# Fused Adaptive Lasso for Spatial and Temporal Quantile Function Estimation via an MM Algorithm

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## 1. Introduction

Meteorological time series forecast has received tremendous attention in the world because of the uncertainty of climate change. As an important branch of nonlinear science, chaos theory, which has rich connotation and extensive denotation, provides a popular way to study this kind of highly complex systems.

SET pair analysis is an effective method of dealing with the certain-uncertain information. Connection degree is the core in the set pair analysis, which is made up of identical degree, discrepant degree and contrary degree to describe the similarity of a set pair from three characteristics (identical, discrepant and contrary characteristic). The set pairs with the higher connection degree are considered as similar. The refined connection degree is used to select the nearest neighbor points.

In this project, we established a New Weighted Local model based on the Refined Connection Degree (NWL-RCD). NWL-RCD provides a more reasonable selection of the nearest neighbor points and avoids uncertainty in human decision-making for the optimal number of the nearest neighbor points existed in the traditional local prediction method. A case of forecasting the extreme minimum temperature are studied by using NWL-RCD. Compared with the simulation results by traditional local linear prediction method (LLP) and rank set pair analysis (R-SPA), it indicates that the forecast precision of this time series is improved by the NWL-RCD with regard to the mean absolute error (MAE) and the root mean square error (RMSE).

## 2. Objective

Our goal is to establish a new weighted local model based on the refined connection degree (NWL-RCD) in order to overcome the drawbacks in nearest neighbor points selection of the traditional local prediction method (LLP).

## 3. Methodology

### 3.1 Set Construction

For time series $x_1, x_2, ...., x_n$, We apply the autocorrelation function and Cao method to obtain delay time $\tau$ and the minimum embedding dimension $m$ respectively. Then we construct the history sets $A_1, A_2, ..., A_{n-T}$and current set $B$. The reconstructed vector $A_k$ are as follows:

$$A_k = (x_k, x_{k+\tau}, ..., x_{k+(m-1)\tau})$$

The reconstructed space is:

| History Set | Elements in Set | Subsequent Value |
|---|---|---|
| $A_1$ | $x_1, x_{1+\tau}, ..., x_{1+(m-1)\tau}$ | $x_{1+(m-1)\tau+1}$ |
| $A_2$ | $x_2, x_{2+\tau}, ..., x_{2+(m-1)\tau}$ | $x_{2+(m-1)\tau+1}$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $A_{n-(m-1)\tau+1}$ | $x_{n-(m-1)\tau+1}, ..., x_{n-1}$ | $x_n$ |
| $B$ | $x_{n-(m-1)\tau}, ..., x_n$ | $\boxed{x_{n+1}}$ |

**Table 1:** *Reconstructed set space*

### 3.2 New Rank Transformation

By Max-Min Normalization method, the rank sets $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_{n-(m-1)\tau+1}, \tilde{B}$ can be obtained from the history sets $A_1, A_2, \dots, A_{n-(m-1)\tau+1}$ and current set $B$ by the following function

$$f(x_i) = \frac{\Delta_{max}^{(i)}}{\Delta_{max}}$$

, where $x_i$ is the $i$th element in A $\Delta_{max}^{(i)} = |\max_{1\le j\le m}(x_i - x_j)|$, and $\Delta_{max} = \max_{1\le i,j\le m}|(x_i - x_j)|, i = 1, 2, \dots, m.$

The refined connection degree of $(A_k, B)$ is

$$u_k = U_{A_k - B} = \frac{1}{m}\sum_{j=1}^{m} i_j = \frac{1}{m}\sum_{j=1}^{m}(-\frac{\Delta_{Bk}^{(j)}}{0.5} + 1) \quad (1)$$

$$\Delta_{Bk}^{(j)} = |f(x_j) - f(b_j)|, x_j \in A_k, b_j \in B$$

### 3.3 Prediction Process

In this study, $x_{k+(m-1)\tau+1}$ is fitted by the linear function,

$$y_k = x_{k+(m-1)\tau+1} = \beta_0 + \sum_{i=1}^{m}\beta_i x_{k+(i-1)\tau} \quad (2)$$

Using the refined connection degree $\{u_k\}$, we select $q$ points which have biggest $u_k, k = 1, \dots, q$ as $q$ nearest neighbor points, then construct the weighted distance matrix $P$

$$P = \begin{pmatrix} p_1 & 0 & \cdots & 0 \\ 0 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_q \end{pmatrix}$$

, where $p_k = \frac{u_k}{\sum_q^{i=1} u_i}, k = 1, 2, \dots, q$

To estimate the coefficients of equation (2), we consider the following optimization problem

$$\beta = argmin\sum_{k=1}^{q} p_k[y_{n_k} - (\beta_0 + \sum_{i=1}^{m}\beta_i x_{n_k+(i-1)\tau})]^2 \quad (3)$$

Applying the weighted least squared method, a matrix form can be described by

$$Y \cdot P^{1/2} = \beta \cdot X \cdot P^{1/2} + \epsilon \quad (4)$$

Then we have the least squared solution with linear fitting

$$\hat{\beta} = Y(XP^{1/2})^{+} \quad (5)$$

Then we can get the estimation of sequential value using function (2)

$$\hat{x}_{n+1} = \beta_0 + \sum_{i=1}^{m}\beta_i x_{n-(m-i)\tau} \quad (6)$$

## 4. Results

The precision of the prediction results is evaluated by the Mean absolute error (MAE) and Root mean square error (RMSE). The definitions of the two measurement indices can be shown as follows:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|\hat{y}_i - y_i| \quad (7)$$

$$RMSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2 \quad (8)$$

In this section, we choose the monthly extreme minimum temperature from Miyun (Beijing) in China during the period 1989-2009, to compare NWL-RCD with LLP and R-SPA. The delaytime $\tau$=2 and embedding dimension $m$=3. Then follow section 3.2 and 3.3, prediction results for the next two years are shown as follows:

| Indices | NWL-RCD | LLP | R-SPA |
|---|---|---|---|
| MAE | 2.49386 | 6.88701 | 8.25140 |
| RMSE | 3.09199 | 13.66410 | 12.19551 |

**Table 2:** *Results comparation*

## 5. Conclusion

Application results in this meteorological time series indicate that NWL-RCD works better than both LLP and R-SPA, since NWL-RCD gives both the lowest MAE and RMSE. For extreme minimum air temperature, compared with the prediction results of LLP and R-SPA, MAE of NWL-RCD is reduced by $63.8\%$ and $69.8\%$, respectively, and RMSE is reduced by $77.4\%$ and $74.6\%$, respectively. So the forecast precision of meteorological time series is improved by NWL-RCD.

Since the weighted matrix is constructed by the distance weighted method, without taking more specific differences of the selected nearest neighbor points into account, the prediction process may be refined if some optimization algorithms are applied to the determination of the weight of each nearest neighbor point.

This new method can be used in predicting other nonlinear time series, and its theory could be further studied.