Selected Topics in Nature-inspired Algorithms

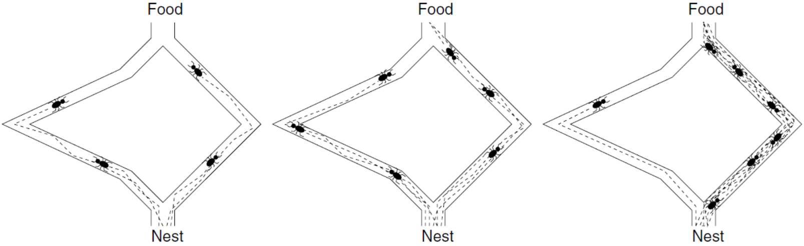
Seminar

Summer 2019

Nico Potyka

Ant Colony Optimization

The Double Bridge Experiment[1]



- argentine ants *iridomyrmex humilis* mark their path to food source via pheromone trails
- in most runs of experiment, nearly all ants take shortest path after a few minutes
- Explanation: pheromone accumulates faster on shorter branch

[1] Goss, S., Aron, S., Deneubourg, J. L., & Pasteels, J. M. (1989). Self-organized shortcuts in the Argentine ant. Naturwissenschaften, 76(12): 579-581.

Ant Colony Optimization: Idea

- ants construct solution for optimization problem through a sequence of decisions
- sequence of decisions can be viewed as path through decision graph (construction graph)
- decision is made probabilistically based on pheromone values
- ants that found good solutions, increase pheromone values
- pheromone also evaporates

Basic Ant Colony Optimization Algorithm

initialize pheromone values

do

  for *ant* $k \in \{1, \ldots, N\}$*:*

    construct a solution

  for all *pheromone values* :

    decrease the value *(evaporation)*

  for all *pheromone values corresponding to good solutions* :

    increase the value *(intensification)*

until *termination condition reached*

return *best solution*

Example: TSP

Solution Construction

- start from city 1
- let $S = \{2, \ldots, n\}$ denote the remaining cities (selectable items)
- ants successively remove elements from S to build up tour
- selection probability depends on pheromone values

Pheromone Matrix

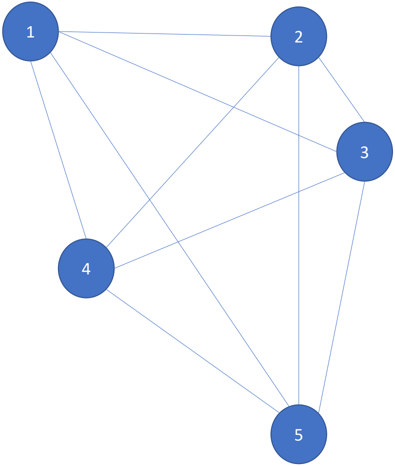
- $n \times n$ pheromone matrix $A = (a_{i,j})$
- entry $a_{i,j}$ is the pheromone value for selecting city j after city i
- initially, we set all entries to 1

Distance Matrix D

0	2	3	2	4
2	0	1	3	4
3	1	0	3	3
2	3	3	0	2
4	4	3	2	1

Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	



Selection Probability

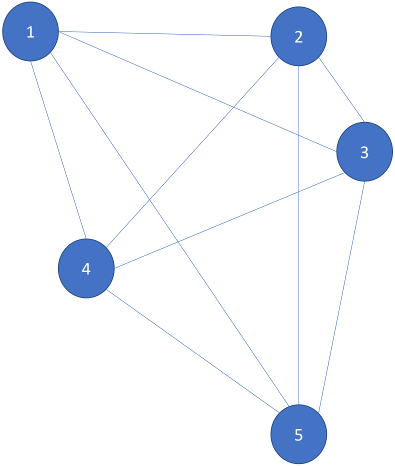
- assume, we selected city i before
- the next city is selected in a pheromone-proportionate manner
- the probability of selecting city j next is

$$p_j = \frac{a_{i,j}}{N},$$

where $N = \sum_{k \in S} a_{i,k}$

Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	



S = {2, 3, 4, 5}

$p_2 = \frac{1}{4}$

$p_3 = \frac{1}{4}$

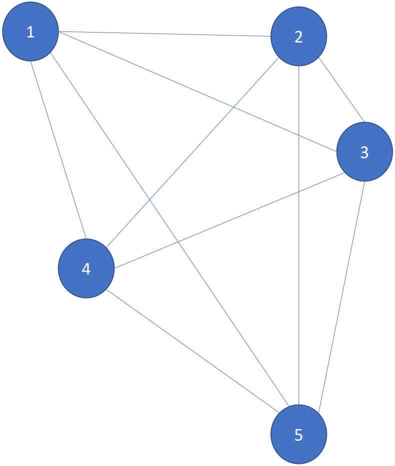
$p_4 = \frac{1}{4}$

$p_5 = \frac{1}{4}$

Solution: ()

Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	



S = {2, 4, 5}

$\mathbf{p_2 = \frac{1}{4}}$

$\color{red}{\mathbf{p_3 = \frac{1}{4}}}$

$\mathbf{p_4 = \frac{1}{4}}$

$\mathbf{p_5 = \frac{1}{4}}$

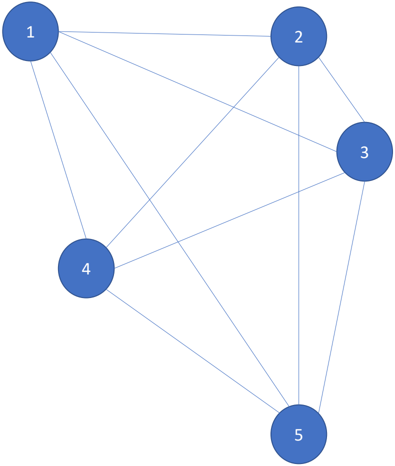
Solution: (3)

Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	

S = {2, 4, 5}

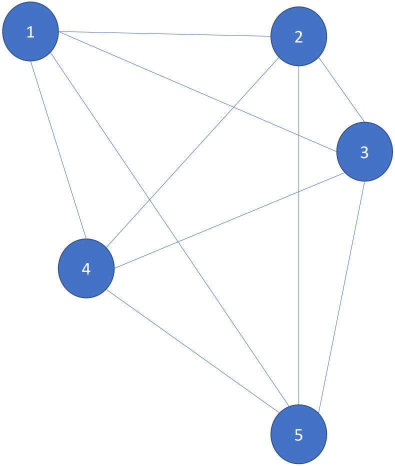
$$p_2 = \frac{1}{3}$$
$$p_4 = \frac{1}{3}$$
$$p_5 = \frac{1}{3}$$



Solution: (3)

Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	



S = {2, 4}

$\mathbf{p_2 = \frac{1}{3}}$

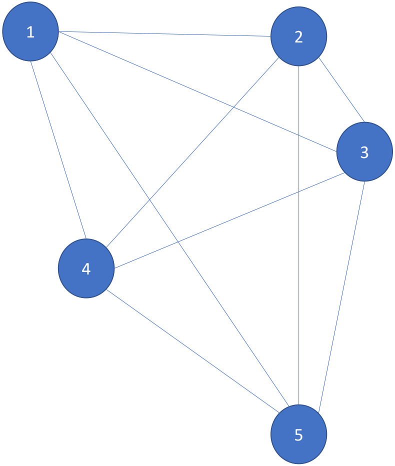
$\mathbf{p_4 = \frac{1}{3}}$

$\color{red}{\mathbf{p_5 = \frac{1}{3}}}$

Solution: (3, 5)

Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	



S = {2, 4}

$\mathbf{p_2 = \frac{1}{2}}$

$\mathbf{p_4 = \frac{1}{2}}$

Solution: (3, 5)

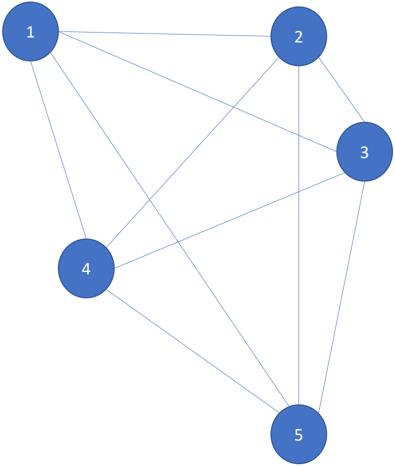
Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	

S = {4}

$p_2 = \frac{1}{2}$

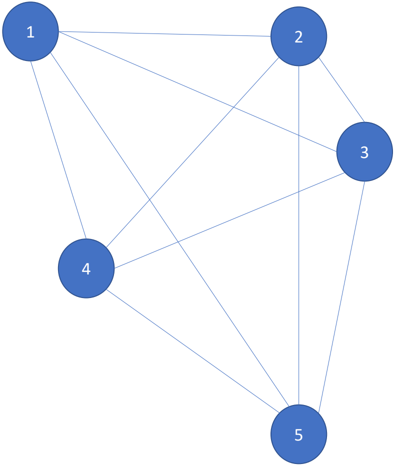
$p_4 = \frac{1}{2}$



Solution: (3, 5, 2)

Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	



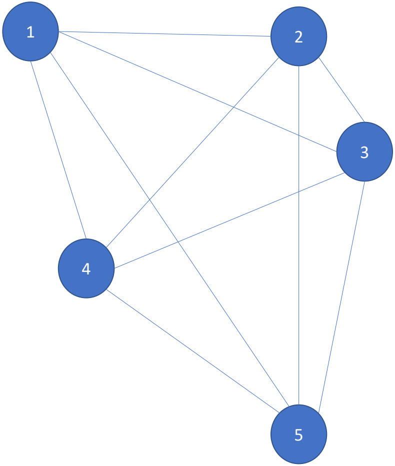
S = {4}

$$p_4 = \frac{1}{1}$$

Solution: (3, 5, 2)

Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	



S = {}

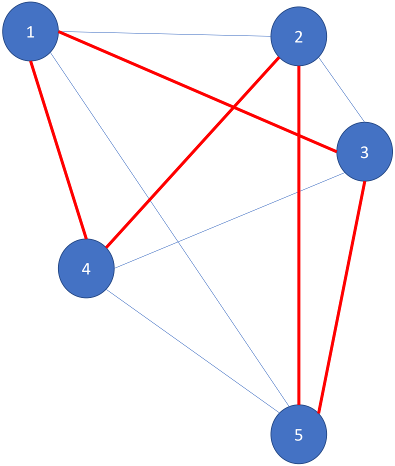
$p_4 = \frac{1}{1}$

Solution: (3, 5, 2, 4)

Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	

S = {}



Solution: (3, 5, 2, 4)

Pheromone Update

1. Evaporation: reduce all values by a fixed proportionate $\rho \in (0, 1)$

$$a_{i,j} := a_{i,j} \cdot \rho \qquad \textit{for all } 1 \leq i, j \leq n$$

Pheromone Update

1. Evaporation: reduce all values by a fixed proportionate $\rho \in (0, 1)$

$$a_{i,j} := a_{i,j} \cdot \rho \qquad \textit{for all } 1 \leq i, j \leq n$$

2. Intensification: increase all values corresponding to best solution s by absolute amount $\Delta > 0$

$$a_{i,j^*} := a_{i,j^*} + \Delta \qquad \textit{for best choice } j^* \textit{ from } i$$

Evaporation ρ=0.9

Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	

	0.9	0.9	0.9	0.9
0.9		0.9	0.9	0.9
0.9	0.9		0.9	0.9
0.9	0.9	0.9		0.9
0.9	0.9	0.9	0.9	

Solution Evaluation

Distance Matrix D

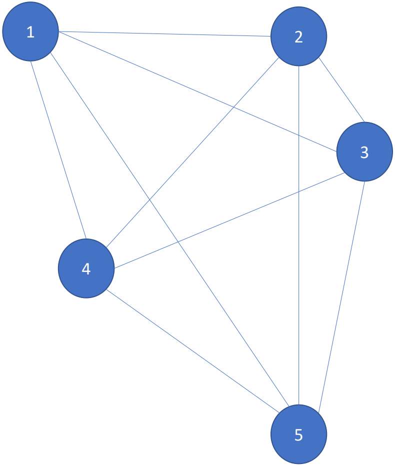
0	2	3	2	4
2	0	1	3	4
3	1	0	3	3
2	3	3	0	2
4	4	3	2	1

(3, 5, 2, 4)

(2, 4, 5, 3)

(3, 4, 5, 2)

(5, 2, 4, 3)



Solution Evaluation

Distance Matrix D

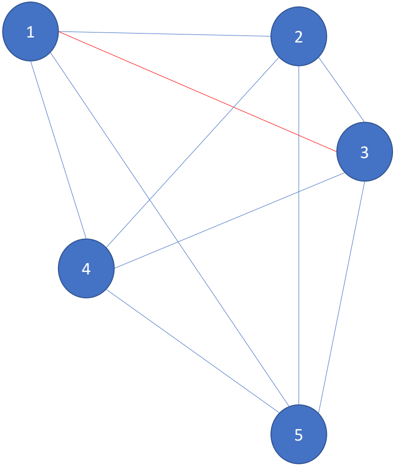
0	2	3	2	4
2	0	1	3	4
3	1	0	3	3
2	3	3	0	2
4	4	3	2	1

(3, 5, 2, 4): 3

(2, 4, 5, 3)

(3, 4, 5, 2)

(5, 2, 4, 3)



Solution Evaluation

Distance Matrix D

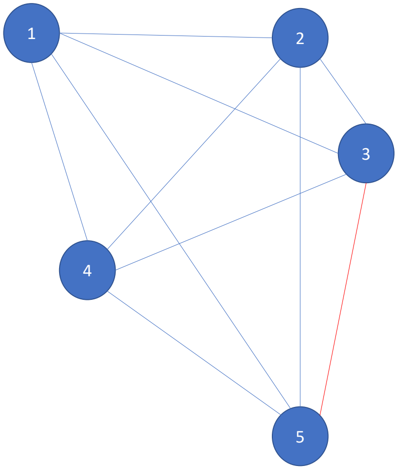
0	2	3	2	4
2	0	1	3	4
3	1	0	3	3
2	3	3	0	2
4	4	3	2	1

(3, 5, 2, 4): 3+3

(2, 4, 5, 3)

(3, 4, 5, 2)

(5, 2, 4, 3)



Solution Evaluation

Distance Matrix D

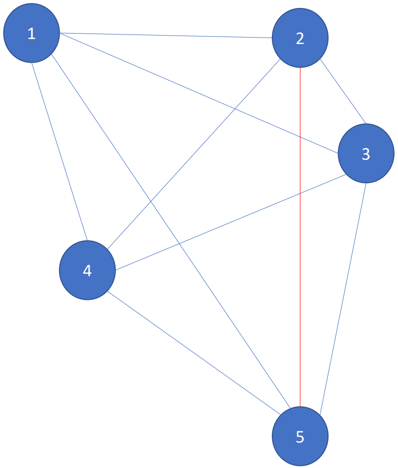
0	2	3	2	4
2	0	1	3	4
3	1	0	3	3
2	3	3	0	2
4	4	3	2	1

(3, 5, 2, 4): 3+3+4

(2, 4, 5, 3)

(3, 4, 5, 2)

(5, 2, 4, 3)



Solution Evaluation

Distance Matrix D

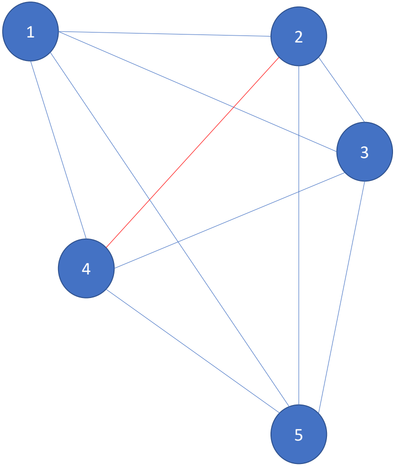
0	2	3	2	4
2	0	1	3	4
3	1	0	3	3
2	3	3	0	2
4	4	3	2	1

(3, 5, 2, 4): 3+3+4+3

(2, 4, 5, 3)

(3, 4, 5, 2)

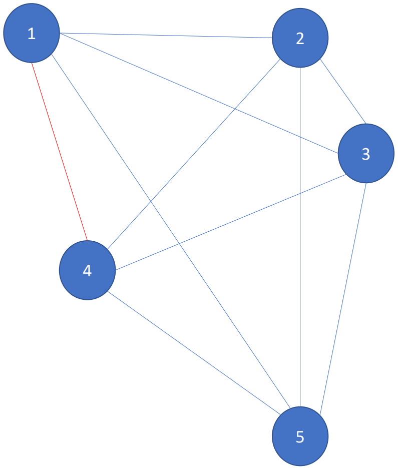
(5, 2, 4, 3)



Solution Evaluation

Distance Matrix D

0	2	3	2	4
2	0	1	3	4
3	1	0	3	3
2	3	3	0	2
4	4	3	2	1



(3, 5, 2, 4): 3+3+4+3+2

(2, 4, 5, 3)

(3, 4, 5, 2)

(5, 2, 4, 3)

Solution Evaluation

Distance Matrix D

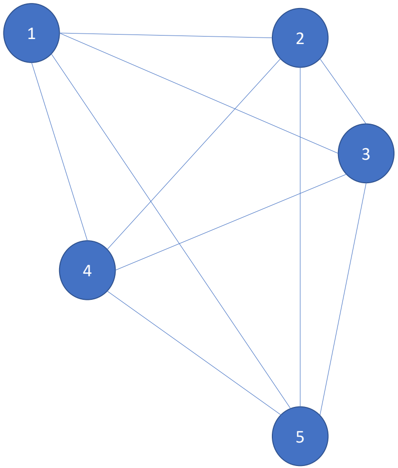
0	2	3	2	4
2	0	1	3	4
3	1	0	3	3
2	3	3	0	2
4	4	3	2	1

(3, 5, 2, 4): 3+3+4+3+2 = 15

(2, 4, 5, 3)

(3, 4, 5, 2)

(5, 2, 4, 3)



Solution Evaluation

Distance Matrix D

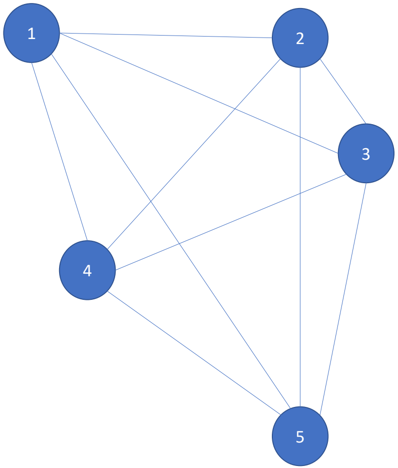
0	2	3	2	4
2	0	1	3	4
3	1	0	3	3
2	3	3	0	2
4	4	3	2	1

(3, 5, 2, 4): 3+3+4+3+2 = 15

(2, 4, 5, 3): 2+3+2+3+3 = 13

(3, 4, 5, 2): 3+3+2+4+2 = 14

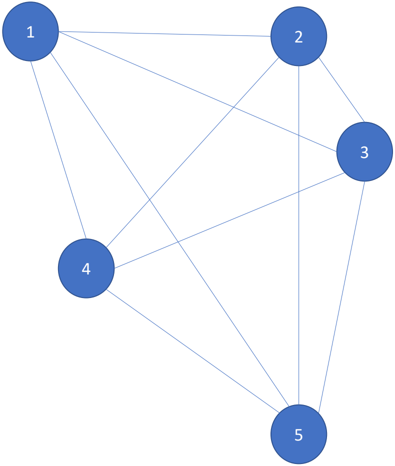
(5, 2, 4, 3): 4+4+3+3+3 = 17



Solution Evaluation

Distance Matrix D

0	2	3	2	4
2	0	1	3	4
3	1	0	3	3
2	3	3	0	2
4	4	3	2	1



(3, 5, 2, 4): 3+3+4+3+2 = 15

(2, 4, 5, 3): 2+3+2+3+3 = 13

(3, 4, 5, 2): 3+3+2+4+2 = 14

(5, 2, 4, 3): 4+4+3+3+3 = 17

Intensification **Δ=0.1**

Best Solution (2, 4, 5, 3)

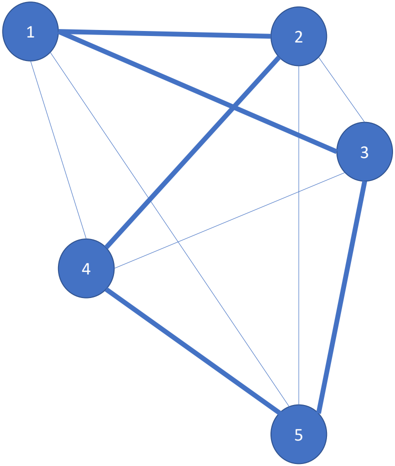
Pheromone Matrix A

	0.9	0.9	0.9	0.9
0.9		0.9	0.9	0.9
0.9	0.9		0.9	0.9
0.9	0.9	0.9		0.9
0.9	0.9	0.9	0.9	

	1	0.9	0.9	0.9
0.9		0.9	1	0.9
1	0.9		0.9	0.9
0.9	0.9	0.9		1
0.9	0.9	1	0.9	

Pheromone Matrix A

	1	0.9	0.9	0.9
0.9		0.9	1	0.9
1	0.9		0.9	0.9
0.9	0.9	0.9		1
0.9	0.9	1	0.9	



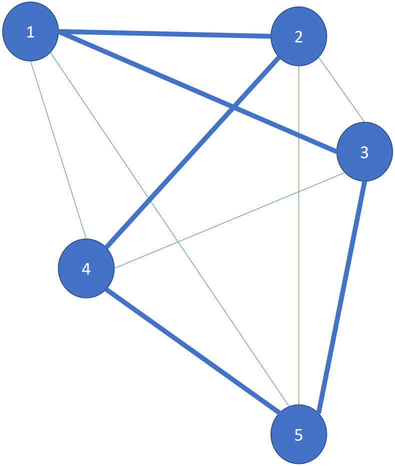
S = {2, 3, 4, 5}

$$p_2 = \frac{1}{3.7} \approx 0.27$$
$$p_3 = \frac{0.9}{3.7} \approx 0.24$$
$$p_4 = \frac{0.9}{3.7} \approx 0.24$$
$$p_5 = \frac{0.9}{3.7} \approx 0.24$$

Solution: ()

Pheromone Matrix A

	1	0.9	0.9	0.9
0.9		0.9	1	0.9
1	0.9		0.9	0.9
0.9	0.9	0.9		1
0.9	0.9	1	0.9	



$S = \{2, 3, 5\}$

$p_2 = \frac{1}{3.7} \approx 0.27$

$p_3 = \frac{0.9}{3.7} \approx 0.24$

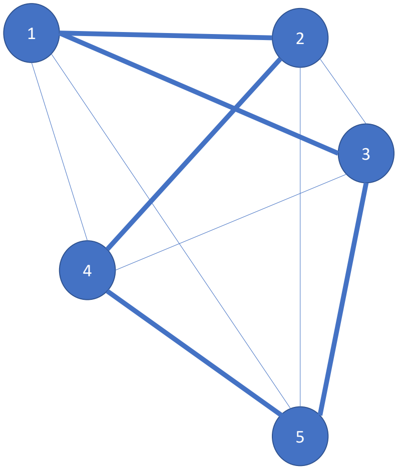
$p_4 = \frac{0.9}{3.7} \approx 0.24$

$p_5 = \frac{0.9}{3.7} \approx 0.24$

Solution: (4)

Pheromone Matrix A

	1	0.9	0.9	0.9
0.9		0.9	1	0.9
1	0.9		0.9	0.9
0.9	0.9	0.9		1
0.9	0.9	1	0.9	



S = {2, 3, 5}

$p_2 = \frac{0.9}{2.8} \approx 0.32$

$p_3 = \frac{0.9}{2.8} \approx 0.32$

$p_5 = \frac{1}{2.8} \approx 0.36$

Solution: (4)

Pheromone Matrix A

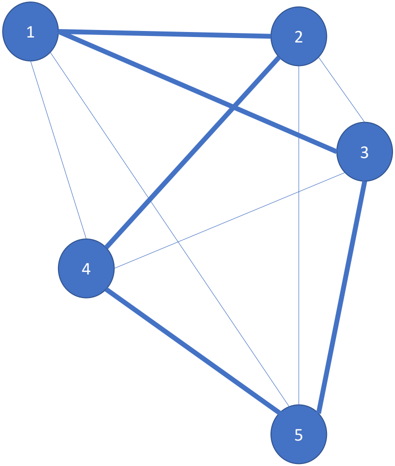
	1	0.9	0.9	0.9
0.9		0.9	1	0.9
1	0.9		0.9	0.9
0.9	0.9	0.9		1
0.9	0.9	1	0.9	

S = {2, 3}

$p_2 = \frac{0.9}{2.8} \approx 0.32$

$p_3 = \frac{0.9}{2.8} \approx 0.32$

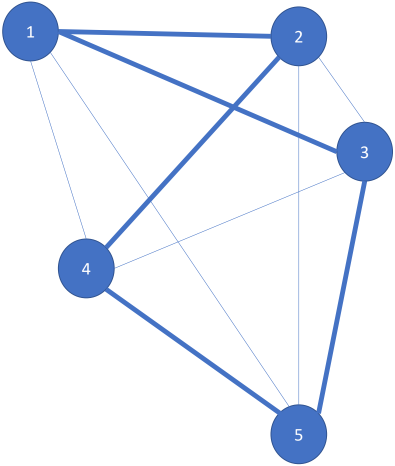
$p_5 = \frac{1}{2.8} \approx 0.36$



Solution: (4,5)

Pheromone Matrix A

	1	0.9	0.9	0.9
0.9		0.9	1	0.9
1	0.9		0.9	0.9
0.9	0.9	0.9		1
0.9	0.9	1	0.9	



S = {2, 3}

$$p_2 = \frac{0.9}{1.9} \approx 0.47$$

$$p_3 = \frac{1}{1.9} \approx 0.53$$

Solution: (4,5)

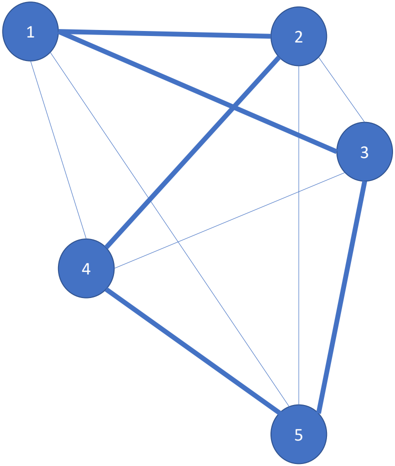
Pheromone Matrix A

	1	0.9	0.9	0.9
0.9		0.9	1	0.9
1	0.9		0.9	0.9
0.9	0.9	0.9		1
0.9	0.9	1	0.9	

S = {3}

$$p_2 = \frac{0.9}{1.9} \approx 0.47$$

$$p_3 = \frac{1}{1.9} \approx 0.53$$



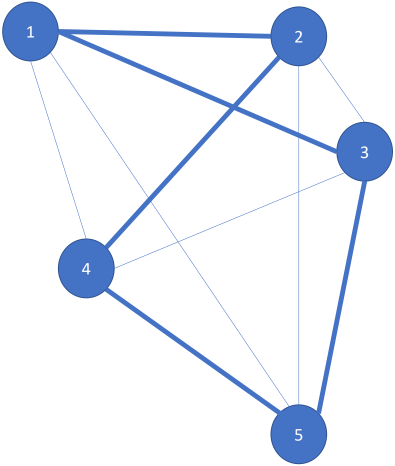
Solution: (4,5,2)

Pheromone Matrix A

	1	0.9	0.9	0.9
0.9		0.9	1	0.9
1	0.9		0.9	0.9
0.9	0.9	0.9		1
0.9	0.9	1	0.9	

S = {3}

$p_3 = \frac{0.9}{0.9}$



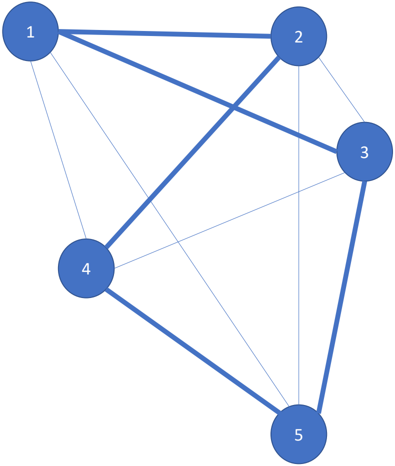
Solution: (4,5,2)

Pheromone Matrix A

	1	0.9	0.9	0.9
0.9		0.9	1	0.9
1	0.9		0.9	0.9
0.9	0.9	0.9		1
0.9	0.9	1	0.9	

S = {}

$p_3 = \frac{0.9}{0.9}$

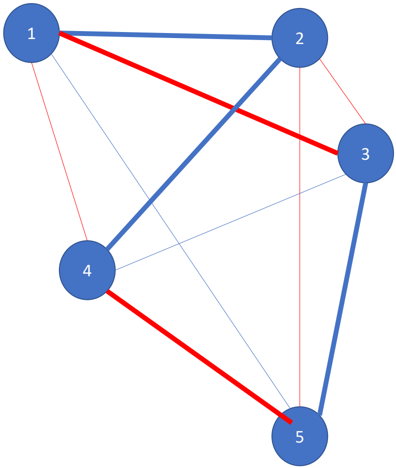


Solution: (4,5,2,3)

Pheromone Matrix A

	1	0.9	0.9	0.9
0.9		0.9	1	0.9
1	0.9		0.9	0.9
0.9	0.9	0.9		1
0.9	0.9	1	0.9	

S = {}



Solution: (4,5,2,3)

Evaporation $\rho=0.9$

Pheromone Matrix A

	1	0.9	0.9	0.9
0.9		0.9	1	0.9
1	0.9		0.9	0.9
0.9	0.9	0.9		1
0.9	0.9	1	0.9	

	0.9	0.81	0.81	0.81
0.81		0.81	0.9	0.81
0.9	0.81		0.81	0.81
0.81	0.81	0.81		0.9
0.81	0.81	0.9	0.81	

Intensification **Δ=0.1**

Best Solution (2, 3, 5, 4)

Pheromone Matrix A

	0.9	0.81	0.81	0.81
0.81		0.81	0.9	0.81
0.9	0.81		0.81	0.81
0.81	0.81	0.81		0.9
0.81	0.81	0.9	0.81	

	1.0	0.81	0.81	0.81
0.81		0.91	0.9	0.81
0.9	0.81		0.81	0.91
0.91	0.81	0.81		0.9
0.81	0.81	0.9	0.91	

Pheromone Matrix A

	1.0	0.81	0.81	0.81
0.81		0.91	0.9	0.81
0.9	0.81		0.81	0.91
0.91	0.81	0.81		0.9
0.81	0.81	0.9	0.91	

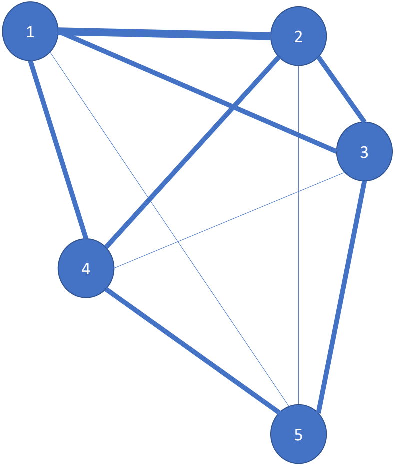
S = {2, 3, 4, 5}

$$p_2 = \frac{1}{3.43} \approx 0.29$$

$$p_3 = \frac{0.81}{3.43} \approx 0.24$$

$$p_4 = \frac{0.81}{3.43} \approx 0.24$$

$$p_5 = \frac{0.81}{3.43} \approx 0.24$$



Solution: ()

Incorporating Heuristic Knowledge

- we often have rules of thumb to find good solutions
- in TSP, close cities may be a better choice than distant cities
- before, the probability of selecting city j after city i was

$$p_j = \frac{a_{i,j}}{N},$$

where $N = \sum_{k \in S} a_{i,k}$

- instead, consider

$$p_j = \frac{1}{N} \frac{a_{i,j}^{\alpha}}{d_{i,j}^{\beta}},$$

where $N = \sum_{k \in S} \frac{a_{i,j}^{\alpha}}{d_{i,j}^{\beta}}$ and $\alpha, \beta \geq 0$

Intuition of α and β

- α controls the influence of pheromone values
- β controls the influence of the distance heuristic
- for $\alpha = 1, \beta = 0$, we get

$$p_j = \frac{1}{N}\frac{a_{i,j}^1}{d_{i,j}^0} = \frac{a_{i,j}}{N},$$

(equal to original approach)

- for $\alpha = 0, \beta = 1$, we get

$$p_j = \frac{1}{N}\frac{a_{i,j}^0}{d_{i,j}^1} = \frac{1}{d_{i,j} \cdot N},$$

(probability decreases proportional to distance)

- if both $\alpha, \beta > 0$, we balance between these extremes

Distance Matrix D

0	2	3	2	4
2	0	1	3	4
3	1	0	3	3
2	3	3	0	2
4	4	3	2	1

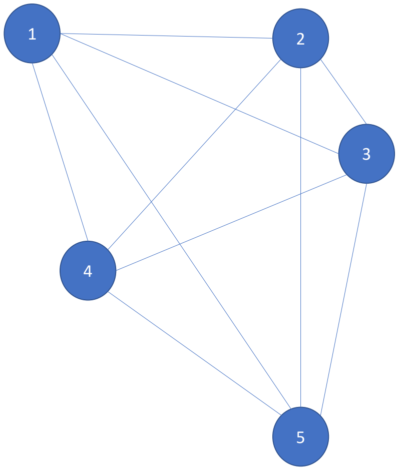
Pheromone Matrix A

	1	1	1	1
1		1	1	1
1	1		1	1
1	1	1		1
1	1	1	1	

S = {2, 3, 4, 5}

$\alpha = 1$
$\beta = 1$

$p_2 \approx 0.32$
$p_3 \approx 0.21$
$p_4 \approx 0.32$
$p_5 \approx 0.16$



Parameters

Parameters

- number of ants N
- initial pheromone values
- evaporation parameter ρ
- intensification parameter Δ
- pheromone weight α
- heuristic weight β
- termination condition
- greedy parameter q

Number of Ants

Tradeoff

- large: good exploration (diversity)
- small: faster runtime per iteration, but search may become too random

Initial Pheromone Values

- just set initial pheromone values to 1
- for other initial values, just rescale intensification parameter

Evaporation Parameter

Tradeoff

- close to 1: pheromone values will decrease slowly (better exploration, longer runtime)
- close to 0: pheromone values will decrease rapidly (less exploration, shorter runtime)

Intensification Parameter

Should be chosen relative to

- initial pheromone values
- evaporation rate

Intensification Parameter

Should be chosen relative to

- initial pheromone values
- evaporation rate

If

- initial pheromone values are 1 and
- evaporation rate is $1 - e$,

then intensification rate should not be much larger than e

Example

Initial pheromone values: 1
Evaporation rate: 0.9
Intensification rate: 0.1

Iteration	Lowest Possible Value	Largest Possible Value
0	1	1
1	0.9	1
2	0.81	1
3	0.729	1
4	0.6561	1

Example

Initial pheromone values: 1
Evaporation rate: 0.9
Intensification rate: 0.2

Iteration	Lowest Possible Value	Largest Possible Value
0	1	1
1	0.9	1.1
2	0.81	1.19
3	0.729	1.271
4	0.6561	1.3439

Example

Initial pheromone values: 1
Evaporation rate: 0.9
Intensification rate: 0.05

Iteration	Lowest Possible Value	Largest Possible Value
0	1	1
1	0.9	0.95
2	0.81	0.8645
3	0.729	0.82805
4	0.6561	0.795245

Example

Initial pheromone values: 1
Evaporation rate: 0.9
Intensification rate: 1

Iteration	Lowest Possible Value	Largest Possible Value
0	1	1
1	0.9	1.9
2	0.81	2.71
3	0.729	3.439
4	0.6561	4.0951

Pheromone and Heuristic Weight

Tradeoff

- $\alpha = \beta$: 'equal balance' between ACO and heuristic
- $\alpha \gg \beta$: basically ACO
- $\beta \gg \alpha$: basically (randomized variant of) Heuristic

Termination Condition

- stop when solutions do not improve anymore
  (for fixed number of iterations)
- stop after fixed number of iterations
- stop when time limit has been reached

Greedy Parameter

when making next decision, with probability

- $q \in [0, 1]$: select strongest edge
- $1 - q$: select edge in strength-proportionate manner

Greedy Parameter

when making next decision, with probability

- $q \in [0, 1]$: select strongest edge
- $1 - q$: select edge in strength-proportionate manner

Tradeoff

- $q = 1$: greedy selection
- $q = 0$: previous ACO

Programming Task Ant Colony Optimization

TSP Programming Task

- solve TSP using our basic ACO algorithm
- as usual, decompose your implementation
  - Initialization (N)
  - Solution Generation (α, β, q)
  - Evaporation (ρ)
  - Intensification (Δ)
- split the work among group members

TSP Programming Task

- make a few experiments with different (reasonable) parameter settings for three benchmark problems
- document your findings and prepare a small presentation (5-10 minutes)
- upload your
  1. slides (structure findings in table or other visualization)
  2. source files/ notebook
  3. assignment of tasks to group members

  in your group folder in stud.ip

Benchmark Problems

You find three benchmark problems with 150 cities each in stud.ip

- Problem 1.tsp
  - Best HC solution found: 6,376 (FCHC, Transposition)
  - Best ACO solution found: 3,632
  - Strict (probably not tight) Lower Bound: 2,502
- Problem 2.tsp
  - Best HC solution found: 4,315 (FCHC, Transposition)
  - Best ACO solution found: 2,878
  - Strict (probably not tight) Lower Bound: 1,971
- Problem 3.tsp
  - Best HC solution found: 4,508 (FCHC, Transposition)
  - Best ACO solution found: 2,617
  - Strict (probably not tight) Lower Bound: 1,728