

# Global Illumination

Multi-Sampling  
Path Tracing

# Simple Sampling

- Josef talked about all of the details behind signals and sampling
- For assignment purposes, things will be a bit simpler...

# Sampling techniques

- Uniform
- Random
  - We will focus on this one today
- Jittered
  - This is not much harder than the above  
(extra credit)

# Random Sampling

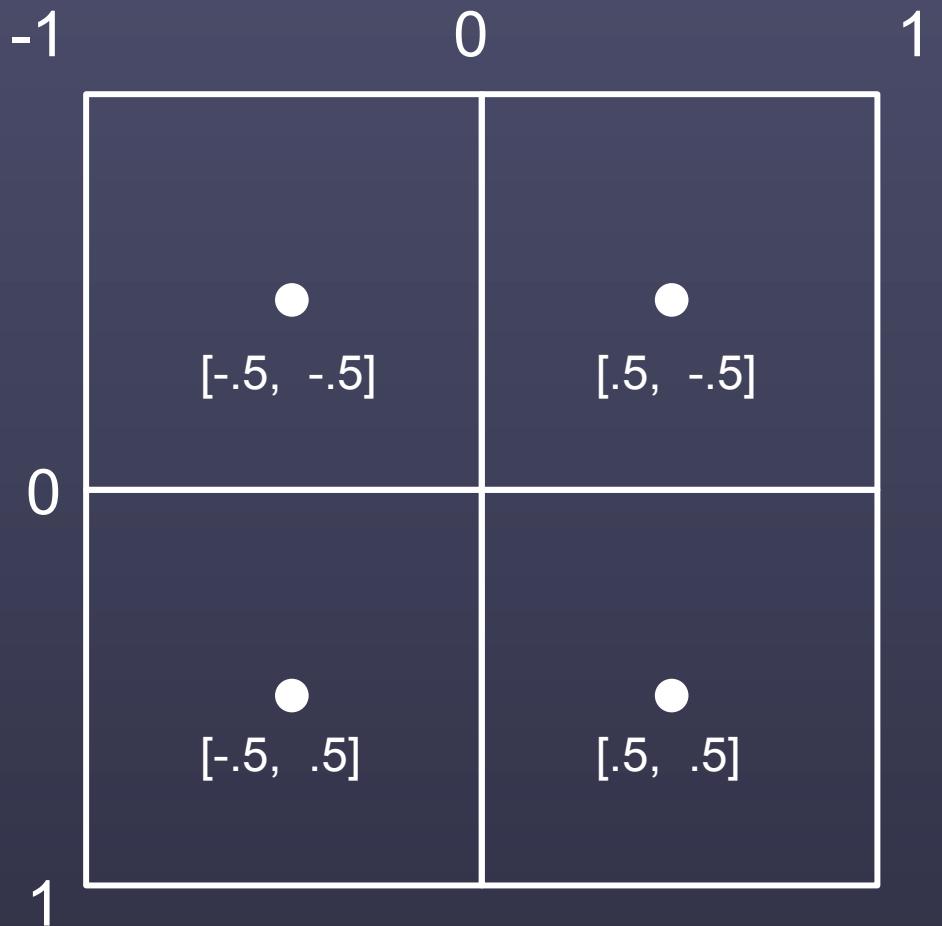
- You will find that it actually works well enough a lot of the time
- Pick a random point within the area being sampled
- Let's start by sampling in pixels

# 2x2 Image

```
for each pixel (i, j)
```

```
x = 2.0f *  
(j - xres/2.f + 0.5f)/xres;
```

```
y = 2.0f *  
(i - yres/2.f + 0.5f)/yres;
```



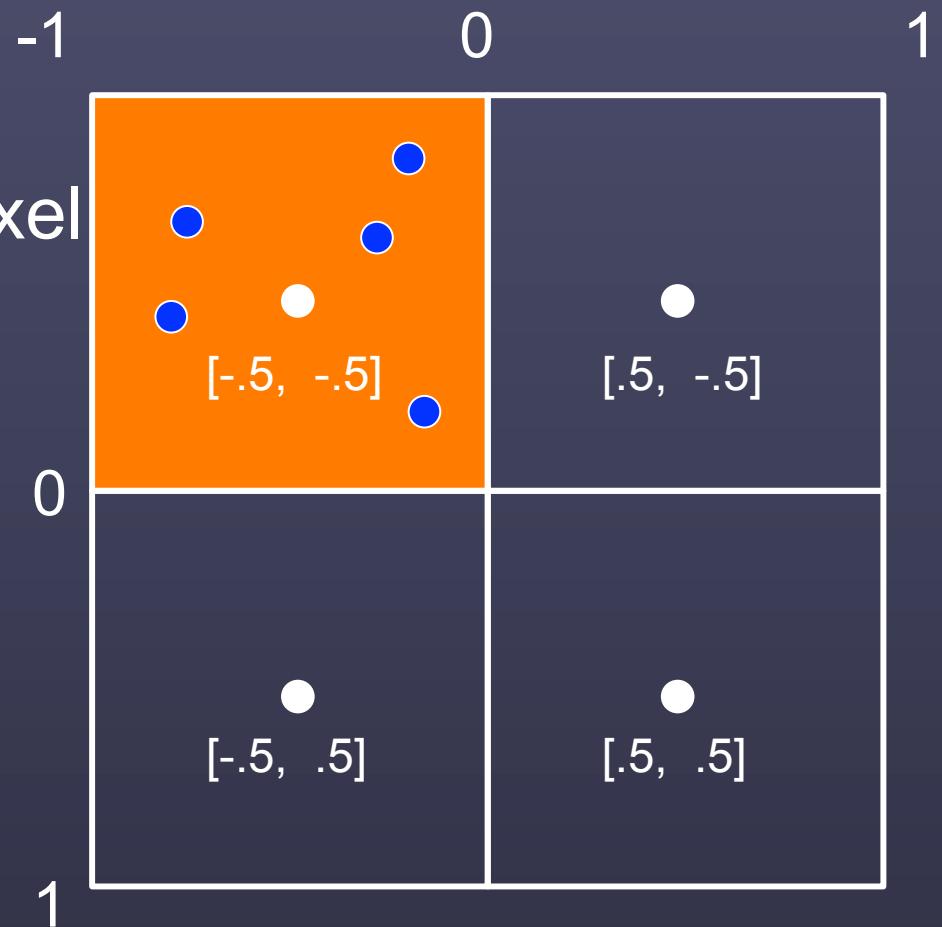
Everything in the range of  $x = [-1 .. 0]$ ,  $y = [-1 .. 0]$  falls within the same pixel

# Sampling a Pixel

Randomly offset (x, y)  
within the area of the pixel

Take as many samples  
as desired

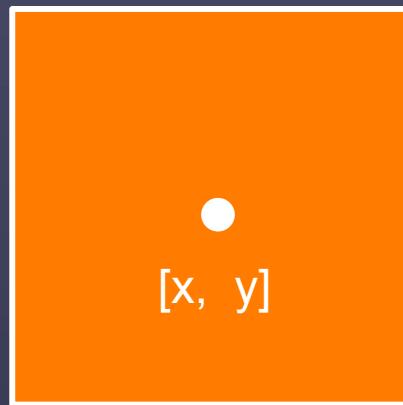
How do we offset?



# In General

$x - 1/\text{width}$ ,  
 $y - 1/\text{height}$

$x + 1/\text{width}$ ,  
 $y - 1/\text{height}$



$x - 1/\text{width}$ ,  
 $y + 1/\text{height}$

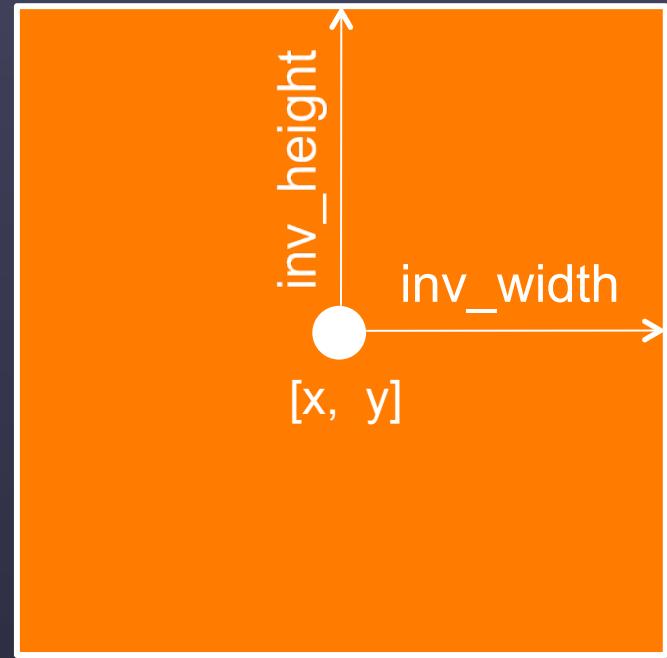
$x + 1/\text{width}$ ,  
 $y + 1/\text{height}$

`inv_width = loadf(0, 2)`

`inv_height = loadf(0, 5)`

# Random Point in Pixel

```
// value between -1 .. 1  
x_off = (trax_rand() - .5f) * 2.f  
y_off = (trax_rand() - .5f) * 2.f  
  
x_off *= inv_width  
y_off *= inv_height  
  
x += x_off  
y += y_off  
  
camera.makeRay(ray, x, y);
```



# Filtering

- Box
  - We will focus on this one today
- Triangle
  - More difficult than the above, but not terribly (requires samples outside pixel)
- Gaussian
  - Same infrastructure as triangle filter, different math

# Box Filter

- Simply means all samples are weighted equally
- For each sample, add ray contribution to the color of the pixel
  - Pixel will likely end up with  $> 1$  intensity
- Then divide color by num\_samples
  - then clamp to 0 .. 1

# Putting it all together

```
for(pixels)
    for(num_samples)
        camera.makeRay(ray, x, y) // x and y
                                    have been
                                    randomly
                                    permuted

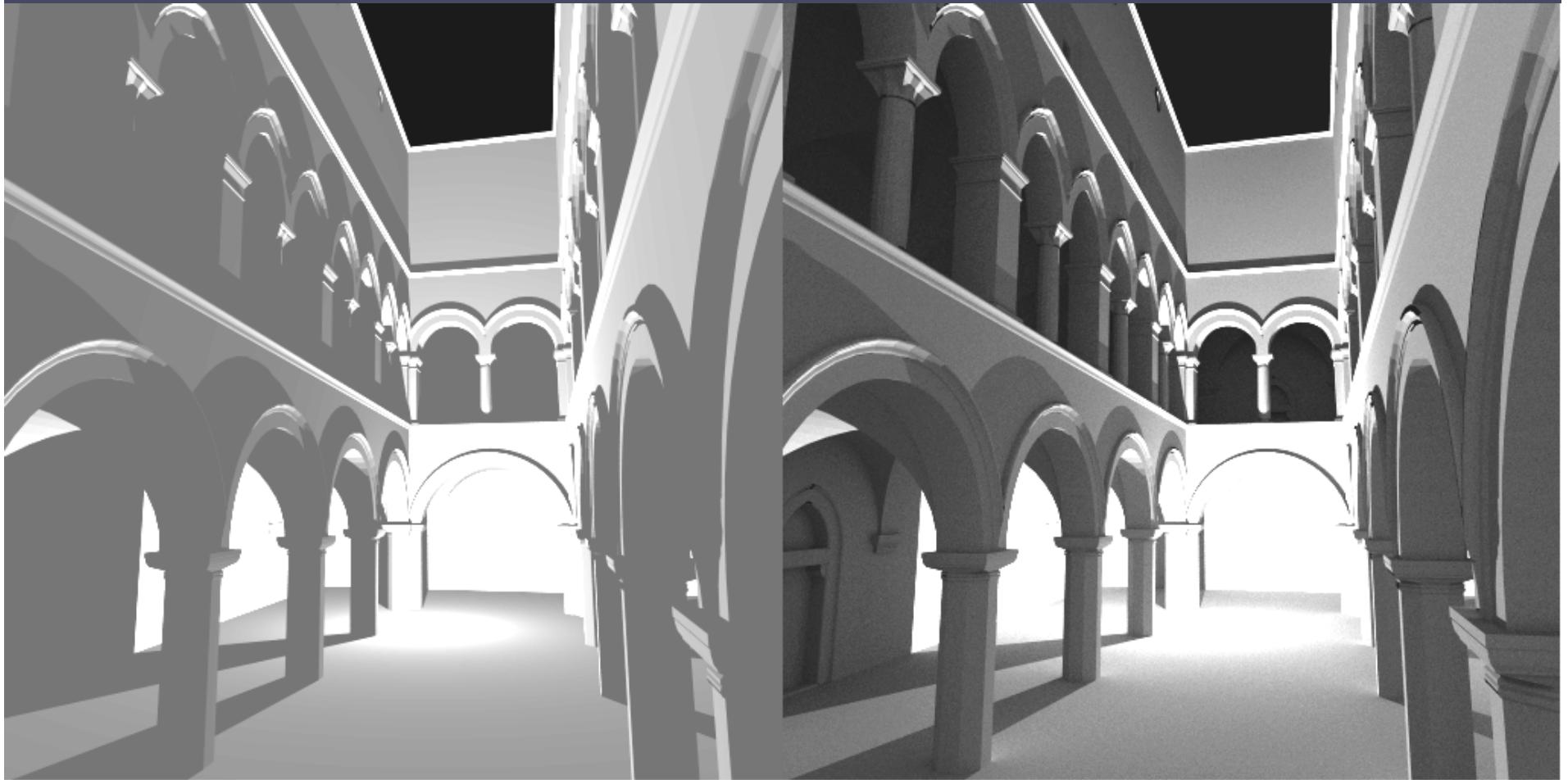
        bvh.intersect(hit, ray)
        result += shade(...) // +=, not =

    result /= num_samples // box filter
    image.set(i, j, result);
```

# Global Illumination

- So far, we have looked at light from specific sources
  - Light source
  - Reflections
  - Refractions
- In reality, it isn't this simply
  - Still using “ambient” term for everything not in direct light

# Global Illumination





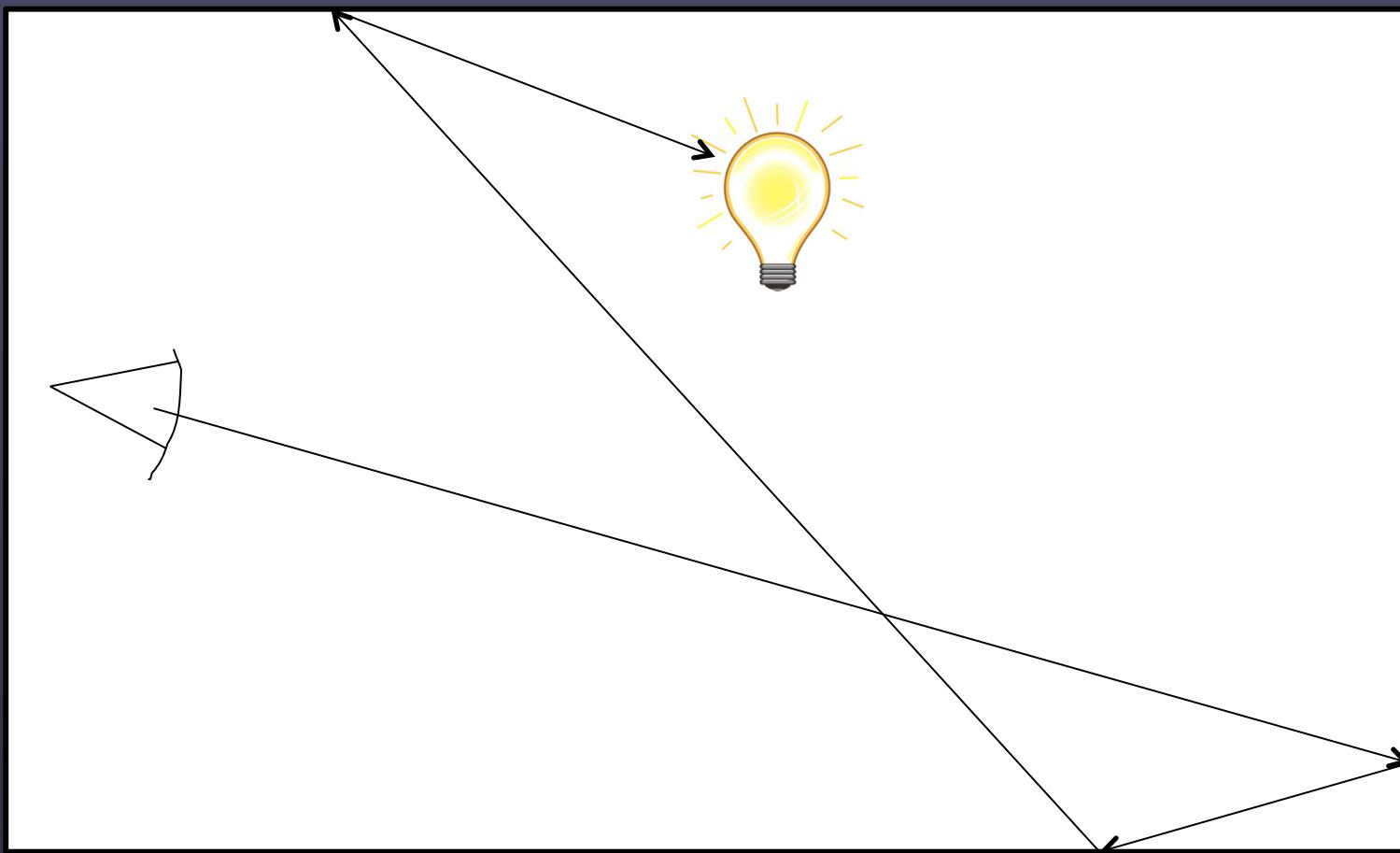
# Global Illumination

- Metropolis
- Ambient Occlusion
- Photon Mapping
- Path Tracing
  - arguably the most straight-forward
- Others...

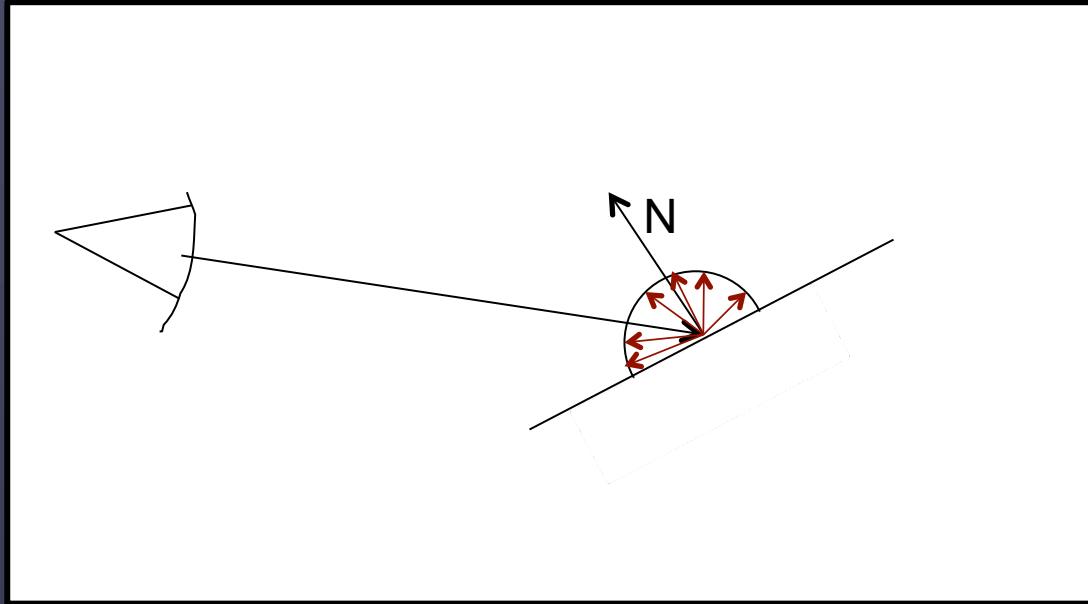
# Path Tracing

- Pure path tracing is the most naïve solution to global illumination
  - Also the most elegant (my opinion)
- Path tracing for Lambertian shading
  1. Cast a ray from the camera
  2. Multiply attenuation by material
    - From that point, cast exactly 1 ray in a random direction
  3. Repeat step 2 until light source is hit
  4. Final color = attenuation \* emitted light

# Path Tracing



# Random reflection direction



- Pick a random direction on the normal hemisphere
- How?

# Orthonormal basis

- First, we need to find a set of orthonormal axes based on the normal
  - This will be sort of like a “camera”
- Set the z axis in our new basis equal to the normal
  - Find any x and y orthogonal to z and unit length (“orthonormal”)
  - How?

# Orthonormal basis

- Remember, cross product returns a vector that is perpendicular to both input vectors

```
Vector Z = normal;  
cross(N, any vector)
```

Pick one of  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$

- This result will be perpendicular to the normal
  - But what if the vector we pick is parallel to the normal?
  - Result will be zero vector

# Orthonormal basis

- Choose axis with smallest component in normal

```
if(N.x < N.y && N.x < N.z)
    { axis = vec(1.0f, 0.0f, 0.0f); }
else if (N.y < N.z)
    { axis = vec(0.0f, 1.0f, 0.0f); }
else
    { axis = vec(0.0f, 0.0f, 1.0f); }
```

# Orthonormal basis

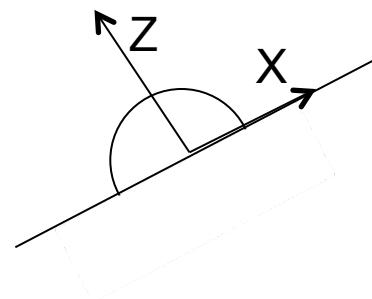
- Last axis is cross product of other two

```
X = normal.cross(axis).normalize()
```

```
Y = normal.cross(X)
```

- Now we have a new axis system
  - X and Y are tangent to the surface
  - Z is normal to the surface

# Orthonormal basis



Y not drawn in 2D example

# Hemisphere sampling

- Pick a random vector on the unit hemisphere defined by our new basis
- Option 1:
  - Define the “hemicube” (half cube) on the surface
  - Randomly pick points inside the cube until we get one that is inside the hemisphere
  - Will be uniformly distributed (actually a bad thing)
- Option 2:
  - Randomly pick points on the unit disc
  - Project out to hemisphere
  - Not uniformly distributed (more later)

# Hemisphere sampling

- Pick random point inside the unit disc:

```
do
{
    u = trax_rand()
    v = trax_rand()
    u *= 2.0f;
    u -= 1.0f;
    v *= 2.0f;
    v -= 1.0f;
    u_2 = u * u;
    v_2 = v * v;
}
while((u_2 + v_2) >= 1.0f);
```

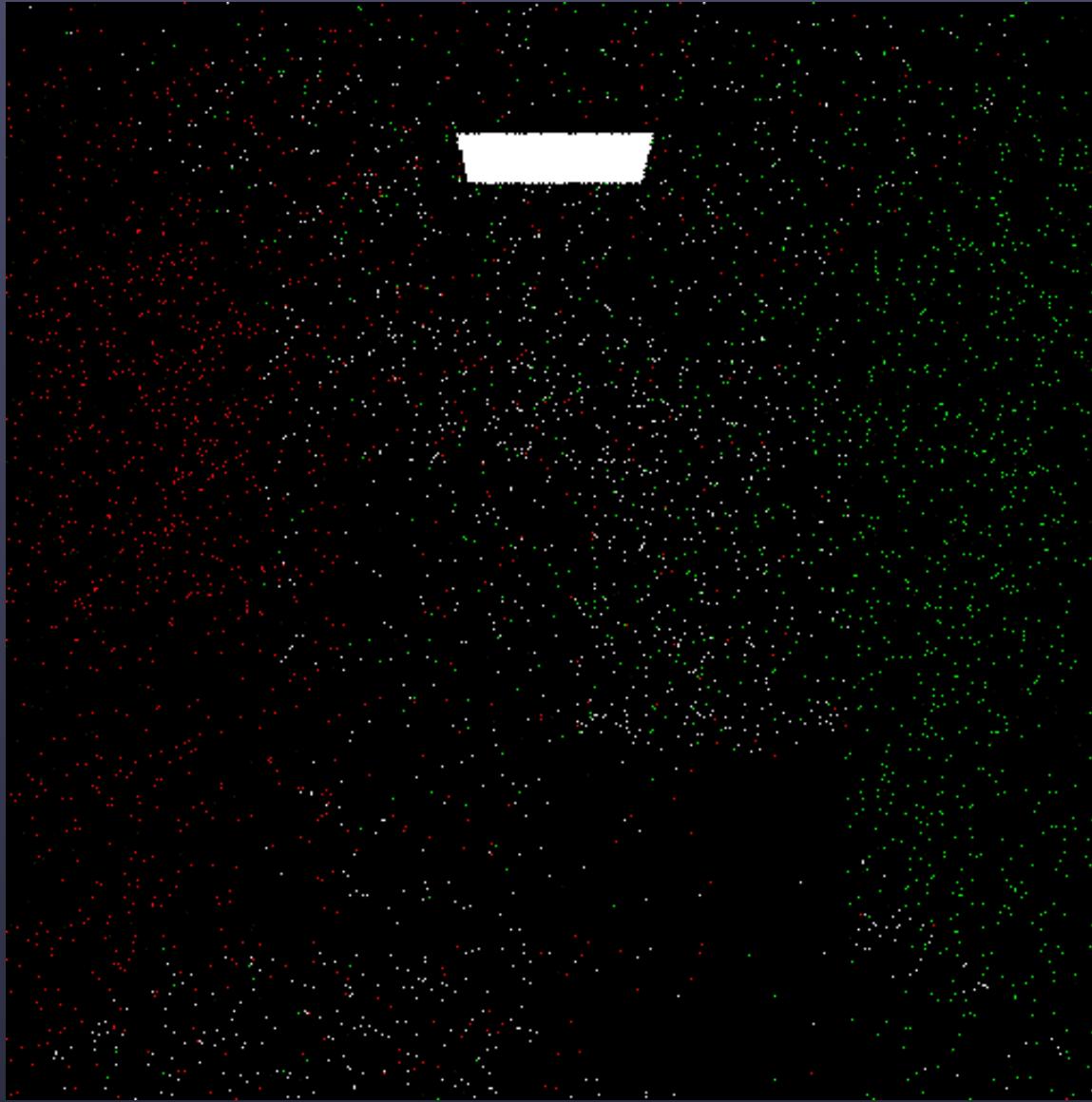
# Path Tracing

- We now have a vector ( $u, v$ ) on the (X, Y) plane
- Need to project up Z axis to make unit length (this will be a point on unit hemisphere)
- We know length needs to be 1.0
  - $w = \sqrt{1 - u^2 - v^2}$
- $\text{refDir} = (X * u) + (Y * v) + (\text{normal} * w)$

# Cosine weighting

- This generates samples weighted more heavily towards the normal
- Specifically, weighted by the cosine of the angle between the reflected ray and the normal
- Lambertian shading says we should multiply incoming light by cosine of angle
  - With samples cosine-weighted, we don't need to

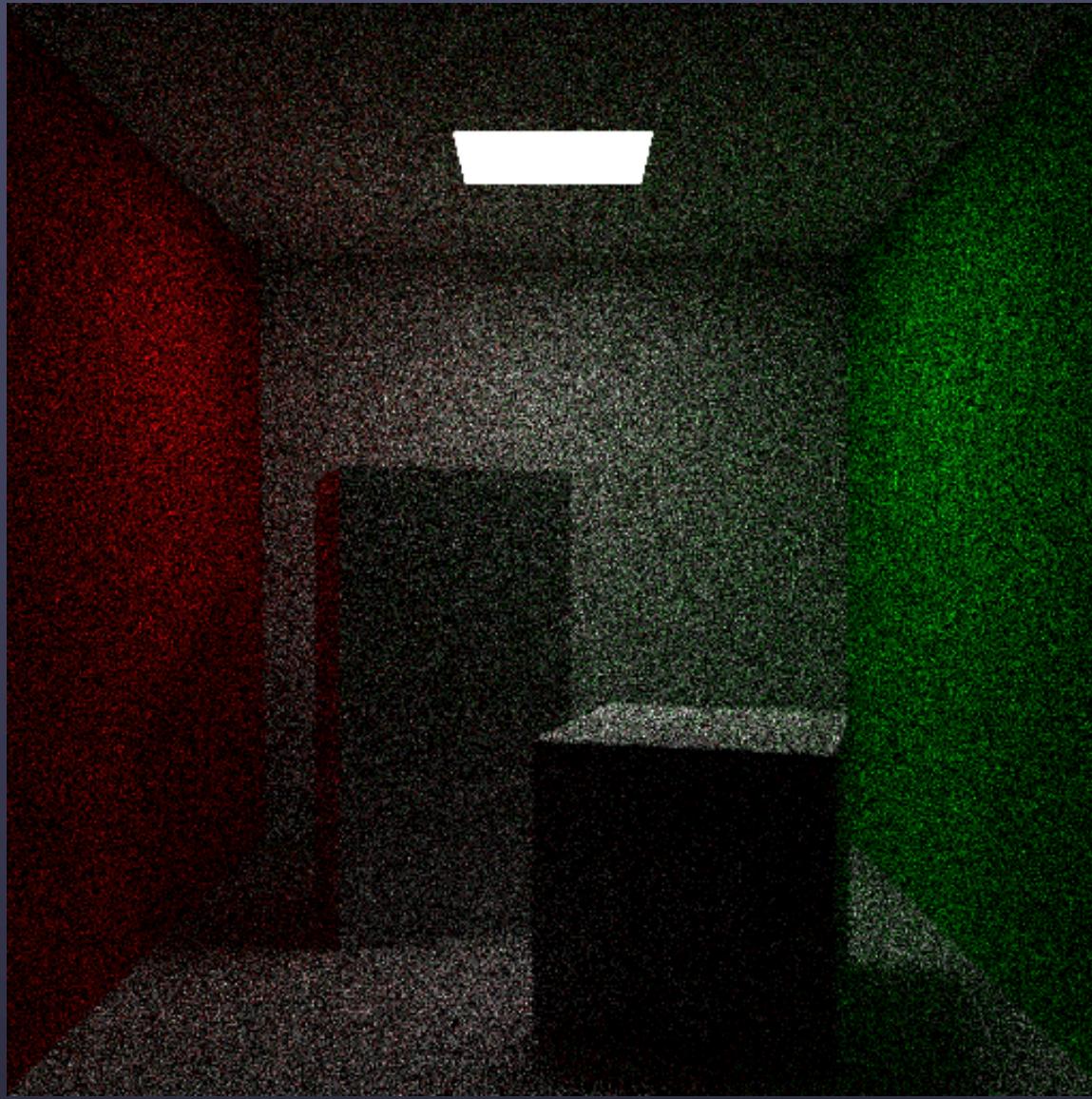
# Path Tracing



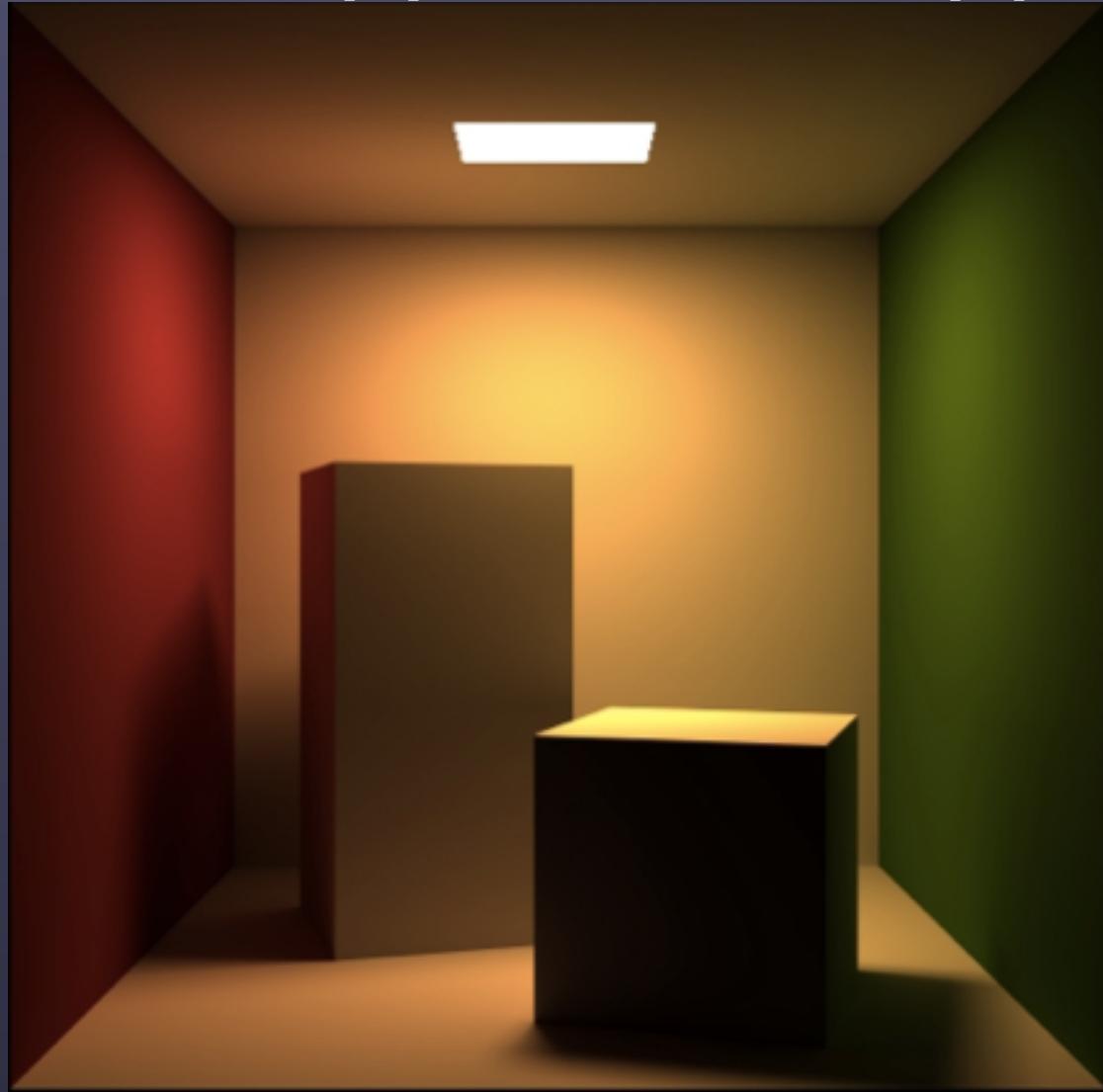
# Path Tracing

- Obviously we need more than 1 sample per pixel
- With more samples, the image begins to converge to the “correct” result
  - In practice, requires more than is reasonable
  - Unless you have hours, even days to wait

# 100 samples per pixel



100k spp, tone mapped



# Sampling

- Two techniques:
  1. Take multiple GI samples per hit point
  2. Take 1 GI sample per hit point, increase samples per pixel
- Option 2 will provide anti-aliasing at the same time
  - But we also may not need that many primary ray samples
- Option 1 muddies implementation a bit

# Path Length

- In an enclosed space, a path may bounce forever
- Need some way of terminating “useless” paths
  - Russian roulette
  - Max depth
  - Min attenuation

# Attenuation

- Every time a ray bounces, it is attenuated by that material (loses energy)
- Start with attenuation (color) of (1, 1, 1), multiply it by color of hit material on each bounce
- If total energy becomes less than small amount, kill the path

# Importance sampling

- Probabilistically, most paths of light will not hit a light source
- These paths don't contribute anything to the image, and are wasted work
- With point light sources, no light will be found whatsoever
- Kajia path tracing combines pure path tracing with direct Lambertian shading

# Kajia path tracing

```
...  
for each sample  
    attenuation = Color(1.f, 1.f, 1.f)  
    Ray r = cameraRay(...)  
    while(depth < max_depth) {  
        HitRecord hit  
        bvh.intersect(hit, ray)  
        result += shade(...) * attenuation  
        attenuation *= mat_color  
        r = hemiRay(...)  
        depth++  
    }
```

# Kajia path tracing

- The previous pseudo code doesn't stop when hitting a light
- In TRaX, we will never hit the light (point lights only)
  - Pure path tracing won't work with point lights
- Must not be recursive!

# Kajia path tracing

- Kajia path tracing samples a random light source directly (not all of them)
  - We only have one anyway
- Sampling light sources directly does not account for visible intensity of light
  - May be obscured slightly
  - May be far away
  - Can't handle transparent materials

# Pure Path Tracing

- Automatically solves various problems
  - Caustics
  - Visible intensity of light sources
- Simplifies architecture
  - No longer need “Light” objects (use emissive term in material)
- Requires bajillions of samples to converge
  - Probability of path hitting a light source is low

# Pure Path Tracing

Total energy in the scene will be low – based on probability of hitting a light...

Need some kind of tone mapping to bring things in to reasonable range



# Tone-Mapped

Exact same  
information as  
previous image



# Free caustics

