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1.

(a)

$a_1 = 2$

$a_2 = 3 \times a_1 + 2 = 8$

$a_3 = 3 \times a_2 + 2 = 26$

$a_4 = 3 \times a_3 + 2 = 80$

$a_5 = 3 \times a_4 + 2 = 242$

(b) guess: $a_n = 3^n - 1$

prove: $a_1 = 3^1 - 1 = 2$

$a_2 = 3^2 - 1 = 8$

$a_3 = 3^3 - 1 = 26$

$a_4 = 3^4 - 1 = 80$

$a_5 = 3^5 - 1 = 243$.

$a_6 = 3^6 - 1 = 728 = 3 \times a_5 + 2$

Therefore, the explicit formula is $a_n = 3^n - 1$.

2.

$A - (B \cup C) = A \cap (B \cup C)^c$ by set difference law

$A \cap (B \cup C)^c = \underline{A \cap (B^c \cap C^c)}$ by De Morgan's law

$(A - B) - C = (A \cap B^c) - C$ by set difference law.

$(A \cap B^c) - C = \underline{(A \cap B^c) \cap C^c}$ by set difference law.

$A \cap (B^c \cap C^c) = (A \cap B^c) \cap C^c$ by associative law.

$(A \cap B^c) \cap C^c = (A \cap B^c) \cap^c$

Therefore, $A - (B \cup C) = (A - B) - C$

3. $S = \{n \in \mathbb{Z} \mid 4a \text{ for } a \in \mathbb{Z}\}$

$T = \{n \in \mathbb{Z} \mid 4b+2 \text{ for } b \in \mathbb{Z}\}$

$E = \{n \in \mathbb{Z} \mid 2c \text{ for } c \in \mathbb{Z}\}$

Let (x,y) be an element of $S \times T$, it could be write as $(2(2a), 2(2b+1))$ for $a, b \in \mathbb{Z}$

Let (f,g) be an element of $E \times E$, it could be write as $(2(c), 2(c))$ for $c \in \mathbb{Z}$

Since $2a$ is another form of c and $(2b+1)$ is another form of c, we can have $S \times T \subseteq E \times E$.

However, for example $\{2,2\}$ is a subset of $E \times E$ but not a subset of $S \times T$, therefore, $S \times T \subset E \times E$.

4. $P(\{0,1\}) = \{\{\emptyset\}, \{0\}, \{1\}, \{0,1\}\}$

$P(\{1,2\}) = \{\{\emptyset\}, \{1\}, \{2\}, \{1,2\}\}$

$P(\{0,1\}) \times P(\{1,2\}) = \{\{\emptyset\}, \{0,1\}, \{0,2\}, \{0,1,2\}, \{1\}, \{1,2\},\}$

(a) take two $f((A_1, B_1))$ and $f((A_2, B_2))$ that are equal to 1, there are many input options would output 1. For example, $f((\{0,1\}, \{1\})) = 1$ $f((\{0,2\}, \{1,2\})) = 1$, $f((\{0,1,2\}, \{1\})) = 1$. Therefore, f is not an injective function.

(b) For $f((A,B)) = 0$, we could find $f((\{\emptyset\}, \{0,1\})) = 0$

For $f((A,B)) = 1$, we could find $f((\{0,1\}, \{1\})) = 1$

For $f((A,B)) = 2$, we could find $f((\{0,1,2\}, \{1,2\})) = 2$

For every $f((A,B)) \in \{0,1,2\}$, we could find at least 1 $f((A,B)) = y$.

Therefore, f is a subjective function.

$\{0,1,2\}$ $\{3,4,5\}$

5. $A = [0,2]$, $B = (2,6)$

$f: A \to B \Rightarrow f(a) = a+3$

Suppose $a_1, a_2 \in A$ and $f(a_1) = f(a_2)$

We have $a_1 + 3 = a_2 + 3$. Thus $a_1 = a_2$, we find an injective function $A \to B$.

$g: B \to A \Rightarrow g(b) = b-3$

Suppose $b_1, b_2 \in B$ and $g(b_1) = g(b_2)$.

We have $b_1 - 3 = b_2 - 3$, Thus $b_1 = b_2$, we find an injective function $B \to A$.

Therefore, base on Schroder - Bernstein Theorem, we find two injective functions f, g for both $A \to B$ and $B \to A$. So, $|A| = |B|$.

6. I. Basis Property

$I(0)$: " $i = 0+1$ and min = minimum value of $A[1]$ "

which is true before the first interaction of the loop.

II. Inductive Property:

Suppose k is a positive integer such that $i \neq n \wedge I(k)$ is true

before an iteration of the loop, $i \neq n$, min = minimum value of $A[1], A[k]$, and $i = k$. Since $i \neq n$, the guard is passed and statement 1 is executed. Before execution of statement 1,

$$min_{old} = \text{minimum of } A[1], A[k]$$

execution of statement 1 has the following effect:

$$min_{new} = \text{minimum of } A[1], A[k], A[k+1]$$

before the execution. $i_{old} = k$

after the execution. $i_{new} = k+1$.

Hence, ($i = k$ and min = minimum value of $A[1], A[k], A[k+1]$

III Eventual falsity of $i \neq n$.

This loop will keep adding 1 to i until $i = n$, then $i = n$ will be false after $(n-1)$ times iteration (because initial $i = 1$, not 0).

IV. Correctness of the Post-Condition.

According to the post-condition, the set A after all execution of the loop should be $A[1], A[2], \dots A[n]$. But if $i \neq n$ becomes false after $(n-1)$ times iteration, $i = n$. And if $I(n)$ is true, $i = m$ and min = minimum value of $A[1], A[2], A[3], \dots A[m+1]$. Since both $i \neq n$ false and $i(m)$ true are satisfied, $n = i = m$ and min = minimum value of $A[1], A[2], A[3], \dots A[m+1]$ as required. □