Vic, Hong 47523483 Totorial Group Number: 1

1. (a) $\lfloor \frac{n+1}{d} \rfloor = \lceil \frac{n}{d} \rceil$

if $d = 1$, $\lfloor \frac{n+1}{1} \rfloor = n+1$, $\lceil \frac{n}{1} \rceil = n$ for every $n \in \mathbb{Z}$, we cannot find any $n \in \mathbb{Z}$ satisfies $\lfloor \frac{n+1}{1} \rfloor = \lceil \frac{n}{1} \rceil$!

Therefore, (a) is disproved by counterexample.

(b) rewrite the statement, $\forall n \in \mathbb{Z}$, if $(3n+2)^2 \equiv 4 \pmod 6$, then n is even

It means, $(3n+2)^2 - 4$ can be divided by 6.

$(3n+2)^2 - 4 = 9n^2 + 12n + 4 - 4 = 9n^2 + 12n = 3(3n^2 + 4n)$

now we have $3(3n^2+4n)$ can be divided by 6, also $6 = 2 \times 3$, we cancel the factor 3 we have $(3n^2+4n)$ can be divide by 2 (or even). ①

Now let n is odd, n^2 must be odd, $3n^2$ must be odd, and $4n$ must be even, thus $(3n^2+4n)$ must be odd, which cause a contradiction to sentence ①, thus n must be even to secure $(3n^2+4n)$ is even.

Therefore, (b) is proved by contraposition and contradiction.

(c) $\forall n \in \mathbb{Z}$, if $(2n+1) \equiv 1 \pmod 3$, then n is odd.

we can have, $\forall n \in \mathbb{Z}$, if $2n$ can be divided by 3, then n is odd.

let $n = 6$, $2n = 2 \times 6 = 12$, which can be divided by 3. however, 6 is even.

Therefore, (c) is disproved by counterexample.

(d) For all $m, n, d \in \mathbb{Z}^+$, if $m|n$ and $d \nmid n$, then $d \nmid m$ ← statement ②

let (d) is false, then "For all $m, n, d \in \mathbb{Z}^+$, if $m|n$ and $d \nmid n$, then $d|m$" is True.

Now we have $n = am$, for $a \in \mathbb{Z}$, $m = bd$, for $b \in \mathbb{Z}$,

thus $n = a(bd) = (ab)d$, means $d|n$, which cause a contradiction of statement ② Thus, statement (d) is True

Therefore, (d) is proved by contradiction.

(e) let $\frac{rs}{3} = \pi$, which is a irrational number.

then $rs = 3\pi$. Because π cannot be divided by any other number, one of the r or s must equal to π to make sure π is exist in "$rs = 3\pi$" sentence.

Therefore, (e) is prove by direct proof.

2. Find $\gcd(-8765, 1234)$

$-8765 = 1234 \times -7 + (-127)$, $\gcd(-8765, 1234) = \gcd(1234, -127)$

$1234 = -127 \times -9 + 91$, $\gcd(1234, -127) = \gcd(-127, 91)$

$-127 = 91 \times -1 + (-36)$, $\gcd(-127, 91) = \gcd(91, -36)$

$91 = -36 \times -2 + 19$, $\gcd(91, -36) = \gcd(-36, 19)$

$-36 = 19 \times -1 + (-17)$, $\gcd(-36, 19) = \gcd(19, -17)$

$19 = -17 \times -1 + 2$, $\gcd(19, -17) = \gcd(-17, 2)$

$-17 = 2 \times -8 + (-1)$, $\gcd(-17, 2) = \gcd(2, -1)$

$2 = -1 \times -2 + 0$, $\gcd(2, -1) = 1$

3. $n = 2 \cdot 3^a \cdot 7^2 \cdot 13^b$

$m = 3^k \cdot 7 \cdot 11 \cdot 13^l$

$\gcd(n,m) = 3^a \cdot 7 \cdot 13^l$

$nm = 2 \cdot 3^{(a+k)} \cdot 7^3 \cdot 11 \cdot 13^{(b+l)}$

$\mathrm{lcm}(n,m) = mn \div \gcd(n,m) = 2 \cdot 3^k \cdot 7^2 \cdot 11 \cdot 13^b$

4 Let $P(n)$ be the predicate " $b_n \leq 3^{n-1}$ "

Basis step: $b_1 = 1 \leq 3^0$ $P(1)$ is true

$b_2 = 3 \leq 3^1$ $P(2)$ is true

$b_3 = 7 \leq 3^2$ $P(3)$ is true.

Inductive Hypothesis: Suppose $P(k)$ is true for all integer k with $1 \leq k \leq n$

$$b_{n+1} = 2b_n + b_{n-2} \leq 2(3^{n-1}) + (3^{n-3}) \quad \left[\begin{array}{l} \text{because } 2b_n < 2(3^{n-1}) \\ \text{and } b_{n-2} \leq (3^{n-3}) \end{array}\right]$$

$$\Rightarrow \leq 2((3^n)(3^{-1})) + (3^n)(3^{-3})$$

$$\Rightarrow \leq (3^n)(2(3^{-1}) + (3^{-3}))$$

$$\Rightarrow \leq (3^n)\underbrace{\left(\frac{2}{3} + \frac{1}{27}\right)}_{\frac{19}{27}} \leq 3^n$$

AS $b_{n+1} \leq 3^{n-1}$, $P(n+1)$ is true

Therefore by principle of strong mathematical induction,

$b_n \leq 3^{n-1}$ is true for each integer $n \geq 1$.

5.

5.1(a)

(i)

Line	n	s	r	d	A
1	41	6	0	2	1
3			1		
6				3	
3			2		
6				4	
3			1		
6				5	
3			1		
6				6	
3			5		
6				7	
3			6		
6				8	
3			1		
6				9	
3			5		
6				10	
3			1		
6				11	
3			8		
6				12	
3			5		
6				13	
3			2		
6				14	
3			13		
6				15	
3			11		
6				16	
3			9		
6				17	
3			7		
6				18	
3			5		
6				19	
3			3		
6				20	
3			1		
6				21	

Line	n	S	r	d
3			20	
6				22
3			19	
6				23
3			18	
6				24
3			17	
6				25
3			16	
6				26
3			15	
6				27
3			14	
6				28
3			13	
6				29
3			12	
6				30
3			11	
6				31
3			10	
6				32
3			9	
6				33
3			8	
6				34
3			7	
6				35
3			6	
6				36
3			5	
6				37
3			4	
6				38
3			3	
6				39
3			2	
6				40
3			1	
6				41

Line	n	s	r	d	A	Output
3			0			
5					0	
9						0

(ii)

Line	n	S	r	d	A	Output
1	445	21	0	2	1	
3			1			
6				3		
3			1			
6				4		
3			1			
6				5		
3			0			
5					0	
9						0

(b)

Line 1: define all the variables

Line 2: d suppose to be a varable to find the smallest factor of S, thus while condition should constraint d is smaller or equal to S.

Line 3: redefine variable "r" as the reminder of n/d

Line4: if (r=0) means if d|n

Line5: if d|n, redefine A = 0

Line6: if d∤n, plus 1 to d to continue find a smallest factor d of n.

Line7: Must after Line 5, end the while loop.

Line9: Give the output A=0, if the smallest factor d had been found.

(c)

The smallest factor of 2431 is 11.

So the while loop will run for 9 times.

(2, 2+1, 2+1+1, 2+1+1+1, 2+1+1+1+1, ... 2+1+1+1+1+1+1+1+1+1.)