Logical Equivalences

Given any statement variables p, q and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double negative laws	$\sim (\sim p) \equiv p$	
Idempotent laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
Universal bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations of $\mathbf{t}$ *and* $\mathbf{c}$	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Valid Argument Forms

<table>
<tr><td>Modus Ponens</td><td>$p \to q$
p
$\therefore q$</td><td>Elimination</td><td>(a) $p \vee q$
$\sim q$
$\therefore p$

(b) $p \vee q$
$\sim p$
$\therefore q$</td></tr>
<tr><td>Modus Tollens</td><td>$p \to q$
$\sim q$
$\therefore \sim p$</td><td>Transitivity</td><td>$p \to q$
$q \to r$
$\therefore p \to r$</td></tr>
<tr><td>Generalization</td><td>(a) p
$\therefore p \vee q$

(b) q
$\therefore p \vee q$</td><td rowspan="2">Proof by
division into cases</td><td rowspan="2">$p \vee q$
$p \to r$
$q \to r$
$\therefore r$</td></tr>
<tr><td>Specialization</td><td>(a) $p \wedge q$
$\therefore p$

(b) $p \wedge q$
$\therefore q$</td></tr>
<tr><td>Conjunction</td><td>p
q
$\therefore p \wedge q$</td><td>Contradiction Rule</td><td>$\sim p \to \mathbf{c}$
$\therefore p$</td></tr>
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