Testing Variance Component in linear mixed models

1. Statement of Problem

In this report, we address the problem of testing variance component in linear mixed models with one variance component. The one-way classification random model can be expressed as following,

$$y_{ij} = \mu + a_i + e_{ij}, \tag{1}$$

where μ is an unknown parameter, a_i and e_{ij} are mutually independent normal distributed with mean 0, unknown variance ${\sigma_a}^2$ a and ${\sigma_e}^2$ respectively, where $i = 1, ...a and j = 1, ..., n_i$. The n-pattern of subgroup sizes is$(n_1, n_2, ..., n_a)$. The model can be rewritten in matrix form as

$$\begin{aligned}
\mathbf{y} &= \mathbf{1_N}\mu + \begin{bmatrix} \mathbf{1_{n1}} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{1_{n2}} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{1_{na}} \end{bmatrix} \mathbf{a} + \mathbf{I_N e} \\
&= \mathbf{X}\mu + \mathbf{Z_1 a} + \mathbf{Z_2 e}.
\end{aligned}$$

The $\mathbf{y}$ is a vector of random variables with mean $\mathbf{1_N}\mu$ and variance-covariance matrix

$$\begin{aligned}
\mathbf{V} &= {\sigma_a}^2\mathbf{V_1} + \sigma_\mathbf{e}^\mathbf{2}\mathbf{V_2} \\
&= {\sigma_a}^2\mathbf{Z_1 Z_1}' + \sigma_\mathbf{e}^\mathbf{2}\mathbf{Z_2 Z_2}' \\
&= {\sigma_a}^2\mathbf{Z_1 Z_1}' + \sigma_\mathbf{e}^\mathbf{2}\mathbf{I}.
\end{aligned}$$

We are interested in the following hypothesis test,

$$H_0 : {\sigma_a}^2 = 0 \quad vs \quad H_a : {\sigma_a}^2 > 0.$$

2. Questions to be Addressed

In this project, we consider three ways to do the likelihood ratio test and compare the performance of each test under different patterns and $\lambda = {\sigma_a}^2/{\sigma_e}^2$.

• Likelihood ratio test (LRT) using the $0.5{\chi_0}^2 + 0.5{\chi_1}^2$ asymptotic null distribution;

• LRT using the exact finite sample null distribution derived in [1];

• Restricted likelihood ratio test (RLRT) using the exact finite sample null distribution derived in [1].

3. Description of the Design of the Experiment

We choose ten different n-patterns in [2] and fix ${\sigma_e}^2$ equal to 1 and let λ be $(0, 0.1, 0.2, 0.5, 1, 2, 5)$. For each combination of n-patterns and λ, we generate 10000 random samples from model $\mathbf{y} \sim \mathbf{Normal}(\mathbf{1_N}, \sigma_\mathbf{a}{}^\mathbf{2}\mathbf{Z_1}\mathbf{Z_1}' + \sigma_\mathbf{e}{}^\mathbf{2}\mathbf{I})$. For all the three tests, we set the significance level as 0.05. Using these simulated samples, we are able to computed the power of each test and plot the power curve. We will conduct the multiple paired t-test to test the significance between three tests.

4. Computational Details

For each simulated sample, the LRT and RLRT values, the critical values as well as p-values can be calculated. If p-value is less than 0.05, then we reject H_0. Therefore, the power for each test can be computed as the following,

$$power = \frac{\sum_{i=1}^{10000} I(p_i < 0.05)}{10000}.$$

The standard error of the power is $SE = \sqrt{\frac{power(1-power)}{10000}}$. To get a general idea of the goodness of three tests, we can plot the power curves of three methods for each n-pattern. We can also do a multiple paired t-test to test the power difference among the three tests.

5. Analysis of Results

Table 1 shows the results of the simulation study. In the method column, test 1 represents LRT using the using the $0.5{\chi_0}^2 + 0.5{\chi_1}^2$ asymptotic null distribution, test 2 represents LRT using the exact finite sample null distribution and test 3 represents RLRT using the exact finite sample null distribution. The entries in the column with $\lambda = 0$ represent test size and the rest columns represent test power. The value in the bracket under each entry represents its standard error.

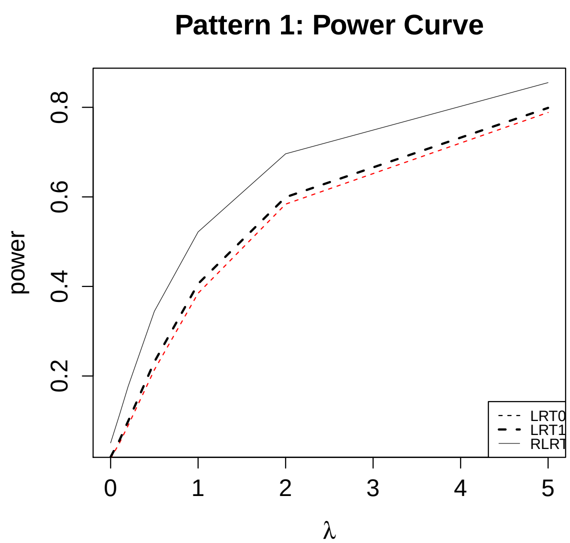
From an over view of the table, we can see that test 3 has the largest rejection rate and the rejection rate of test 2 is slightly higher than test 1. The power of all three methods increases with λ or ${\sigma_a}^2$. The size of test 1 and test 2 are always smaller than the significance level 0.05, while the size of test 3 are around 0.05.

Table 1: Powers of three tests

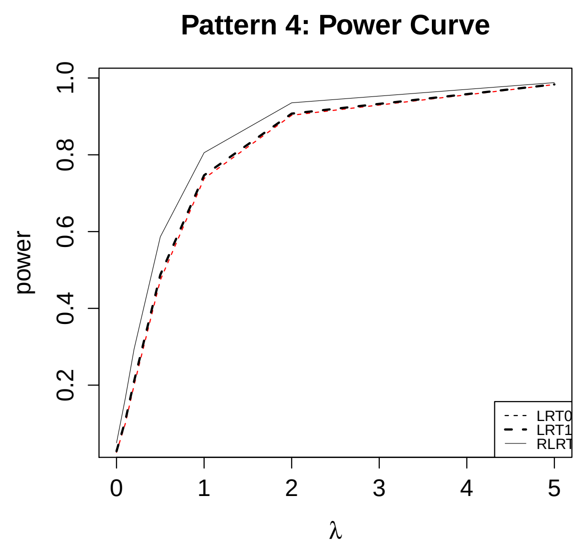
		$\lambda = {\sigma_a}^2$						
n-pattern		0	0.1	0.2	0.5	1.0	2.0	5.0
$P_1 = (3, 5, 7)$	test 1	0.0159	0.0498	0.0895	0.2137	0.3848	0.5838	0.7885
		(0.0016)	(0.0026)	(0.0033)	(0.0044)	(0.0049)	(0.0048)	(0.0038)
	test 2	0.0192	0.0558	0.0989	0.2316	0.4059	0.5994	0.7988
		(0.00137)	(0.0022)	(0.0029)	(0.0042)	(0.0049)	(0.0049)	(0.0040)
	test 3	0.0505	0.1123	0.1766	0.3445	0.5218	0.6962	0.8552
		(0.00218)	(0.0031)	(0.0038)	(0.0047)	(0.0049)	(0.0045)	(0.0035)
$P_2 = (1, 5, 9)$	test 1	0.0137	0.0426	0.0781	0.1805	0.3171	0.4854	0.6963
		(0.0015)	(0.0024)	(0.0031)	(0.0041)	(0.0048)	(0.0049)	(0.0044)
	test 2	0.0183	0.0503	0.0907	0.1973	0.3391	0.5044	0.7113
		(0.00134)	(0.0021)	(0.0028)	(0.0039)	(0.0047)	(0.0049)	(0.0045)
	test 3	0.0525	0.1081	0.1705	0.3105	0.4664	0.6250	0.7981
		(0.00223)	(0.0031)	(0.0037)	(0.0046)	(0.0049)	(0.0048)	(0.0040)
$P_4 = (3, 3, 5, 5, 7, 7)$	test 1	0.0252	0.0995	0.1996	0.4735	0.7382	0.903	0.9829
		(0.0019)	(0.0035)	(0.0044)	(0.0049)	(0.0040)	(0.0026)	(0.0011)
	test 2	0.0280	0.1070	0.2097	0.4873	0.7466	0.9072	0.9833
		(0.0016)	(0.0030)	(0.0040)	(0.0049)	(0.0043)	(0.0029)	(0.0012)
	test 3	0.0495	0.1650	0.2934	0.5863	0.8054	0.9354	0.9880
		(0.0021)	(0.0037)	(0.0045)	(0.0049)	(0.0039)	(0.0024)	(0.0010)
$P_5 = (1, 1, 5, 5, 9, 9)$	test 1	0.0189	0.1013	0.2006	0.4552	0.6906	0.855	0.962
		(0.00090)	(0.0024)	(0.0034)	(0.0048)	(0.0048)	(0.0038)	(0.0021)
	test 2	0.0232	0.1150	0.2173	0.4774	0.7060	0.8631	0.9651
		(0.0015)	(0.0031)	(0.0041)	(0.0049)	(0.0045)	(0.0034)	(0.0018)
	test 3	0.0510	0.1776	0.2999	0.5749	0.7743	0.9011	0.9760
		(0.0021)	(0.0038)	(0.0045)	(0.0049)	(0.0041)	(0.0029)	(0.0015)

$P_7 = (1,1,1,1,13,13)$	test 1	0.0176	0.087	0.1706	0.3638	0.5477	0.7271	0.9024
		(0.0008)	(0.0023)	(0.0033)	(0.0045)	(0.0049)	(0.0046)	(0.0032)
	test 2	0.0213	0.1046	0.1919	0.3924	0.5730	0.7460	0.9107
		(0.0014)	(0.0030)	(0.0039)	(0.0048)	(0.0049)	(0.0043)	(0.0028)
	test 3	0.0501	0.1768	0.2875	0.4976	0.6735	0.8252	0.9467
		(0.0021)	(0.0038)	(0.0045)	(0.0049)	(0.0046)	(0.0037)	(0.0022)
$P_8 =$	test 1	0.0263	0.1382	0.2974	0.6617	0.8875	0.9784	0.9986
$(3,3,3,5,5,5,7,7,7)$		(0.0010)	(0.0030)	(0.0043)	(0.0048)	(0.0032)	(0.0015)	(0.0003)
	test 2	0.0275	0.1735	0.3649	0.7272	0.9140	0.9838	0.9988
		(0.0016)	(0.0037)	(0.0048)	(0.0044)	(0.0028)	(0.0012)	(0.0003)
	test 3	0.0517	0.2398	0.4436	0.7851	0.9393	0.9887	0.9988
		(0.0022)	(0.0042)	(0.0049)	(0.0041)	(0.0023)	(0.0010)	(0.0003)
$P_9 =$	test 1	0.0235	0.1443	0.3036	0.6444	0.8585	0.9611	0.9954
$(1,1,1,5,5,5,9,9,9)$		(0.0010)	(0.0029)	(0.0041)	(0.0049)	(0.0038)	(0.0022)	(0.0007)
	test 2	0.0274	0.1590	0.3231	0.6628	0.8657	0.9637	0.9955
		(0.0016)	(0.0036)	(0.0046)	(0.0047)	(0.0034)	(0.0018)	(0.0006)
	test 3	0.0537	0.2256	0.4075	0.7314	0.9022	0.9751	0.9973
		(0.0022)	(0.0041)	(0.0049)	(0.0044)	(0.0029)	(0.0015)	(0.0005)
$P_{11} =$	test 1	0.0134	0.1345	0.2526	0.4779	0.6667	0.8303	0.9619
$(1,1,1,1,1,1,1,19,19)$		(0.0008)	(0.0028)	(0.0039)	(0.0049)	(0.0048)	(0.0040)	(0.0021)
	test 2	0.0205	0.1609	0.2832	0.5095	0.6881	0.8454	0.9660
		(0.0014)	(0.0036)	(0.0045)	(0.0049)	(0.0046)	(0.0036)	(0.0018)
	test 3	0.0510	0.2392	0.3786	0.6025	0.7665	0.8993	0.9811
		(0.0021)	(0.0042)	(0.0048)	(0.0048)	(0.0042)	(0.0030)	(0.0013)
$P_{12} = (2,10,18)$	test 1	0.01	0.0773	0.1548	0.3448	0.5181	0.6802	0.8428
		(0.0006)	(0.0021)	(0.0032)	(0.0045)	(0.0049)	(0.0048)	(0.0038)
	test 2	0.0176	0.1047	0.1907	0.3852	0.5566	0.7117	0.8587
		(0.0013)	(0.0030)	(0.0039)	(0.0048)	(0.0049)	(0.0045)	(0.0034)
	test 3	0.0541	0.1782	0.2969	0.5019	0.6537	0.7875	0.9024
		(0.0022)	(0.0038)	(0.0045)	(0.0049)	(0.0047)	(0.0040)	(0.0029)
$P_{13} = (3,15,27)$	test 1	0.0098	0.1203	0.2324	0.4519	0.626	0.7714	0.8937
		(0.0006)	(0.0027)	(0.0038)	(0.0048)	(0.0049)	(0.0044)	(0.0032)
	test 2	0.0179	0.1547	0.2788	0.5011	0.6655	0.798	0.9056
		(0.0013)	(0.0036)	(0.0044)	(0.0049)	(0.0047)	(0.0040)	(0.0029)
	test 3	0.0505	0.2439	0.3879	0.5976	0.7434	0.852	0.9369
		(0.0021)	(0.0042)	(0.0048)	(0.0049)	(0.0043)	(0.0035)	(0.0024)

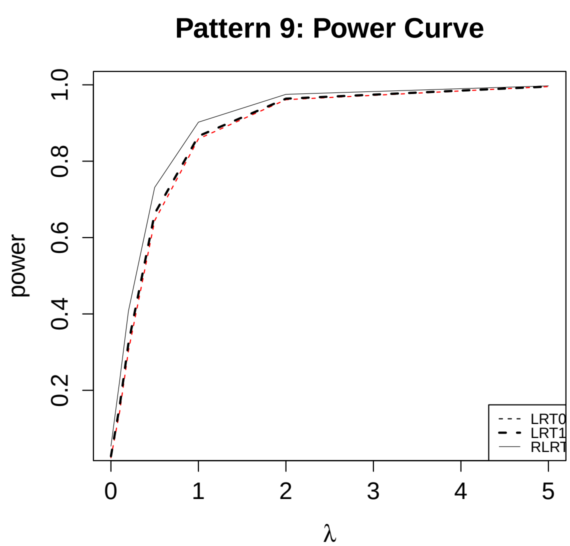
Figure 1 shows the power curves of pattern 1, 4, 9 and 12. Based on these plots, patterns with the same sample size, the power curves increase faster when the subgroup number is larger and also the difference among the three tests becomes less significant. All the other power curve plots are in Appendix 7.1.



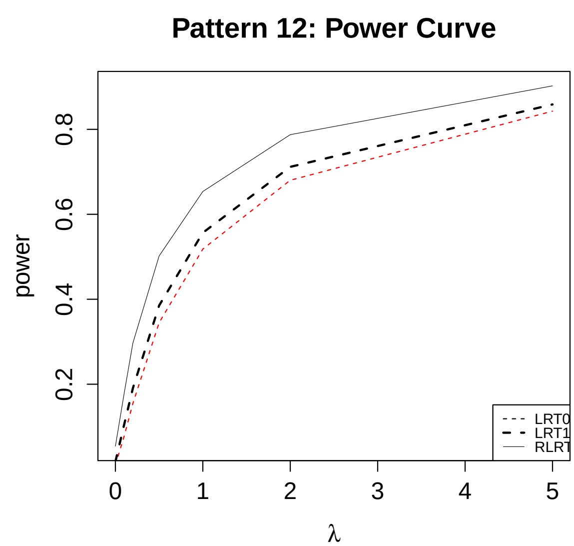
(a)



(b)



(c)



(d)

Figure 1: The power curve of pattern 1 (a), pattern 4 (b), pattern 9 (c), and pattern 12 (d).

The results of multiple paired t-test are listed below. Only for pattern 8 and 9, the differences among the three tests are not significant. All the other patterns show significant results. This is probably due to the fact that the sample sizes of pattern 8 and 9 are large enough and the variation of subgroup sizes are bigger than the other patterns.

Pattern 1

	test1	test2
test2	0.0069	-
test3	0.0019	0.0016

Pattern 2

	test1	test2
test2	0.0019	-
test3	0.00102	0.00093

Pattern 4

	test1	test2
test2	0.035	-
test3	0.034	0.034

Pattern 5

	test1	test2
test2	0.015	-
test3	0.016	0.016

Pattern 7

	test1	test2
test2	0.0028	-
test3	0.0018	0.0017

Pattern 8

	test1	test2
test2	0.108	-
test3	0.099	0.107

Pattern 9

	test1	test2
test2	0.082	-
test3	0.069	0.066

Pattern 11

	test1	test2
test2	0.0121	-
test3	0.0084	0.0076

Pattern 12

	test1	test2
test2	0.0011	-
test3	0.0014	0.0016

Pattern 13

	test1	test2
test2	0.005	-
test3	0.004	0.0039

6. Conclusions

This simulation study shows that on testing the variance component in a linear mixed model LRT and RLRT using the exact finite sample null distribution are more powerful than LRT using asymptotic null distribution. The performance of LRT with finite sample null distribution and LRT using asymptotic null distribution are similar, but the previous one is slightly more powerful.

7.1. Appendix: Power curve plots

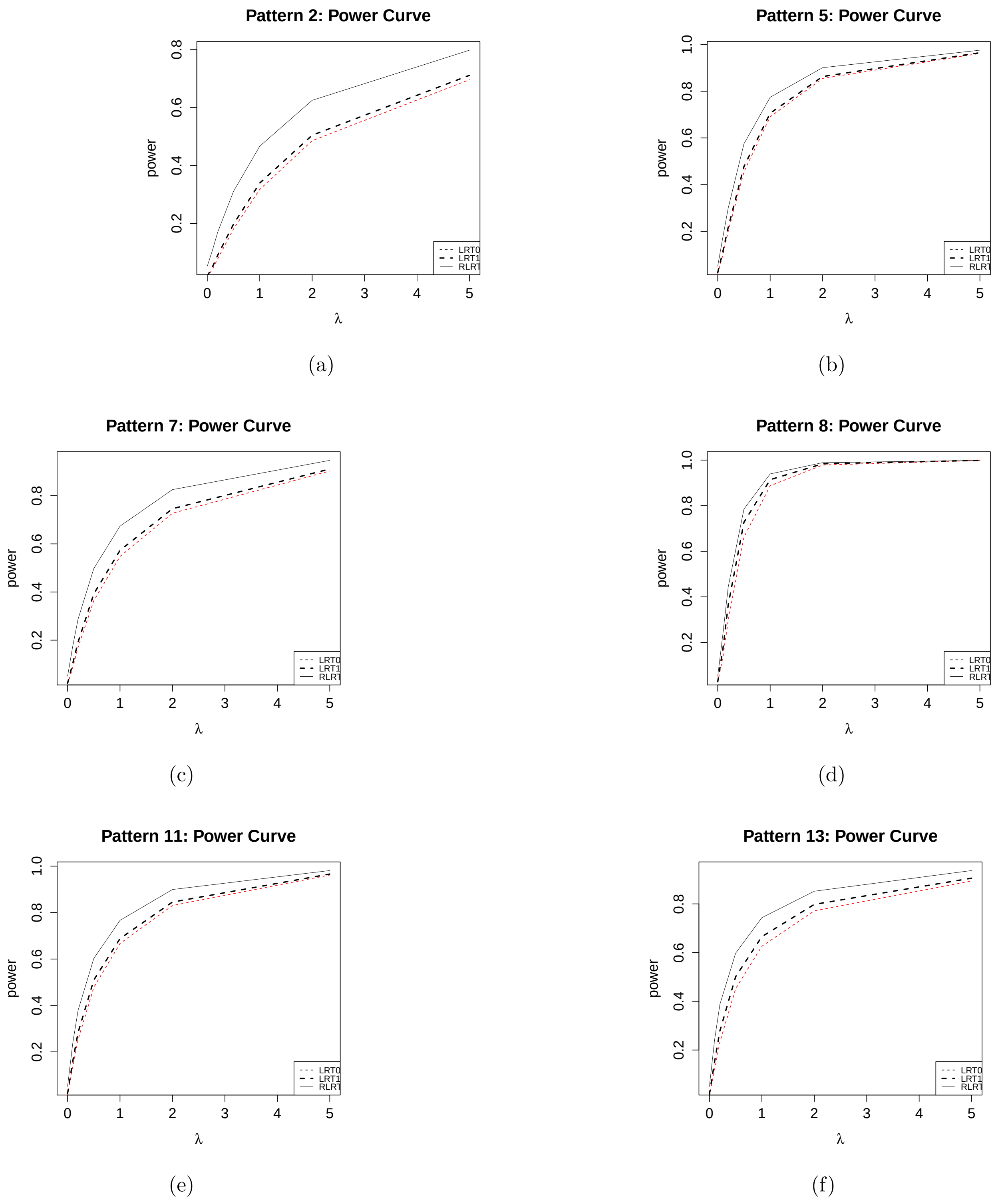


Figure 2: The power curve of pattern 2 (a), pattern 5 (b), pattern 7 (c), pattern 8 (d), pattern 11 (e) and pattern 13 (f).

7.2. Appendix: R code

```
# clean the memory
rm(list=ls())
library(RLRsim)
library(Matrix)
library(MASS)
library(nlme)
library(RLRsim)

fun <- function(x){
  z <- rep(1,x)
  return(z)
}

lrt.test <- function(p,sigma){
  total <- 10000 # sample number
  n <- sum(p)
  z1 <- bdiag(sapply(p,fun))
  v1 <- crossprod(t(z1),t(z1))
  v2 <- diag(1,n)
  mu <- as.vector(rep(1,n))
  group <- rowSums(t(c(1:length(p))*t(z1)))
  v <- sigma*v1+v2
  ######################################
  rej1 <- 0
  rej2 <- 0
  rej3 <- 0
  for(i in 1:total){
  set.seed(758*i)
  y <- mvrnorm(1, mu, v)
  data <- data.frame(y,group)
  names(data)<-c("y", "group")
  data$group<-as.factor(data$group)
  fit0 <- lm(y  ~ 1, data=data, method='qr')
  fita <- lme(y ~1,data=data,random=~1|group,method="ML")
```

```
  fitaR <- lme(y ~1,data=data,random=~1|group,method="REML")
  #test1####################################################
  pvalue <- 1/2 - 1/2*pchisq(-2*logLik(fit0)[1] +
                             2*logLik(fita)[1],df=1,lower.tail=TRUE)
  if (pvalue < 0.05) {
    rej1 <- rej1 + 1
  }
  power1 <- rej1/total
  sd1 <- sqrt(power1*(1-power1)/total)
  #test2#####################################################
  LRT <- exactLRT(m=fita,m0=fit0)
  if (LRT$p < 0.05) {
    rej2 <- rej2 + 1
  }
  power2 <- rej2/total
  sd2 <-sqrt( power2*(1-power2)/total)
  #test3#####################################################
  RLRT <- exactRLRT(m=fitaR,mA=fitaR,m0=fit0)
  if (RLRT$p < 0.05) {
    rej3 <- rej3 + 1
  }
  power3 <- rej3/total
  sd3 <- sqrt(power3*(1-power3)/total)
 }#output#######################################
  power <- c(power1,power2,power3)
  sd <- c(sd1,sd2,sd3)
  return(list(power=power,sd=sd))
}

sigma <- c(0,0.1,0.2,0.5,1,2,5)

#pattern 1 power
pp1 <- NULL
sd1 <- NULL
for(i in 1:7){
  temp1 <- lrt.test(c(3,5,7),sigma[i])$power
```

```
  pp1 <- cbind(pp1,temp1)
  temp2 <- lrt.test(c(3,5,7),sigma[i])$sd
  sd1 <- cbind(sd1,temp2)
}

#power curve
plot(sigma,pp1[3,],type="l",xlab=expression(lambda),ylab="power",lwd=0.5,lty=1,main="Pat
lines(sigma,pp1[1,],col="red",lwd=1,lty=2)
lines(sigma,pp1[2,],lwd=2,lty=2)
legend("topleft",c("LRT0","LRT1","RLRT"),lwd=c(1,2,0.5),lty=c(2,2,1),cex=0.65)

#use the multiple paired t test to test the significance of the power difference
power1 <- c(pp1[1,2:7],pp1[2,2:7],pp1[3,2:7])
group1<-c(rep("test1",6),rep("test2",6),rep("test3",6))
pairwise.t.test(power1,group1,p.adjust="bonf",paired=TRUE)

#pattern 2 power
pp2 <- NULL
sd2 <- NULL
for(i in 1:7){
  temp1 <- lrt.test(c(1,5,9),sigma[i])$power
  pp2 <- cbind(pp2,temp1)
  temp2 <- lrt.test(c(1,5,9),sigma[i])$sd
  sd2 <- cbind(sd2,temp2)
}

#power curve
plot(sigma,pp2[3,],type="l",xlab=expression(lambda),ylab="power",lwd=0.5,lty=1,main="Pat
lines(sigma,pp2[1,],col="red",lwd=1,lty=2)
lines(sigma,pp2[2,],lwd=2,lty=2)
legend("topleft",c("LRT0","LRT1","RLRT"),lwd=c(1,2,0.5),lty=c(2,2,1),cex=0.65)

#use the multiple paired t test to test the significance of the power difference
power2 <- c(pp2[1,2:7],pp2[2,2:7],pp2[3,2:7])
group2<-c(rep("test1",6),rep("test2",6),rep("test3",6))
pairwise.t.test(power1,group1,p.adjust="bonf",paired=TRUE)
```

```
#pattern 4 power
pp4 <- NULL
sd4 <- NULL
for(i in 1:7){
  temp1 <- lrt.test(c(3,3,5,5,7,7),sigma[i])$power
  pp4 <- cbind(pp4,temp1)
  temp2 <- lrt.test(c(3,3,5,5,7,7),sigma[i])$sd
  sd4 <- cbind(sd4,temp2)
}

#power curve
plot(sigma,pp4[3,],type="l",xlab=expression(lambda),ylab="power",lwd=0.5,lty=1,main="Pat
lines(sigma,pp4[1,],col="red",lwd=1,lty=2)
lines(sigma,pp4[2,],lwd=2,lty=2)
legend("topleft",c("LRT0","LRT1","RLRT"),lwd=c(1,2,0.5),lty=c(2,2,1),cex=0.65)

#use the multiple paired t test to test the significance of the power difference
power4 <- c(pp4[1,2:7],pp4[2,2:7],pp4[3,2:7])
group4<-c(rep("test1",6),rep("test2",6),rep("test3",6))
pairwise.t.test(power1,group1,p.adjust="bonf",paired=TRUE)

#pattern 5 power
pp5 <- NULL
sd5 <- NULL
for(i in 1:7){
  temp1 <- lrt.test(c(1,1,5,5,9,9),sigma[i])$power
  pp5 <- cbind(pp5,temp1)
  temp2 <- lrt.test(c(1,1,5,5,9,9),sigma[i])$sd
  sd5 <- cbind(sd5,temp2)
}

#power curve
plot(sigma,pp5[3,],type="l",xlab=expression(lambda),ylab="power",lwd=0.5,lty=1,main="Pat
lines(sigma,pp5[1,],col="red",lwd=1,lty=2)
lines(sigma,pp5[2,],lwd=2,lty=2)
```

```
legend("topleft",c("LRT0","LRT1","RLRT"),lwd=c(1,2,0.5),lty=c(2,2,1),cex=0.65)

#use the multiple paired t test to test the significance of the power difference
power5 <- c(pp5[1,2:7],pp5[2,2:7],pp5[3,2:7])
group5<-c(rep("test1",6),rep("test2",6),rep("test3",6))
pairwise.t.test(power1,group1,p.adjust="bonf",paired=TRUE)

#pattern 7 power
pp7 <- NULL
sd7 <- NULL
for(i in 1:7){
  temp1 <- lrt.test(c(1,1,1,1,13,13),sigma[i])$power
  pp7 <- cbind(pp7,temp1)
  temp2 <- lrt.test(c(1,1,1,1,13,13),sigma[i])$sd
  sd7 <- cbind(sd7,temp2)
}

#power curve
plot(sigma,pp7[3,],type="l",xlab=expression(lambda),ylab="power",lwd=0.5,lty=1,main="Pat
lines(sigma,pp7[1,],col="red",lwd=1,lty=2)
lines(sigma,pp7[2,],lwd=2,lty=2)
legend("topleft",c("LRT0","LRT1","RLRT"),lwd=c(1,2,0.5),lty=c(2,2,1),cex=0.65)

#use the multiple paired t test to test the significance of the power difference
power7 <- c(pp7[1,2:7],pp7[2,2:7],pp7[3,2:7])
group7 <-c(rep("test1",6),rep("test2",6),rep("test3",6))
pairwise.t.test(power1,group1,p.adjust="bonf",paired=TRUE)

#pattern 8 power
pp8 <- NULL
sd8 <- NULL
for(i in 1:7){
  temp1 <- lrt.test(c(3,3,3,5,5,5,7,7,7),sigma[i])$power
  pp8 <- cbind(pp8,temp1)
  temp2 <- lrt.test(c(3,3,3,5,5,5,7,7,7),sigma[i])$sd
  sd8 <- cbind(sd8,temp2)
```

```
}

#power curve
plot(sigma,pp8[3,],type="l",xlab=expression(lambda),ylab="power",lwd=0.5,lty=1,main="Pat
lines(sigma,pp8[1,],col="red",lwd=1,lty=2)
lines(sigma,pp8[2,],lwd=2,lty=2)
legend("topleft",c("LRT0","LRT1","RLRT"),lwd=c(1,2,0.5),lty=c(2,2,1),cex=0.65)

#use the multiple paired t test to test the significance of the power difference
power8 <- c(pp8[1,2:7],pp8[2,2:7],pp8[3,2:7])
group8 <-c(rep("test1",6),rep("test2",6),rep("test3",6))
pairwise.t.test(power1,group1,p.adjust="bonf",paired=TRUE)

#pattern 9 power
pp9 <- NULL
sd9 <- NULL
for(i in 1:7){
  temp1 <- lrt.test(c(1,1,1,5,5,5,9,9,9),sigma[i])$power
  pp9 <- cbind(pp9,temp1)
  temp2 <- lrt.test(c(1,1,1,5,5,5,9,9,9),sigma[i])$sd
  sd9 <- cbind(sd9,temp2)
}

#power curve
plot(sigma,pp9[3,],type="l",xlab=expression(lambda),ylab="power",lwd=0.5,lty=1,main="Pat
lines(sigma,pp9[1,],col="red",lwd=1,lty=2)
lines(sigma,pp9[2,],lwd=2,lty=2)
legend("topleft",c("LRT0","LRT1","RLRT"),lwd=c(1,2,0.5),lty=c(2,2,1),cex=0.65)

#use the multiple paired t test to test the significance of the power difference
power9 <- c(pp9[1,2:7],pp9[2,2:7],pp9[3,2:7])
group9 <-c(rep("test1",6),rep("test2",6),rep("test3",6))
pairwise.t.test(power1,group1,p.adjust="bonf",paired=TRUE)

#pattern 11 power
pp11 <- NULL
```

```
sd11 <- NULL
for(i in 1:7){
  temp1 <- lrt.test(c(1,1,1,1,1,1,1,19,19),sigma[i])$power
  pp11 <- cbind(pp11,temp1)
  temp2 <- lrt.test(c(1,1,1,1,1,1,1,19,19),sigma[i])$sd
  sd11 <- cbind(sd11,temp2)
}

#power curve
plot(sigma,pp11[3,],type="l",xlab=expression(lambda),ylab="power",lwd=0.5,lty=1,main="Pa
lines(sigma,pp11[1,],col="red",lwd=1,lty=2)
lines(sigma,pp11[2,],lwd=2,lty=2)
legend("topleft",c("LRT0","LRT1","RLRT"),lwd=c(1,2,0.5),lty=c(2,2,1),cex=0.65)

#use the multiple paired t test to test the significance of the power difference
power11 <- c(pp11[1,2:7],pp11[2,2:7],pp11[3,2:7])
group11 <-c(rep("test1",6),rep("test2",6),rep("test3",6))
pairwise.t.test(power1,group1,p.adjust="bonf",paired=TRUE)

#pattern 12 power
pp12 <- NULL
sd12 <- NULL
for(i in 1:7){
  temp1 <- lrt.test(c(2,10,18),sigma[i])$power
  pp12 <- cbind(pp12,temp1)
  temp2 <- lrt.test(c(2,10,18),sigma[i])$sd
  sd12 <- cbind(sd12,temp2)
}

#power curve
plot(sigma,pp12[3,],type="l",xlab=expression(lambda),ylab="power",lwd=0.5,lty=1,main="Pa
lines(sigma,pp12[1,],col="red",lwd=1,lty=2)
lines(sigma,pp12[2,],lwd=2,lty=2)
legend("topleft",c("LRT0","LRT1","RLRT"),lwd=c(1,2,0.5),lty=c(2,2,1),cex=0.65)

#use the multiple paired t test to test the significance of the power difference
```

```
power12 <- c(pp12[1,2:7],pp12[2,2:7],pp12[3,2:7])
group12 <-c(rep("test1",6),rep("test2",6),rep("test3",6))
pairwise.t.test(power1,group1,p.adjust="bonf",paired=TRUE)

#pattern 13 power
pp13 <- NULL
sd13 <- NULL
for(i in 1:7){
  temp1 <- lrt.test(c(3,15,27),sigma[i])$power
  pp13 <- cbind(pp13,temp1)
  temp2 <- lrt.test(c(3,15,27),sigma[i])$sd
  sd13 <- cbind(sd13,temp2)
}

#power curve
plot(sigma,pp13[3,],type="l",xlab=expression(lambda),ylab="power",lwd=0.5,lty=1,main="Pa
lines(sigma,pp13[1,],col="red",lwd=1,lty=2)
lines(sigma,pp13[2,],lwd=2,lty=2)
legend("topleft",c("LRT0","LRT1","RLRT"),lwd=c(1,2,0.5),lty=c(2,2,1),cex=0.65)

#use the multiple paired t test to test the significance of the power difference
power13 <- c(pp13[1,2:7],pp13[2,2:7],pp13[3,2:7])
group13 <-c(rep("test1",6),rep("test2",6),rep("test3",6))
pairwise.t.test(power1,group1,p.adjust="bonf",paired=TRUE)
```

References

[1] C.M. Crainiceanu and D. Ruppert. Likelihood ratio tests in linear mixed models with one variance component. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 66(1):165185, 2003.

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