The Python based statistics and LaTeX report coursework for [WL]

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This is my submission for the Python statistical analysis and LaTeX report writing. This is new to me so may be a bit rough/I did last last time and so this time I'll build on my feedback. I've never used these things before/I'm still poor at something. Here I work through the questions outlines in the booklet using Python and LaTeX hence gaining a better understanding of how to use them. I'll report a range of findings based on the supplied data. This course-work has really helped me gain an insight into how all this works and will aid me in my future lab reports across all time and space. I could do with a really strong cup of coffee.

YEAR 1 SEMESTER 1

Data Set 1

Figure 1 presents a graph of the first 50 electrical measurements including, where appropriate, good use the SI-Prefixes. Note to be nice we run the 'measurement number' axis from 0 to 51. Also note, the nice scale on the y-axis which is symmetric about the mean value of the measurements.

What's important is to produce a good legible graph that allows the casual reader to get a good feel for your experimental results. So easy to read font size for both the axis labels and the numerical text. The choise of unit is also important to keep the numerical values presented in a nice human range, typically from 1 to 1000. Any lower and drop the SI Prefix any higher and raise the SI Prefix.

TABLE I. The mean $\langle I \rangle$, standard deviation σ and standard error on the mean s_{mean} for a set of n electrical current measurements.

n	$\langle I \rangle$ / mA	σ / mA	s_{mean} / mA
3	506	43	25
5	491	49	22
10	476	51	16
50	506	66	9.3
100	497	63	6.3
10000	502.72	60	0.60

Table I presents the numerical results required for the analysis of the electrical current measurements. They show as the number of data point analysed increases so the mean of the current $\langle I \rangle$ and the standard deviation converge onto their true values, whereas the standard error on the mean drops since $s_{\text{mean}} \propto \sqrt{N}$. Also note the use of the SI Prefix so as to present these numerical values nicely. This analysis also highlights that taking more and more data does help the precision of the final mean results, but at ever diminishing cost.

Aside: If you were inclined you could also investigate the dependence of the standard error s_{mean} as a function of the number of data points analysed. A plot should show a square-root dependence. You could go further and plot that logarithmically and show that the power law was indeed near 0.5 and its uncertainty. You don't have to do that for this exercise, but you could for fun.

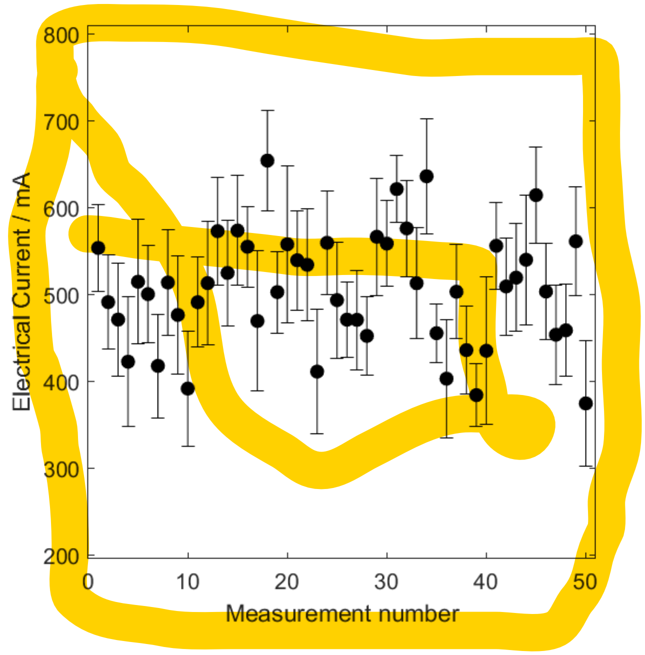


FIG. 1. The first 50 electrical current measurements. See table I for statistical values.

The current is flowing thought a $R = 100\ \Omega$ resistor. The power dissipated in the resistor p is given by $p = I^2 R$. Hence here we have $p = (25.273 \pm 0.061)$ W.

Data Set 2

This section presents an analyses of a Hook's law spring obeying the relationship

$$F = -k(x - x_0) \tag{1}$$

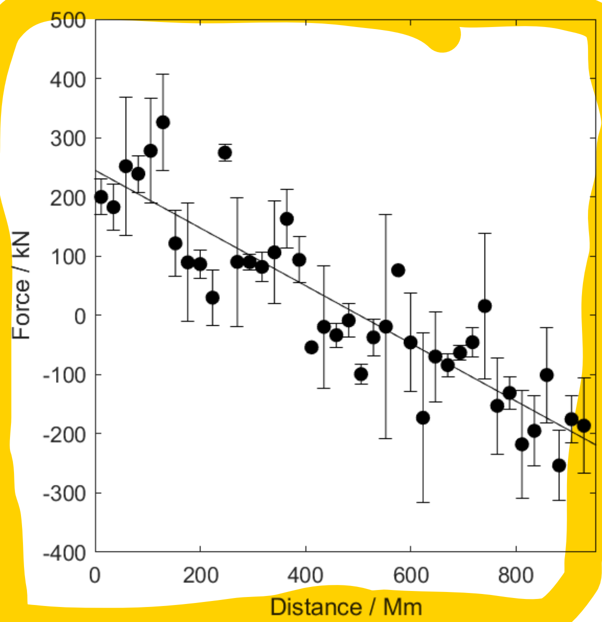


FIG. 2. The restoring force generated by the controlled expansion and compression of a Hook's law spring. Line indicates Linear last square fit with parameters as presented in the main text.

where F is the measured force, k is the spring constant, x is the experimentally controlled position of one end of the spring as measured in the laboratory, and x_0 is the equilibrium length of the spring. Forty pairs of F and x were measured, each with their own standard deviation in the force measurement. Figure 2 presents these data.

To determine k and the intercept with the Force axis $F_{x=0}$, a linear least squares fit was performed giving $k = (487 \pm 40)\ \mu\text{N/m}$ and $F_{x=0} = (245 \pm 22)$ kN (see figure 2).

YEAR 1 SEMESTER 2

Data Set 1

Table II presents the numerical results required for the analysis of the electrical current measurements. Here we have used the uncertainties σ_i of each data point to generate a weight $w_i = 1/\sigma_i^2$ to compute weighted values of the mean of the current $\langle I \rangle$ and standard error on the mean s_{mean}. Where as usual,

$$\langle I \rangle = \frac{\sum_{i=n}^{i=1} w_i I_i}{\sum_{i=n}^{i=1} w_i} \tag{2}$$

and

$$s_{\text{mean}} = \frac{1}{\sqrt{\sum_{i=n}^{i=1} w_i}}. \tag{3}$$

TABLE II. The weighted mean $\langle I \rangle$ and weighted standard error on the mean s_{mean} for a set of n electrical current measurements.

n	$\langle I \rangle$ / mA	s_{mean} / mA
3	512	32
5	501	27
10	482	19
50	505.4	7.9
100	497.8	5.7
10000	503.42	0.56

Note that there is no standard deviation for this set of results, as instead each data point has its own unique value. Mostly this will only happen if you have a set of data, that has all the same uncertainties, but pass it through a mathematical formula so that the final set of values ends up with unequal uncertainties. At that point you may want to use the weighted mean and standard error.

The analysis of the uncertainties in your experiments can nearly be as subtle and complex as you wish. It takes time and practise to work out what uncertainty goes where and what's important and what's not. These labs and this assessment allows you to at least practice the process. What we ask for in these [WL] labs is that you get all the basic uncertainty analysis right and use the most appropriate techniques. As you go further then you can delve deeper into the statistical underpinning of all this work.

Uncertainty propagations equations

Here we have been given two equations to analyse for propagation of an uncertainty Δx in x through to an uncertainty Δy in y.

Equation 1

Given that a and b are constants, for the equation,

$$y = \frac{1}{a}\log_{10}(b + x) \tag{4}$$

the uncertainty Δx in x propagates to y as

$$\Delta y = \frac{1}{ab\log_e(10) + ax\log_e(10)}\Delta x. \tag{5}$$

Equation 2

Given that a and b are constants, for the equation,

$$y = a\cos(bx) \tag{6}$$

the uncertainty Δx in x propagates to y as

$$\Delta y = -ab\sin(bx)\Delta x. \tag{7}$$

Equilibrium Spring position

By combining the spring constant k, force $F_{x=0}$ at $x = 0$ with their associated uncertainties, the x position of the equilibrium spring position is $x_0 = (503 \pm 60)$ Mm.

YEAR 2 SEMESTER 1

The normal relationship between the position of body h experiencing a constant force with resultant acceleration a with the time t of its fall is

$$h = \frac{1}{2}at^2. \tag{8}$$

Here we measure the position and time for a body undergoing free fall and experiencing the effects of air drag and turbulence. Figure 3a presents 35 measurements and their individual standard deviations.

As expected the relationship is nonlinear. But rather than just fitting equation 8 which is just the integrated form of Newton's 2nd law, here we fit a more generic power law

$$h = \frac{1}{2}at^p \tag{9}$$

to determine the power p dependence of the position with the fall-time. To extract the power law here we perform a weighted linear least squares straight line to the data after taking its logarithm.

To ensure the logarithms return positive values (for ease of plotting) the data is divided by $h_0 = 0.1$ m for the distance measurements, and $t_0 = 0.1$ s for the time measurements. Using equation 9 to link h_0 and t_0 gives $a_0 = 2h_0/t_0^p$. Thus dividing both sides of equation 9 by h_0 it becomes

$$\frac{h}{h_0} = \frac{a}{a_0}\left(\frac{t}{t_0}\right)^p \tag{10}$$

and taking base 10 logarithms gives

$$\log_{10}\left(\frac{h}{h_0}\right) = \log_{10}\left(\frac{a}{a_0}\right) + p\log_{10}\left(\frac{t}{t_0}\right). \tag{11}$$

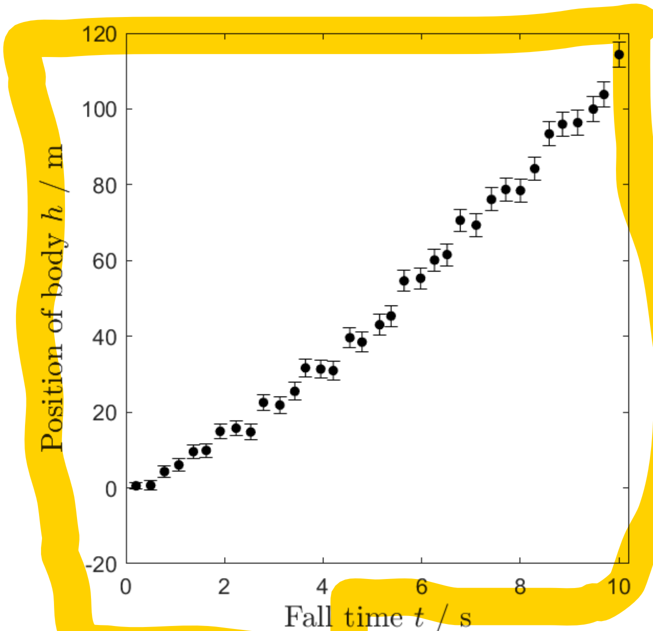
Figure 3b presents the same data but after taking their logarithms with appropriate divisors. It also shows a weighted linear least squares fit giving a power law exponent $p = 1.318 \pm 0.022$. Note, the reason for taking base 10 logarithms is to match the logarithmic scale of normal log-graph paper if you should ever use it.

If we wanted to (not here) we could then use the intercept $C = \log_{10}(a/a_0)$ of the logarithmic graph to determine the acceleration constant a through the relationship

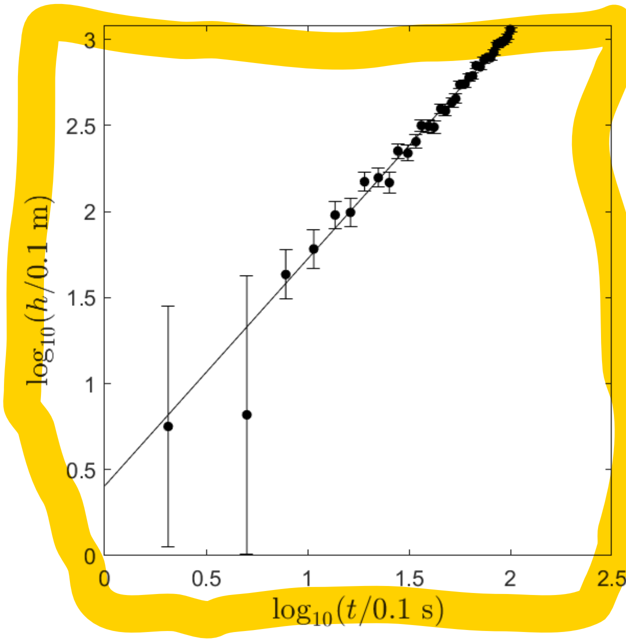
$$a = a_0 10^C \tag{12}$$

leading to

$$a = 2\frac{h_0}{t_0^p}10^C. \tag{13}$$



(a) Linear scale.



(b) Logarithmic scale.

FIG. 3. The position is a body undergoing constant acceleration measured at several time intervals.

Here h_0 and t_0 are well defined, but to determine the uncertainty on a, both the uncertainty on the intercept C and the uncertainty on the power law p would have to be used. Tricky but I could easily do it if I wished.

In an actual report I would first show that the power law gave strong evidence that the theory model, here given in equation 8, was a good description of the experiment. Then I could just use that equation to fit $y = h$

and $x = v^2$ to perform another linear least square fit to determine a. Thus I would have shown that the model fits to my data. Then under the limitation of that model extracted some physical parameter of interest. The power-law fitting step adds maturity to the data analysis and also gives the report something more to present in the discussion, especially if it does deviate from the model.