$ | $0.045 \\ 0.057$ |

**Table 2.** Theoretical near-IR RRL period-luminosity relation coefficients for  $\omega$  Cen (Marconi et al. 2015), for relations of the form  $M=a+b\times\log P+c\times$  [Fe/H] with intrinsic dispersion  $\sigma$ .

| Band  | Mode | a      | b      | c     | σ     |
|-------|------|--------|--------|-------|-------|
| J     | RRab | -0.510 | -1.980 | 0.170 | 0.060 |
|       | RRc  | -1.070 | -2.460 | 0.150 | 0.040 |
| H     | RRab | -0.760 | -2.240 | 0.190 | 0.040 |
|       | RRc  | -1.310 | -2.700 | 0.160 | 0.020 |
| $K_s$ | RRab | -0.820 | -2.270 | 0.180 | 0.030 |
|       | RRc  | -1.370 | -2.720 | 0.150 | 0.020 |

distance moduli we fit in Section 5. in Our final RRL sample consists of 24 stars, or 12 in each pulsation mode.

#### 4 PERIOD-LUMINOSITY RELATIONS

We fit PL relations using the theoretical near-infrared PL relation parameters presented in Marconi et al. (2015) for the  $JHK_s$  bands, and the empirical PL relation parameters derived from photometry of RRLs in the globular cluster M4 (NGC 6121) from Neeley et al. (2015) for the IRAC bands. With the use of preexisting PL relation coefficients, the distance modulus becomes the only free parameter in our fit. We fit all distance moduli using an unweighted least-squares method, and fit the distance modulus to each pulsation mode in each wavelength separately.

For the mid-IR we use the PL relations from Neeley et al. (2015), as described in Table 1. These relations take the form

$$M = a + b \times (\log(P) + P_0) \tag{1}$$

where a and b are empirically derived coefficients and  $P_0$  is the absolute value of the logarithm of the mean period of the M4 RRL sample. We calculate the absolute PL zero-points adopting Neeley et al.'s M4 distance modulus of  $\mu=11.399$  mag.

The  $JHK_s$  RRL PL relations are described in Table 2. The relations take the form

$$M = a + b \times \log P + c \times [Fe/H] \tag{2}$$

where a, b, and c are theoretically derived coefficients.

The theoretical PL relations for the near-IR have a metallicity-dependent term; however, we do not have known metallicities for all RRL in our sample. We therefore use the average [Fe/H] of the RRLs for which there are known metallicities. Using spectroscopic metallicities from Sollima et al. (2006), we obtain an average [Fe/H] of -1.67.

#### 5 DISTANCE MODULI

We combine the uncorrected distance moduli from each bandpass to obtain a mean reddening value and reddening-corrected distance modulus. We fit the near-infrared reddening law from Cardelli et al. (1989) to the  $JHK_s$  data and the mid-infrared law from Indebetouw et al. (2005) to [3.6] and [4.5] simultaneously, assuming a ratio of total to selective absorption  $R_V=3.1$ . The resulting fit is shown in Figure 3. We derive a true mean dereddened distance modulus of  $\langle \mu_0 \rangle = 13.781 \pm 0.018$  with  $E(B-V) = 0.066 \pm 0.030$  using the weighted mean RRab + RRc distances. The individual uncorrected distance moduli  $\mu_0$ , and PL residuals are shown in Table 3.

It is apparent from Figure 3 that there are large discrepancies in the distance moduli in [3.6] and [4.5] for the two pulsation modes; these contribute to the relatively low E(B-V) value and high dereddened distance modulus (Lub 2002; Del Principe et al. 2006). If we remove the RRc's and fit the extinction curve only to the RRab's, as shown in Figure 4, we obtain a better fit of all points to the extinction curve than when we use the mean. From these distance moduli we derive a true dereddened distance modulus of  $\langle \mu_0 \rangle = 13.739 \pm 0.024$  with  $E(B-V) = 0.110 \pm 0.042$ , both of which are closer to accepted values than the values derived from the weighted mean distance moduli. All individual corrected distance moduli from this fit are shown in Table 4.

Given the large errors in the [4.5] distances, we also fit the extinction curve to the  $JHK_s$  and [3.6] distances only, excluding [4.5] entirely; this was found to have a negligible effect on the final distance modulus and reddening for both the mean and RRab-only measurements.

#### 6 METALLICITY

Theoretical models suggest that the metallicity dependence of the RRL PL relation should decrease monotonically from the optical to the near-infrared (Bono et al. 2001; Catelan et al. 2004). Observational evidence corroborates this; previous investigations performed on WISE data suggest no obvious metallicity dependence in the mid-IR PL relations (Madore et al. 2013).

 $\omega$  Cen is ideal for examining the RRL periodluminosity-metallicity relation, because **need source for canonical metallicity spread** A metallicity spread this wide is not found in any other Galactic globular cluster. One of the advantages of using globular clusters to calibrate PL coefficients is that all stars in a cluster can be considered to be at the same distance from Earth. The dispersion in the PL relation is a combination of the a) the intrinsic dispersion of the PL relation, b) the photometric uncertainties, and c) dispersion induced by other complicating factors such as the spread in metalicity. Since we have measured the intrinsic dispersion of the RRL PL in [3.6] and [4.5] from the cluster M4 (Neeley et al. 2015) and our photometric uncertainties are well understood, we can isolate the remaining scatter due to astrophysical sources such as metallicity.

We know from mid-IR spectra that a significant CO feature sits within the IRAC [4.5] filter. In the case of Cepheids, Monson et al. (2012) have shown that this has a significant

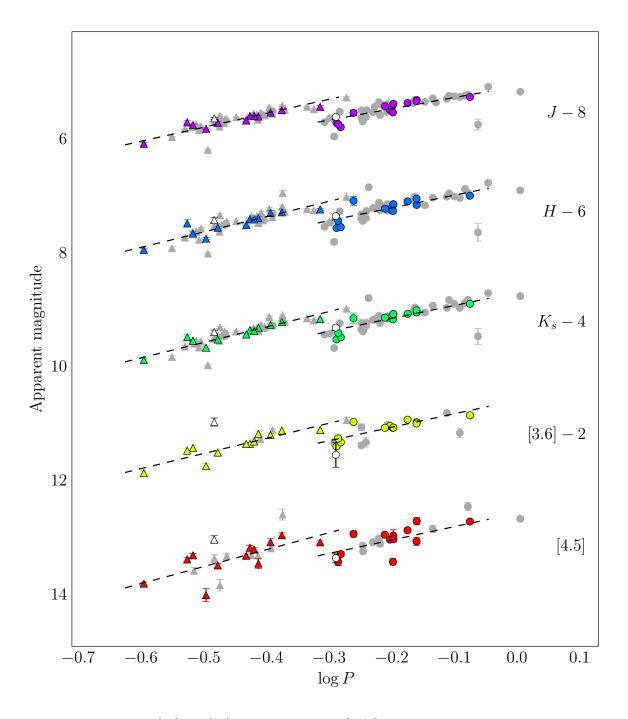


Figure 2. PL relations for  $JHK_s$ , [3.6], and [4.5] photometry assuming [Fe/H]= -1.567. Here circles represent RRab stars, triangles represent RRcs, colored points are the final consistent sample with photometry in all 5 wavebands, grey points are stars that did not appear in all bands, and the unfilled points are stars rejected from the final sample based on  $2\sigma$  clipping of the residuals of [3.6] vs. the residuals of H and H.

effect on the [4.5] magnitudes, and is metallicity dependent. However, this effect decreases with increasing temperatures, turning off completely above 6000 K where all the CO has been destroyed (Monson et al. 2012). As even the coolest RRL have temperatures over 6000 K (Iben 1971), we expect to see no such CO absorption in the [4.5] PL relation. However, we can directly and empirically test this prediction.

# 6.1 Metallicity Contribution to the Overall Dispersion

We can place an upper limit on the contribution of metallicity to the  $\omega$  Cen PL dispersion using the known variances of the observed distribution of  $\omega$  Cen PL residuals  $\sigma^2_{\rm observed}$ , the intrinsic PL width  $\sigma^2_{\rm intrinsic}$ , and the scatter induced by

Table 3. Uncorrected distance moduli  $\mu_0$ , corrected distance moduli  $\mu_0$ , and the PL dispersion  $\sigma$ . The corrected distance moduli  $\mu_0$  are equal to  $\mu - A_{\lambda}A_V$ , where  $A_V$  is derived from fitting the reddening laws to the mean distance moduli.

| Band  | $\mu$ , RRab       | $\mu$ , RRc        | $\mu$ , mean       | $\mu_0$ , RRab     | $\mu_0$ , RRc      | $\mu_0$ , mean     | $\sigma_{\rm PL}$ , RRab | $\sigma_{\rm PL}$ , RRc |
|-------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------------|-------------------------|
| J     | $13.839 \pm 0.022$ | $13.850 \pm 0.019$ | $13.844 \pm 0.015$ | $13.779 \pm 0.035$ | $13.790 \pm 0.033$ | $13.784 \pm 0.031$ | 0.077                    | 0.063                   |
| H     | $13.796 \pm 0.030$ | $13.827 \pm 0.026$ | $13.811 \pm 0.020$ | $13.758 \pm 0.035$ | $13.790 \pm 0.031$ | $13.773 \pm 0.026$ | 0.108                    | 0.085                   |
| $K_s$ | $13.790 \pm 0.024$ | $13.814 \pm 0.021$ | $13.802 \pm 0.016$ | $13.767 \pm 0.026$ | $13.791 \pm 0.023$ | $13.778 \pm 0.019$ | 0.084                    | 0.070                   |
| [3.6] | $13.751 \pm 0.024$ | $13.838 \pm 0.031$ | $13.793 \pm 0.019$ | $13.738 \pm 0.025$ | $13.825 \pm 0.031$ | $13.779 \pm 0.020$ | 0.088                    | 0.102                   |
| [4.5] | $13.768 \pm 0.048$ | $13.890 \pm 0.061$ | $13.826 \pm 0.038$ | $13.756 \pm 0.048$ | $13.878 \pm 0.061$ | $13.815 \pm 0.039$ | 0.173                    | 0.202                   |

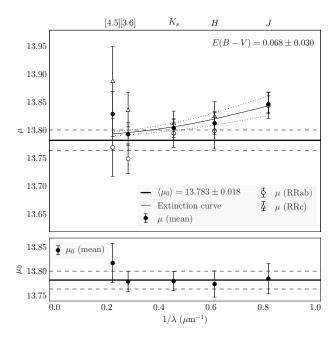
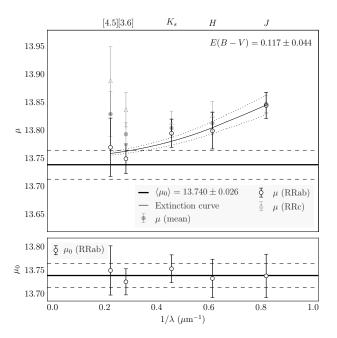


Figure 3. Top: Uncorrected distance moduli for the final sample of  $JHK_s$ , [3.6], and [4.5] photometry. Filled circles are the mean distance moduli using both RRab and RRc stars, open circles are the distance moduli using only RRab stars, and open triangles are distance moduli using only RRc stars. Here the NIR and MIR reddening laws are fit to the mean distance moduli. The solid and dashed horizontal lines are the mean corrected distance modulus and its  $1\sigma$  errors respectively. Bottom: reddening-corrected distance moduli and mean corrected distance modulus. Errors on the corrected distance moduli are the quadrature sum of the uncorrected distance moduli errors and the reddening error at the requisite wavelength.

Table 4. Corrected distance moduli  $\mu_0$  using the  $A_V$  value derived from fitting the reddening laws to only the RRab distance moduli.

| Band  | $\mu_0$ , RRab     | $\mu_0$ , RRc      | $\mu_0$ , mean     |
|-------|--------------------|--------------------|--------------------|
| J     | $13.739 \pm 0.044$ | $13.750 \pm 0.043$ | $13.744 \pm 0.041$ |
| H     | $13.733 \pm 0.039$ | $13.764 \pm 0.035$ | $13.748 \pm 0.031$ |
| $K_s$ | $13.751 \pm 0.028$ | $13.775 \pm 0.026$ | $13.763 \pm 0.022$ |
| [3.6] | $13.729 \pm 0.026$ | $13.815 \pm 0.032$ | $13.770 \pm 0.021$ |
| [4.5] | $13.749 \pm 0.049$ | $13.871 \pm 0.061$ | $13.807 \pm 0.039$ |



**Figure 4.** Same as Figure 3, with the reddening laws fit to the RRab distances only instead of the mean.

**Table 5.** The standard deviation of the observed spread of the PL residuals  $\sigma_{\rm observed}$  ( $\sigma_{\rm PL}$  in Table 3) and its components:  $\sigma_{\rm intrinsic}$  ( $\sigma$  in Table 1),  $\sigma_{\rm phot}$ , and  $\sigma_{\rm [Fe/H]}$  for all IRAC PL relations.

| Band                             | Mode                | $\sigma_{ m observed}$  | $\sigma_{ m intrinsic}$ | $\sigma_{ m phot}$    | $\sigma_{ m [Fe/H]}$  |
|----------------------------------|---------------------|-------------------------|-------------------------|-----------------------|-----------------------|
| [3.6]<br>[3.6]<br>[4.5]<br>[4.5] | RRab<br>RRc<br>RRab | 0.088<br>0.102<br>0.173 | 0.040 $0.079$ $0.045$   | 0.048 $0.041$ $0.046$ | 0.062 $0.050$ $0.160$ |

photometric error  $\sigma_{\rm phot}^2$ . The scatter contributed by metallicity can therefore be constrained as follows:

$$\sigma_{\text{[Fe/H]}} \le \sqrt{\sigma_{\text{observed}}^2 - \sigma_{\text{intrinsic}}^2 - \sigma_{\text{phot}}^2}$$
 (3)

The calculated  $\sigma_{\rm [Fe/H]}$  values for all IRAC PL relations are shown in Table 5, along with with  $\sigma_{\rm observed}$  and the other scatter components.

We use these measurements of  $\sigma_{\rm [Fe/H]}$  to assess the  $\gamma$  parameter for  $\omega$  Cen, where

$$\gamma = \frac{\Delta \text{mag}}{\Delta [\text{Fe/H}]},\tag{4}$$

**Table 6.** Metallicity standard deviations and  $\gamma$  values.

| Band                    | Mode                       | $\sigma_{ m [Fe/H]spect}$        | $\sigma_{ m [Fe/H]phot}$         | $\gamma_{ m spect}$              | $\gamma_{ m phot}$               |
|-------------------------|----------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| [3.6]<br>[3.6]<br>[4.5] | RRab<br>RRc<br>RRab<br>RRc | 0.250<br>0.132<br>0.250<br>0.132 | 0.254<br>0.243<br>0.254<br>0.243 | 0.247<br>0.383<br>0.642<br>1.421 | 0.243<br>0.208<br>0.632<br>0.772 |

similar to  $\gamma$  used to quantify the effect of metallicity on the zero-point of the Cepheid PL relation (Kennicutt et al. 1998; Scowcroft et al. 2009). Here we calculate  $\gamma$  in units of mag dex<sup>-1</sup> by dividing the standard deviation of the metallicity component of the PL scatter by the standard deviation of the metallicity distribution for both (Sollima et al. 2006) and photometric (Rey et al. 2000) metallicities; the results for each PL are shown in Table 6. We use all available metallicity values for each pulsation mode when taking the standard deviation, as we do not have metallicity values for all the stars in our samples, although the metallicity values we do have trace the overall metallicity distributions fairly well (see Figure 5).

For [3.6] we find  $\gamma \le 0.247$  mag dex<sup>-1</sup> for RRab's, and  $\gamma \le 0.383$  mag dex<sup>-1</sup> for RRc's.

#### 6.2 PL Residuals and Individual Metallicities

 $\omega$  Cen provides a second approach to testing for a metallicity effect on the RRL PL relation. The cluster is extremely well studied and many of its RRL have spectroscopic or photometric metallicities in the literature (e.g. Sollima et al. 2006; Rey et al. 2000).

Theory predicts a linear metallicity term in the PLZ relation for the near-IR,  $c \times$  [Fe/H]. We thus fit a relation of the form

$$\Delta \text{mag} = \gamma \times [\text{Fe/H}] + d \tag{5}$$

to the [3.6] and [4.5] PL residuals and metallicity values for stars with known individual metallicity values, as shown in Figure 6. The scatter in the [3.6] and [4.5] PL relations is higher for  $\omega$  Cen than it is for M4 (Neeley et al. 2015; Braga et al. 2015); however, when we examine [Fe/H] vs. the residuals of each PL relation,  $\gamma$  is within  $1\sigma$  of zero for all fits, indicating that there is no significant metallicity dependence in the PL residuals, consistent with predictions. More accurate metallicities will be required to constrain any potential effect further.

#### 7 DISCUSSION

While we find high upper bounds on the contribution of metallicity to the PL scatter in the IRAC passbands, particularly [4.5], it is unlikely that this scatter is due to metallicity alone. Gratton et al. (1986) and Lee (1991) demonstrate that RRL luminosity is dependent on the horizontal branch morphology as well as metallicity; it is also well known that  $\omega$  Cen contains RRLs in multiple evolutionary states (Sollima et al. 2008; Navarrete et al. 2015).

Results from the GAIA mission (Lindegren & Perryman 1996) are expected to improve the overall characterization

of the RRL PL relation dramatically. Trigonometric parallaxes and spectrophotometric metallicities of Galactic RRLs from *GAIA* will increase the number of calibrators for the absolute RRL PL relations by an order of magnitude (Liu et al. 2012, overview paper). something about omega cen parallaxes and metallicities too

We also anticipate that the NIRCam instrument on JWST (Burriesci 2005; Gardner et al. 2006) will provide substantial improvements over IRAC for investigations of this nature. The NIRCam filters F356W and F444W will provide data in passbands comparable to IRAC's [3.6] and [4.5] at an order of magnitude higher resolution (0.065 arcsec/pixel), which will significantly decrease photometric error due to crowding and therefore allow us to obtain data for all known RRLs in  $\omega$  Cen.

#### 8 CONCLUSIONS

We derive a true mean dereddened distance modulus for  $\omega$  Cen of  $\langle \mu_0 \rangle = 13.739 \pm 0.024$  with reddening  $E(B-V) = 0.110 \pm 0.042$  using distance moduli derived from RRab PL relations in  $JHK_s$ , [3.6], and [4.5]. Using the mean of RRab and RRc distance moduli in the same passbands, we derive a true mean dereddened distance modulus of  $\langle \mu_0 \rangle = 13.781 \pm 0.018$  with reddening  $E(B-V) = 0.066 \pm 0.030$ . The RRab results are closer to literature values.

We also constrain the contribution of metallicity, quantified as  $\gamma = \Delta {\rm mag}/\Delta {\rm [Fe/H]},$  to the scatter of the  $\omega$  Cen PL relations by subtracting the variances of the intrinsic scatter and photometric error from the observed PL variance and taking the square root of the result, and dividing that by the standard deviation of the distribution of metallicity values. We also measure  $\gamma$  directly using the slope of the linear fit to the PL residuals vs. [Fe/H].

### ACKNOWLEDGEMENTS

We thank Eric Persson for his many contributions to this project.

This work is based on observations made with the Spitzer Space Telescope, which is operated by the Jet Propulsion Laboratory, California Institute of Technology under a contract with NASA. Support for this work was provided by NASA through an award issued by JPL/Caltech.

This work was also supported in part by the Claremont-Carnegie Astrophysics Research Program.

This publication makes use of data products from the Two Micron All Sky Survey, which is a joint project of the University of Massachusetts and the Infrared Processing and Analysis Center/California Institute of Technology, funded by the National Aeronautics and Space Administration and the National Science Foundation.

This research has made use of the NASA/IPAC Extragalactic Database (NED), which is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

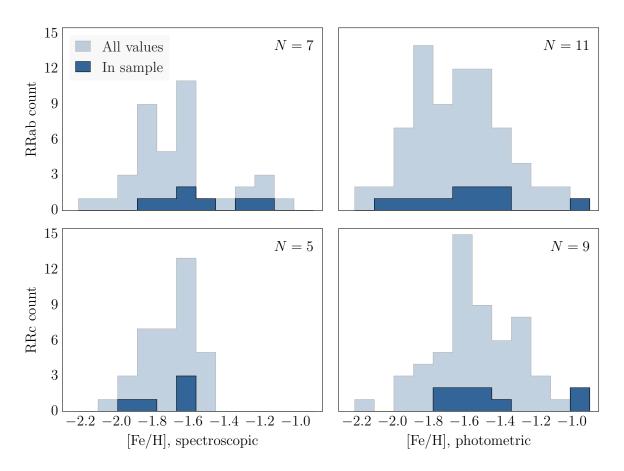


Figure 5. Histograms of spectroscopic (Sollima et al. 2006, right column) and photometric (Rey et al. 2000, left column) [Fe/H] values for RRab (top row) and RRc (bottom row). The light blue histograms represent all known metallicity values for each type, and the dark blue are the metallicity values in our final RRL sample. The number N at the top right corner of each subplot is the number of stars in our final sample that have known metallicity values for the given type and metallicity catalog.

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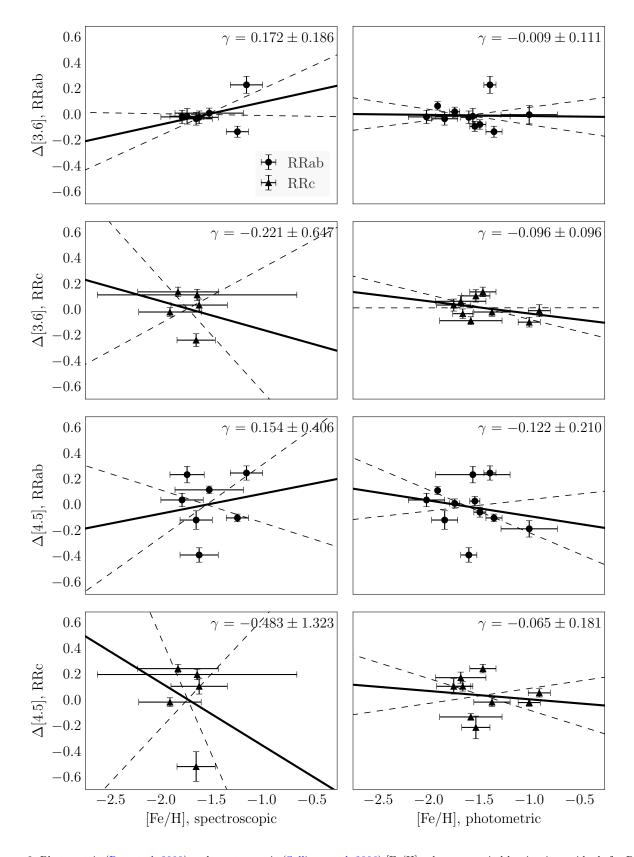


Figure 6. Photometric (Rey et al. 2000) and spectroscopic (Sollima et al. 2006) [Fe/H] values vs. period-luminosity residuals for RRab and RRc in [3.6] and [4.5]. Solid lines are the line of best fit with slope  $\gamma$ , and dashed lines are the  $1\sigma$  confidence intervals. The  $\gamma$  parameter from equation 4 in the top right corner of each subplot. All  $\gamma$  values are consistent with zero.

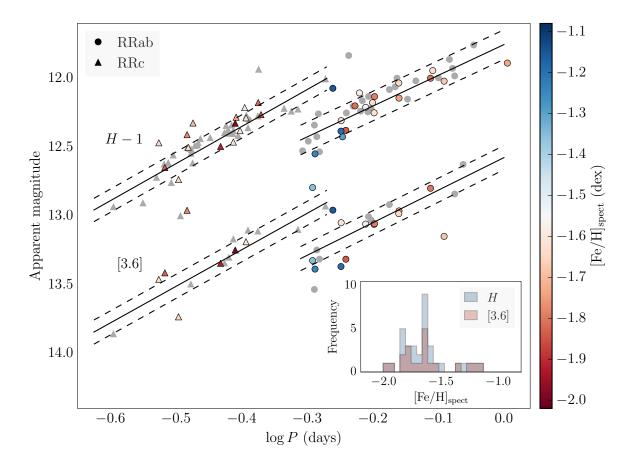


Figure 7. The *H*-band PL relation with fundamentalized RRc periods, with color indicating spectroscopic (Sollima et al. 2006, top) and photometric (Rey et al. 2000, bottom) metallicity values. Gray points have no known metallicity values.

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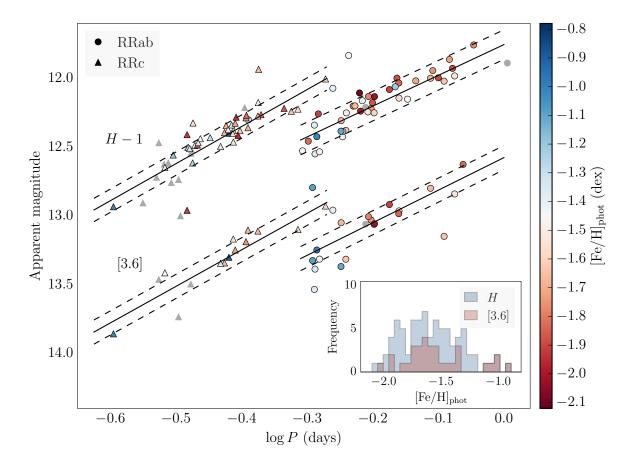


Figure 8. The H-band PL relation with fundamentalized RRc periods, with color indicating spectroscopic (Sollima et al. 2006, top) and photometric (Rey et al. 2000, bottom) metallicity values. Gray points have no known metallicity values.

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# APPENDIX A: RRL PHOTOMETRY

Table A1: Parameters for 99 RRLs in  $\omega$  Cen. First five columns are star ID, right ascension and declination, pulsation mode, and period in days from Kaluzny et al. (2004). Columns 6 through 11 are  $JHK_s$  apparent magnitudes and errors from FourStar data. Columns 12 through 17 are 3.6  $\mu$ m and 4.5  $\mu$ m apparent magnitudes, errors, and PL residuals ( $\Delta$ [3.6] and  $\Delta$ [4.5]) from IRAC data. Columns 18-21 are photometric ([Fe/H], p) and spectroscopic ([Fe/H], s) metallicities and errors from Rey et al. (2000) and Sollima et al. (2006) respectively.

| Math      |    |             |             |      |          | тевре  | corvery.   |        |            |        |                |        |                  |               |        |                  |               |           |                           |           |                           |
|--|----|-------------|-------------|------|----------|--------|------------|--------|------------|--------|----------------|--------|------------------|---------------|--------|------------------|---------------|-----------|---------------------------|-----------|---------------------------|
| 1   13/261/28   13/261/28   13/261/28   13/26   13/2   | ID | RA (J2000)  | Dec (J2000) | Mode | P (days) | J      | $\sigma_J$ | H      | $\sigma_H$ | $K_s$  | $\sigma_{K_S}$ | [3.6]  | $\sigma_{[3.6]}$ | $\Delta[3.6]$ | [4.5]  | $\sigma_{[4.5]}$ | $\Delta[4.5]$ | [Fe/H], p | $\sigma_{\rm [Fe/H]}$ , p | [Fe/H], s | $\sigma_{\rm [Fe/H]}$ , s |
| 1.   1.   1.   1.   1.   1.   1.   1.  | 3  | 13:25:56.15 | -47:25:53.8 | RRab | 0.841    | 13.247 | 0.017      | 12.982 | 0.018      | 12.882 | 0.017          | 12.841 | 0.039            | -0.087        | 12.708 | 0.036            | 0.035         | -1.540    | 0.050                     | _         |                           |
| 1327-069   | 4  | 13:26:12.93 | -47:24:18.8 | RRab | 0.627    | 13.475 | 0.016      | 13.219 | 0.021      | 13.133 | 0.020          | 13.030 | 0.036            | 0.026         | 13.026 | 0.035            | 0.016         | -1.740    | 0.050                     | _         | _                         |
| Section   Sect   | 5  | 13:26:18.33 | -47:23:12.4 | RRab | 0.515    | 13.700 | 0.017      | 13.549 | 0.020      | 13.507 | 0.027          | 13.387 | 0.043            | -0.129        | 13.340 | 0.030            | -0.096        | -1.350    | 0.080                     | -1.240    | 0.110                     |
| 19   13255958   47726406   1782606   4772606   18726   | 7  | 13:27:00.90 | -47:14:00.5 | RRab | 0.713    | 13.333 | 0.009      | 13.151 | 0.031      | 13.036 | 0.018          | _      | _                | _             | _      | _                | _             | -1.460    | 0.080                     | _         | _                         |
| 13296999   | 8  | 13:27:48.45 | -47:28:20.3 | RRab | 0.521    | 13.505 | 0.015      | 13.258 | 0.017      | 13.223 | 0.014          | _      | _                | _             | _      | _                | _             | -1.910    | 0.280                     | _         | _                         |
| 1   13269.59   47239.16   RR   0.56   1.58   0.01   1.397   0.02   1.396   0.02   1.316   0.02   1.316   0.02   1.316   0.02   1.316   0.02   1.316   0.02   1.316   0.02   1.316   0.02   1.316   0.02   1.316   0.02   1.316   0.02     | 9  | 13:25:59.58 | -47:26:24.0 | RRab | 0.523    | 13.776 | 0.017      | 13.534 | 0.021      | 13.470 | 0.016          | 13.315 | 0.036            | -0.072        | 13.279 | 0.039            | -0.051        | -1.490    | 0.060                     | _         | _                         |
| 13   13   13   13   13   13   13   13  | 10 | 13:26:06.99 | -47:24:36.6 | RRc  | 0.375    | 13.579 | 0.014      | 13.395 | 0.023      | 13.345 | 0.019          | 13.342 | 0.037            | -0.026        | 13.168 | 0.037            | 0.112         | -1.660    | 0.100                     | _         | _                         |
| 13/25/88, 18   47/25/216   8   18   0.669   13.333   0.19   13.681   0.029   13.088   0.19   1.2988   0.039   0.19   0.1890   0.045   0.119   0.130   0.165    | 11 | 13:26:30.59 | -47:23:01.6 | RRab | 0.565    | 13.481 | 0.014      | 13.307 | 0.028      | 13.219 | 0.025          | 13.050 | 0.058            | _             | _      | _                | _             | -1.670    | 0.130                     | -1.610    | 0.220                     |
| 14   132959,714   47399,99   47399,99   47399,99   47399,99   47399,99   47399,99   47399,99   47393,99   47   | 12 | 13:26:27.21 | -47:24:06.2 | RRc  | 0.387    | 13.590 | 0.018      | 13.379 | 0.028      | 13.305 | 0.025          | 13.168 | 0.048            | 0.112         | 13.448 | 0.088            | -0.208        | -1.530    | 0.140                     | _         | _                         |
| 13.26.27.11   47.24.38.0   81.8   0.811   13.24   0.18   13.24   0.18   13.24   0.18   13.24   0.18   13.27   0.18   0.18   13.27   0.18   0.18   13.27   0.18   0.18   13.27   0.18   13.27   0.18   0.18   13.27   0.18   13.27   0.18   0.18   13.27   0.18   13.27   0.18   0.18   13.27   0.18   0.18   13.27   0.18   13.27   0.18   0.18   13.28   0.18   13.28   0.1   | 13 | 13:25:58.18 | -47:25:21.6 | RRab | 0.669    | 13.353 | 0.019      | 13.081 | 0.022      | 13.058 | 0.017          | 12.918 | 0.032            | 0.072         | 12.860 | 0.031            | 0.117         | -1.910    | 0.000                     | _         | _                         |
| 18.273.769   | 14 | 13:25:59.74 | -47:39:09.6 | RRc  | 0.377    | 13.588 | 0.011      | 13.343 | 0.020      | 13.365 | 0.016          | _      | _                | _             | 13.299 | 0.045            | _             | -1.710    | 0.130                     | _         | _                         |
| 18.274-5.11   -47-245.65   Rab   0.622   13.371   0.01   13.131   0.024   13.07   0.016   13.006   0.034   0.016   12.940   0.029   0.122  | 15 | 13:26:27.11 | -47:24:38.0 | RRab | 0.811    | 13.245 | 0.018      | 13.020 | 0.031      | 12.954 | 0.025          | 13.149 | 0.084            | _             | _      | _                | _             | -1.640    | 0.390                     | -1.680    | 0.180                     |
| 132714.00   47.28.06.3   Rhab   0.616   13.410   0.015   13.210   0.036   13.125   0.025   13.09   0.037   0.047   0.033   0.047   0.033   13.240   0.032   0.032   0.032   0.032   0.032   0.032   0.032   0.032   0.032   0.032   0.032   0.032   0.032   0.033   0.032   0.033   0.032   0.033   0.032   0.033   0.032   0.033   0.032   0.033   0.032   0.033   0.032   0.033      | 16 | 13:27:37.69 | -47:37:34.8 | RRc  | 0.330    | 13.680 | 0.015      | 13.502 | 0.022      | 13.437 | 0.018          | _      | _                | _             | _      | _                | _             | -1.290    | 0.080                     | -1.650    | 0.460                     |
| 132611.17  | 18 | 13:27:45.11 | -47:24:56.6 | RRab | 0.622    | 13.371 | 0.010      |        | 0.024      |        | 0.016          | 13.006 | 0.043            | _             | _      | _                | _             | -1.780    | 0.280                     | _         | _                         |
| 132741.04  | 20 | 13:27:14.05 | -47:28:06.3 | RRab | 0.616    | 13.410 | 0.015      |        | 0.036      |        | 0.025          | 13.060 | 0.039            | 0.016         | 12.940 | 0.029            | 0.122         | _         | _                         | -1.520    | 0.340                     |
| 13.264.6.50   4.724.9.5   5.   |    |             |             |      |          |        |            |        |            |        |                | 13.301 | 0.047            | -0.003        | 13.200 | 0.032            | 0.061         | -0.900    |                           |           | _                         |
| 1827/98.32   | 22 | 13:27:41.04 | -47:34:07.6 | RRc  | 0.396    | 13.572 | 0.012      | 13.380 | 0.016      | 13.288 | 0.017          | _      | _                | _             | _      | _                | _             | -1.630    | 0.170                     | -1.600    | 0.990                     |
| 30   13.261.5.0 4   -47.29.56.0   RR   0.40   13.521   0.02   13.287   0.04   13.251   0.030   0.188   0.047   0.041   13.071   0.060   0.12   1.750   0.160   0.280   0.280   0.280   0.280   0.285   |    |             |             |      |          |        |            |        |            |        |                | 13.325 | 0.064            | _             | _      | _                | _             |           |                           | -1.350    | 0.580                     |
| 132703.32  |    | 13:27:38.32 |             |      | 0.462    |        |            |        |            |        |                | _      | _                | _             | _      |                  |               | -1.860    | 0.030                     |           | _                         |
| 33   13:25:51.60   -47:29:05.8   RRab   0.602   13:38   0.015   13:106   0.022   13:09   0.019       13:006   0.035     -2:090   0.230   -1.580   0.420  |    |             |             |      |          |        |            |        |            |        |                | 13.188 | 0.047            | 0.041         | 13.071 |                  | 0.112         |           |                           |           | 0.280                     |
| 1326-07.21   -47.331-04   RR   0.734   13.273   0.14   13.018   0.014   12.916   0.013   |    |             |             |      |          |        |            |        |            |        |                | _      | _                | _             | _      |                  | _             |           |                           | _         | _                         |
| 35   13:26:53.21   47:22:34.7   RRc   0.387   13:58   0.012   13:48   0.012   13:49   0.013   0.014   0.013   0.014    |    |             |             |      |          |        |            |        |            |        |                | _      | _                | _             |        |                  |               |           |                           |           | 0.420                     |
| 36         13:27:10.11         47:15:29.8         RRc         0.380         13.534         0.07         13:372         0.019         13:370         0.014         —         —         —         —         —         1.1990         0.230         —         1.550         0.00         0.00         0.017         1.316         0.056         —         1.600         0.080         —         —         —         —         —         —         1.000         0.025         —         1.020         0.019         0.025         —         —         —         —         —         —         —         —         —         1.000         0.025         —         —         —         —         —         1.30  |    |             |             |      |          |        |            |        |            |        |                | _      | _                | _             | 12.838 | 0.065            | _             |           |                           |           | _                         |
| Section   Sect   |    |             |             |      |          |        |            |        |            |        |                | _      | _                | _             | _      | _                |               |           |                           |           | 0.360                     |
| 39   13.27.59.77   |    |             |             |      |          |        |            |        |            |        |                | _      | _                | _             | _      | _                |               |           |                           |           | _                         |
| 40 13-26-22.45   |    |             |             |      |          |        |            |        |            |        |                |        |                  |               |        |                  |               |           |                           |           | 0.400                     |
| 44 13:26:22.39   |    |             |             |      |          |        |            |        |            |        |                |        |                  |               |        |                  |               |           |                           |           |                           |
| 45   |    |             |             |      |          |        |            |        |            |        |                |        |                  |               |        |                  |               |           |                           |           |                           |
| 46 13:25:30.23   |    |             |             |      |          |        |            |        |            |        |                | _      | _                | _             |        |                  |               |           |                           | -1.290    | 0.350                     |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$   |    |             |             |      |          |        |            |        |            |        |                | _      | _                | _             |        |                  |               |           |                           | _         | _                         |
| 49 13:26:07.78   |    |             |             |      |          |        |            |        |            |        |                |        |                  |               |        |                  |               |           |                           | _         | _                         |
| 50         13:25:53.94         -47:27:35.8         RRc         0.386         13.647         0.014         13.402         0.015         13.362         0.014         —         —         —         13.305         0.056         —         -1.590         0.190         —         —           51         13:26:23.54         -47:18:47.7         RRab         0.573         13.281         0.016         12.994         0.017         12.954         0.015         12.799         0.030         —         —         —         —         -1.660         0.120         -1.800         0.230           56         13:25:55.55         -47:18:47.7         RRab         0.568         13.643         0.009         13.385         0.017         —         —         —         —         -1.660         0.150         —         —         —         —         —         —         -1.600         0.150         —  |    |             |             |      |          |        |            |        |            |        |                | 13.099 | 0.030            | -0.080        |        |                  |               |           |                           | _         | _                         |
| 51         13:26:42.66         -47:24:21.4         RRab         0.574         13.597         0.014         13.378         0.033         13.270         0.029         13.315         0.083         —         —         —         —         -1.640         0.210         -1.840         0.230           54         13:26:23.54         -47:18:47.7         RRab         0.773         13.281         0.016         12.998         0.017         12.954         0.015         12.799         0.030         —         —         —         —         -1.660         0.120         -1.800         0.230           56         13:25:55.53         -47:36:50.5         RRab         0.794         13.234         0.015         12.995         0.018         12.882         0.014         —         —         —         —         —         -1.690         0.140         —         —           58         13:26:13.05         -47:24:03.0         RRc         0.370         13.660         0.017         13.495         0.018         13.421         0.021         13.345         0.033         -0.031         13.309         0.034         -0.011         -1.370         0.180         -1.910         0.310           58         13:26:18.43   |    |             |             |      |          |        |            |        |            |        |                | _      | _                |               |        |                  |               |           |                           | _         | _                         |
| 54         13:26:23.54         -47:18:47.7         RRab         0.773         13:281         0.016         12:998         0.017         12:954         0.015         12:799         0.030         —         —         —         —         -1.660         0.120         -1.800         0.230           56         13:25:55.55         3         -47:37:44.1         RRab         0.568         13:643         0.009         13:386         0.022         13:353         0.017         —         —         —         —         -1.260         0.150         —         —           57         13:27:49.38         -47:24:03.0         RRb         0.794         13:269         0.018         12.882         0.014         —         —         —         —         -1.800         0.140         —         —           58         13:26:18.43         -47:29:46.7         RRb         0.519         13:72         0.023         13:421         0.021         13:345         0.033         -0.013         13:438         0.021         13:345         0.071         0.004         13:418         0.064         -0.181         -1.000         0.280         —         —           63         13:26:18.43         -47:36:54.1         RRab <td></td> <td>0.020</td>  |    |             |             |      |          |        |            |        |            |        |                |        |                  |               |        |                  |               |           |                           |           | 0.020                     |
| 56         13:25:55.53         -47:37:44.1         RRab         0.568         13.643         0.009         13.386         0.022         13.353         0.017         —         —         —         13.232         0.035         —         -1.260         0.150         —         —           57         13:27:49.38         -47:36:50.5         RRab         0.794         13.234         0.015         12.995         0.018         12.882         0.014         —         —         —         —         —         -1.890         0.140         —         —           58         13:26:13.05         -47:24:03.0         RR         0.370         13.660         0.017         13.495         0.018         13.421         0.021         13.345         0.033         -0.013         13.309         0.034         -0.011         -1.370         0.180         -1.910         0.310           59         13:26:18.43         -47:29:46.7         RRab         0.519         13.727         0.023         13.424         0.043         13.431         0.043         13.418         0.063         13.418         0.064         -0.181         -1.000         0.090         —         —           63         13:26:18.43         -47:26:25.2  |    |             |             |      |          |        |            |        |            |        |                |        |                  |               |        |                  |               |           |                           |           |                           |
| 57         13:27:49.38         -47:36:50.5         RRab         0.794         13:295         0.018         12.882         0.014         —         —         —         —         —         -1.890         0.140         —         —         —         —         —         —         -1.890         0.140         —<   |    |             |             |      |          |        |            |        |            |        |                |        |                  |               |        |                  |               |           |                           |           | 0.230                     |
| 58         13:26:13.05         -47:24:03.0         RRc         0.370         13:660         0.017         13:495         0.018         13:421         0.021         13:345         0.033         -0.013         13:309         0.034         -0.011         -1.370         0.180         -1.910         0.310           59         13:26:18.43         -47:29:46.7         RRab         0.519         13:727         0.023         13:424         0.031         13:28         0.071         0.004         13:418         0.064         -0.181         -1.000         0.280         —         —           63         13:25:07.96         -47:36:54.1         RRab         0.826         13:23         0.017         12.862         0.017         12.869         0.012         —         —         —         —         —         —         -1.730         0.090         —         —           64         13:26:02.22         -47:36:91.2         RRc         0.344         13:638         0.013         13:438         0.022         13:407         0.022         —         —         —         —         —         -1.660         0.230         —         —         —         —         -1.660         0.340         —         —         <  |    |             |             |      |          |        |            |        |            |        |                |        |                  |               | 13.232 |                  |               |           |                           |           |                           |
| 59 13:26:18.43 -47:29:46.7 RRab 0.519 13.727 0.023 13.424 0.043 13.391 0.033 13.248 0.071 0.004 13.418 0.064 -0.181 -1.000 0.280 — — — 63 13:25:07.96 -47:36:54.1 RRab 0.826 13.223 0.017 12.862 0.017 12.869 0.012 — — — — — — — — — — — — — — 1.730 0.090 — — — — 64 13:26:02.22 -47:36:19.2 RRc 0.344 13.638 0.013 13.438 0.022 13.407 0.022 — — — — 13.314 0.044 — -1.460 0.230 — — — — 66 13:26:38.08 -47:22:52.2 RRc 0.407 13.542 0.011 13.359 0.022 13.264 0.020 13.103 0.035 — — — — -1.680 0.340 — — — — — — — — 67 13:26:28.62 -47:18:46.9 RRab 0.564 13.610 0.014 13.384 0.016 13.326 0.015 13.368 0.047 — — — — — -1.100 0.000 -1.190 0.230 — — — — — — — — — — — — — — — — — — —  |    |             |             |      |          |        |            |        |            |        |                |        |                  | 0.012         | 12 200 |                  |               |           |                           |           | 0.210                     |
| 63 13:25:07.96   |    |             |             |      |          |        |            |        |            |        |                |        |                  |               |        |                  |               |           |                           |           | 0.510                     |
| 64 13:26:02.22 -47:36:19.2 RRc 0.344 13.638 0.013 13.438 0.022 13.407 0.022 13.314 0.0441.460 0.230 66 13:26:33.08 -47:22:25.2 RRc 0.407 13.542 0.011 13.359 0.022 13.264 0.020 13.103 0.035 1.680 0.340 67 13:26:28.62 -47:18:46.9 RRab 0.564 13.610 0.014 13.384 0.016 13.326 0.015 13.368 0.047 1.100 0.000 -1.190 0.230 13:25:11.02 -47:37:33.5 RRab 0.635 1.500 0.010 1.500 0.010 1.500 0.010 1.500 0.010 0.000 0.0   |    |             |             |      |          |        |            |        |            |        |                |        |                  | 0.004         | 10.410 |                  |               |           |                           |           | _                         |
| 66 13:26:33.08 -47:22:25.2 RRc 0.407 13.542 0.011 13.359 0.022 13.264 0.020 13.103 0.035 — — — — -1.680 0.340 — — — 67 13:26:28.62 -47:18:46.9 RRab 0.564 13.610 0.014 13.384 0.016 13.326 0.015 13.368 0.047 — — — — -1.100 0.000 -1.190 0.230 0.0000 0.000 0.0000 0.000 0.00 |    |             |             |      |          |        |            |        |            |        |                |        |                  | _             | 13 314 |                  |               |           |                           |           | _                         |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$   |    |             |             |      |          |        |            |        |            |        |                |        |                  |               |        |                  |               |           |                           |           | _                         |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$   |    |             |             |      |          |        |            |        |            |        |                |        |                  |               | _      | _                |               |           |                           |           | 0.230                     |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$   |    |             |             |      |          |        |            |        |            |        |                |        |                  | _             | _      | _                | _             |           |                           |           |                           |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$   |    |             |             |      |          |        |            |        |            |        |                |        |                  | _             | _      | _                | _             |           |                           |           | _                         |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$   |    |             |             |      |          |        |            | 13.282 |            |        |                |        | _                | _             |        |                  |               |           |                           |           | 0.300                     |
| 73  13:25:53.75  -47:16:10.8  RRab  0.575  13.480  0.018  13.251  0.017  13.215  0.016  -  -  -  -  -  -  -  -  -  |    |             |             |      |          |        |            |        |            |        |                |        | _                | _             | _      | _                | _             |           |                           |           | -                         |
|  |    |             |             |      |          |        |            |        |            |        |                | _      | _                | _             | _      | _                | _             |           |                           | _         | _                         |
|  | 74 | 13:27:07.22 | -47:17:33.9 | RRab | 0.503    | 13.622 | 0.008      | 13.457 | 0.016      | 13.405 | 0.015          | _      | _                | _             | _      | _                | _             | -1.830    | 0.360                     |           | _                         |
| 75 13:27:19.70 -47:18:46.5 RRc 0.422 13.410 0.011 13.175 0.028 13.137 0.025 — — — — — — — — — — — — — — — — — — —  |    |             |             |      |          |        |            |        |            |        |                | _      | _                | _             | _      | _                | _             |           |                           | -1.820    | 0.990                     |
| 76 13:26:57.23 -47:20:07.7 RRc 0.338 13.634 0.012 13.488 0.017 13.449 0.020 — — — — — — — — — — — — — — — — — —  |    |             |             |      |          |        |            |        |            |        |                | _      | _                | _             | _      | _                | _             |           |                           |           | _                         |
| $77  13:27:20.89  -47:22:05.6  \text{RRc} \qquad 0.426  13.474  0.013  13.264  0.028  13.199  0.021  -  -  -  -  -  -  -  -  -  $  | 77 |             |             | RRc  |          | 13.474 | 0.013      | 13.264 |            |        |                | _      | _                | _             | _      | _                | _             |           |                           | -1.840    | 0.430                     |

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| Tab | le A1 - Contina | ued from previo | us page |          |        |            |        |            |        |                |        |                  |               |        |                  |               |           |                           |           |                           |
|-----|-----------------|-----------------|---------|----------|--------|------------|--------|------------|--------|----------------|--------|------------------|---------------|--------|------------------|---------------|-----------|---------------------------|-----------|---------------------------|
| ID  | RA (J2000)      | Dec (J2000)     | Mode    | P (days) | J      | $\sigma_J$ | H      | $\sigma_H$ | $K_s$  | $\sigma_{K_S}$ | [3.6]  | $\sigma_{[3.6]}$ | $\Delta[3.6]$ | [4.5]  | $\sigma_{[4.5]}$ | $\Delta[4.5]$ | [Fe/H], p | $\sigma_{\rm [Fe/H]}$ , p | [Fe/H], s | $\sigma_{\rm [Fe/H]}$ , s |
| 79  | 13:28:24.99     | -47:29:25.2     | RRab    | 0.608    | 13.382 | 0.010      | 13.162 | 0.016      | 13.123 | 0.015          | _      | _                | _             | _      | _                | _             | -1.390    | 0.180                     | _         |                           |
| 81  | 13:27:36.68     | -47:24:48.3     | RRc     | 0.389    | 13.542 | 0.013      | 13.326 | 0.033      | 13.286 | 0.025          | 13.248 | 0.076            | _             | _      | _                | _             | -1.720    | 0.310                     | -1.990    | 0.430                     |
| 82  | 13:27:35.61     | -47:26:30.3     | RRc     | 0.336    | 13.579 | 0.016      | 13.324 | 0.024      | 13.296 | 0.018          | _      | _                | _             | 13.827 | 0.104            | _             | -1.560    | 0.200                     | -1.710    | 0.560                     |
| 83  | 13:27:08.42     | -47:21:34.1     | RRc     | 0.357    | 13.603 | 0.010      | 13.431 | 0.024      | 13.370 | 0.022          | _      | _                | _             | _      | _                | _             | -1.300    | 0.220                     | _         | _                         |
| 84  | 13:24:47.45     | -47:29:56.5     | RRab    | 0.580    | _      | _          | 12.833 | 0.017      | 12.781 | 0.016          | _      | _                | _             | _      | _                | _             | -1.470    | 0.100                     | _         | _                         |
| 85  | 13:25:06.49     | -47:23:34.0     | RRab    | 0.743    | 13.344 | 0.011      | _      | _          | _      | _              | _      | _                | _             | _      | _                | _             | -1.870    | 0.310                     | _         | _                         |
| 94  | 13:25:57.06     | -47:22:46.1     | RRc     | 0.254    | 14.070 | 0.024      | 13.934 | 0.022      | 13.870 | 0.027          | 13.858 | 0.038            | -0.092        | 13.799 | 0.029            | -0.014        | -1.000    | 0.110                     | _         | _                         |
| 95  | 13:25:24.95     | -47:28:53.2     | RRc     | 0.405    | 13.497 | 0.015      | 13.269 | 0.017      | 13.264 | 0.017          | _      | _                | _             | 13.178 | 0.024            | _             | -1.840    | 0.550                     | _         | _                         |
| 97  | 13:27:08.49     | -47:25:30.9     | RRab    | 0.692    | 13.302 | 0.010      | 13.143 | 0.029      | 13.034 | 0.022          | 12.964 | 0.061            | -0.008        | 12.702 | 0.064            | 0.240         | -1.560    | 0.370                     | -1.740    | 0.170                     |
| 101 | 13:27:30.24     | -47:29:51.0     | RRc     | 0.341    | 13.708 | 0.016      | 13.484 | 0.030      | 13.436 | 0.023          | _      | _                | _             | _      | _                | _             | -1.880    | 0.320                     | _         | _                         |
| 102 | 13:27:22.11     | -47:30:12.3     | RRab    | 0.691    | 13.320 | 0.012      | 13.033 | 0.022      | 12.993 | 0.020          | 12.984 | 0.049            | -0.028        | 13.056 | 0.072            | -0.113        | -1.840    | 0.130                     | -1.650    | 0.160                     |
| 103 | 13:27:14.29     | -47:28:36.3     | RRc     | 0.329    | 13.620 | 0.018      | 13.409 | 0.040      | 13.377 | 0.034          | 12.960 | 0.071            | _             | 13.024 | 0.066            | _             | -1.920    | 0.110                     | -1.780    | 0.270                     |
| 104 | 13:28:07.76     | -47:33:44.9     | RRab    | 0.867    | 13.732 | 0.096      | 13.626 | 0.154      | 13.452 | 0.141          | _      | _                | _             | _      | _                | _             | -1.830    | 0.180                     | _         | _                         |
| 105 | 13:27:46.02     | -47:32:43.9     | RRc     | 0.335    | 13.768 | 0.014      | 13.615 | 0.020      | 13.533 | 0.018          | _      | _                | _             | _      | _                | _             | -1.240    | 0.180                     | _         | _                         |
| 107 | 13:27:14.05     | -47:30:57.9     | RRab    | 0.514    | 13.597 | 0.017      | 13.340 | 0.038      | 13.301 | 0.030          | 13.535 | 0.219            | _             | 13.351 | 0.076            | _             | -1.360    | 0.110                     | _         | _                         |
| 115 | 13:26:12.30     | -47:34:17.5     | RRab    | 0.630    | 13.401 | 0.012      | 13.176 | 0.017      | 13.103 | 0.013          | _      | _                | _             | _      | _                | _             | -1.870    | 0.010                     | -1.640    | 0.320                     |
| 117 | 13:26:19.91     | -47:29:21.0     | RRc     | 0.422    | 13.480 | 0.020      | 13.274 | 0.043      | 13.202 | 0.031          | 13.110 | 0.044            | 0.071         | 12.949 | 0.043            | 0.179         | -1.680    | 0.250                     | _         | _                         |
| 120 | 13:26:25.52     | -47:32:48.6     | RRab    | 0.549    | 13.525 | 0.049      | 13.072 | 0.079      | 13.135 | 0.094          | 12.958 | 0.066            | 0.236         | 12.927 | 0.055            | 0.253         | -1.390    | 0.060                     | -1.150    | 0.160                     |
| 121 | 13:26:28.17     | -47:31:50.5     | RRc     | 0.304    | 13.741 | 0.016      | 13.648 | 0.033      | 13.531 | 0.026          | 13.414 | 0.037            | 0.144         | 13.302 | 0.033            | 0.249         | -1.460    | 0.130                     | -1.830    | 0.400                     |
| 122 | 13:26:30.31     | -47:33:02.2     | RRab    | 0.635    | 13.369 | 0.018      | 13.132 | 0.042      | 13.062 | 0.024          | 13.057 | 0.052            | -0.013        | 12.987 | 0.052            | 0.043         | -2.020    | 0.180                     | -1.790    | 0.210                     |
| 123 | 13:26:51.17     | -47:37:13.2     | RRc     | 0.474    | 13.462 | 0.016      | 13.239 | 0.019      | 13.174 | 0.017          | _      | _                | _             | _      | _                | _             | -1.640    | 0.010                     | _         | _                         |
| 124 | 13:26:54.49     | -47:39:07.5     | RRc     | 0.332    | 13.708 | 0.013      | 13.510 | 0.018      | 13.482 | 0.023          | _      | _                | _             | _      | _                | _             | -1.330    | 0.230                     | _         | _                         |
| 125 | 13:26:48.92     | -47:41:03.7     | RRab    | 0.593    | 13.420 | 0.015      | 13.200 | 0.016      | 13.153 | 0.015          | _      | _                | _             | _      | _                | _             | -1.670    | 0.220                     | -1.810    | 0.380                     |
| 126 | 13:28:08.03     | -47:40:46.7     | RRc     | 0.342    | 13.642 | 0.011      | 13.467 | 0.017      | 13.370 | 0.016          | _      | _                | _             | _      | _                | _             | -1.310    | 0.130                     | _         | _                         |
| 127 | 13:25:19.36     | -47:28:37.6     | RRc     | 0.305    | _      | _          | _      | _          | 13.579 | 0.018          | _      | _                | _             | 13.573 | 0.063            | _             | -1.590    | 0.080                     | _         | _                         |
| 128 | 13:26:17.75     | -47:30:13.0     | RRab    | 0.835    | 13.207 | 0.018      | 12.927 | 0.032      | 12.810 | 0.020          | _      | _                | _             | 12.445 | 0.074            | _             | -1.880    | 0.040                     | _         | _                         |
| 130 | 13:26:09.93     | -47:13:40.0     | RRab    | 0.493    | 13.688 | 0.021      | 13.527 | 0.032      | 13.418 | 0.025          | _      | _                | _             | _      | _                | _             | -1.460    | 0.170                     | _         | _                         |
| 147 | 13:27:15.86     | -47:31:09.2     | RRc     | 0.423    | 13.397 | 0.012      | 12.934 | 0.041      | 13.083 | 0.022          | _      | _                | _             | 12.585 | 0.096            | _             | -1.660    | 0.140                     | _         | _                         |
| 149 | 13:27:32.94     | -47:13:43.6     | RRab    | 0.683    | 13.354 | 0.015      | 13.061 | 0.035      | 13.024 | 0.024          | _      | _                | _             | _      | _                | _             | -1.210    | 0.240                     | _         | _                         |
| 150 | 13:27:40.21     | -47:36:00.1     | RRab    | 0.899    | 13.068 | 0.019      | 12.757 | 0.025      | 12.692 | 0.018          | _      | _                | _             | _      | _                | _             | -1.760    | 0.340                     | _         | _                         |
| 151 | 13:28:25.40     | -47:16:00.2     | RRab    | 0.408    | 13.501 | 0.013      | 13.301 | 0.020      | 13.265 | 0.016          | _      | _                | _             | _      | _                | _             | -1.300    | 0.240                     | _         | _                         |
| 163 | 13:25:49.42     | -47:20:21.5     | RRc     | 0.313    | 13.763 | 0.019      | 13.557 | 0.016      | 13.545 | 0.025          | _      | _                | _             | _      | _                | _             | -1.180    | 0.270                     | _         | _                         |
| 168 | 13:25:52.78     | -47:32:02.9     | RRc     | 0.321    | 14.176 | 0.015      | 14.000 | 0.020      | 13.960 | 0.018          | _      | _                | _             | _      | _                | _             | _         | _                         | _         | _                         |
| 169 | 13:27:20.47     | -47:23:59.1     | RRc     | 0.319    | 13.805 | 0.013      | 13.735 | 0.019      | 13.652 | 0.025          | 13.734 | 0.050            | -0.232        | 14.001 | 0.116            | -0.512        | _         | _                         | -1.650    | 0.190                     |
| 184 | 13:27:28.50     | -47:31:35.4     | RRc     | 0.303    | 13.778 | 0.012      | 13.624 | 0.028      | 13.536 | 0.019          | _      | _                | _             | _      | _                | _             | _         | _                         | _         | _                         |
| 185 | 13:26:04.13     | -47:21:45.0     | RRc     | 0.333    | 13.701 | 0.016      | 13.545 | 0.018      | 13.508 | 0.023          | 13.496 | 0.036            | -0.043        | 13.479 | 0.033            | -0.046        | _         | _                         | _         | _                         |
| 261 | 13:27:15.41     | -47:21:29.5     | RRc     | 0.403    | 13.431 | 0.009      | 13.212 | 0.019      | 13.113 | 0.020          | _      | _                | _             | _      | _                | _             | _         | _                         | -1.500    | 0.350                     |
| 263 | 13:26:13.13     | -47:26:09.7     | RRab    | 1.012    | 13.155 | 0.017      | 12.888 | 0.017      | 12.746 | 0.016          | _      | _                | _             | 12.660 | 0.034            | _             | _         | _                         | -1.730    | 0.190                     |
| 274 | 13:26:43.73     | -47:22:48.2     | RRc     | 0.311    | 13.828 | 0.011      | 13.758 | 0.023      | 13.650 | 0.022          | _      | _                | _             | _      | _                | _             | _         | _                         | _         | _                         |
| 276 | 13:27:16.51     | -47:33:17.6     | RRc     | 0.308    | 13.727 | 0.021      | 13.614 | 0.046      | 13.533 | 0.024          | _      | _                | _             | _      | _                | _             | _         | _                         | _         | _                         |
| 280 | 13:27:09.33     | -47:23:05.7     | RRc     | 0.282    | 13.951 | 0.012      | 13.905 | 0.026      | 13.816 | 0.029          | _      | _                | _             | _      | _                | _             | _         | _                         | _         | _                         |
| 285 | 13:25:40.20     | -47:34:48.4     | RRc     | 0.329    | 13.687 | 0.017      | 13.504 | 0.027      | 13.503 | 0.015          | _      | _                | _             | 13.358 | 0.074            | _             | _         | _                         | _         | _                         |
| 288 | 13:28:10.32     | -47:23:47.8     | RRc     | 0.295    | 13.809 | 0.011      | 13.719 | 0.016      | 13.635 | 0.019          | _      | _                | _             | _      | _                | _             | _         | _                         | _         | _                         |
| 289 | 13:28:03.68     | -47:21:27.9     | RRc     | 0.308    | 13.743 | 0.013      | 13.618 | 0.015      | 13.584 | 0.022          | _      | _                | _             | _      | _                | _             | _         | _                         | _         | _                         |
| 291 | 13:26:38.52     | -47:33:28.0     | RRc     | 0.334    | 13.674 | 0.018      | 13.518 | 0.044      | 13.444 | 0.026          |        |                  |               |        |                  |               | _         | _                         |           |                           |
| 357 | 13:26:17.77     | -47:30:23.4     | RRc     | 0.298    | 13.692 | 0.027      | 13.468 | 0.064      | 13.468 | 0.045          | 13.462 | 0.044            | 0.120         | 13.375 | 0.041            | 0.204         |           |                           | -1.640    | 0.990                     |

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