## Pattern Recognition

(EE5907R)

Jiashi FENG

Email: elefjia@nus.edu.sg

#### **Outlines**

- Unsupervised Feature Extraction (PCA, NMF,...)
- Supervised Feature Extraction (LDA, GE, ...)
- Clustering and Applications
- Gaussian Mixture Model and Boosting
- Support Vector Machine
- Deep Learning

#### Generative vs. Discriminative

- We want to classify the data x into labels y. A generative model learns the joint probability distribution p(x,y) and a discriminative model learns the conditional probability distribution p(y|x).
- Suppose we have the following data in the form (x, y): (1,0), (1,0), (2,0), (2, 1)
- p(x,y) is

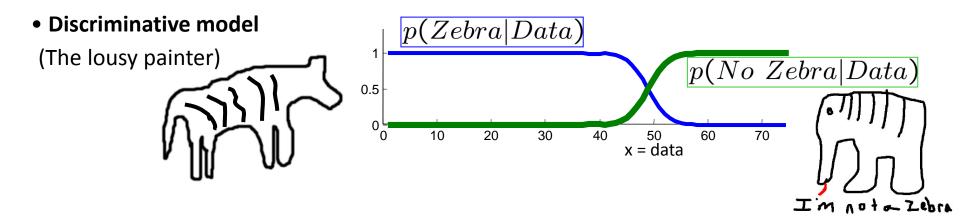
	y = 0	y = 1	
x = 1	1/2	0	
x = 2	1/4	1/4	

• p(y|x) is

	$y = 0 \qquad \qquad y = 1$		
x = 1	1	0	
x = 2	1/2	1/2	

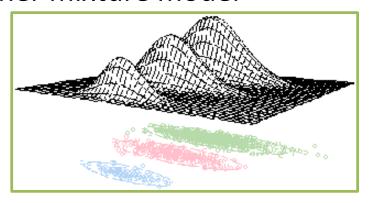
#### **Generative vs. Discriminative**

• Generative model  $p(Data, No\ Zebra)$  (The artist)  $p(Data, No\ Zebra)$  of p(Data, Zebra) of p(Data,

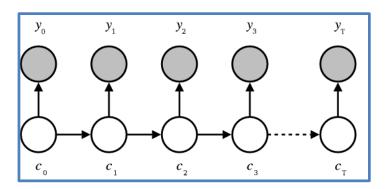


#### **Generative Models**

Gaussian Mixture Model and other mixture model

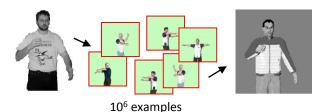


Hidden Markov Model



#### **Discriminative Models**

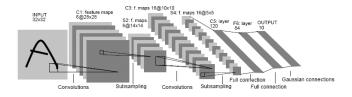
#### Nearest neighbor



Shakhnarovich, Viola, Darrell 2003 Berg, Berg, Malik 2005

•••

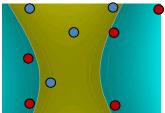
#### **Neural Networks**



LeCun, Bottou, Bengio, Haffner 1998 Rowley, Baluja, Kanade 1998

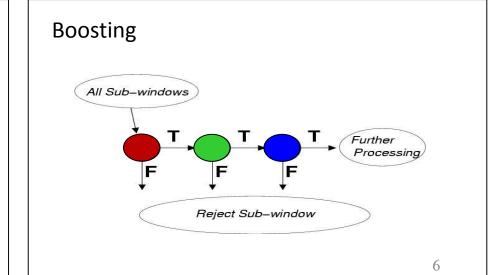
...

#### **Support Vector Machines**



Guyon, Vapnik Heisele, Serre, Poggio, 2001

• • •



## Generative: Gaussian Mixture Model (GMM)

#### **Mixture Models**

• Formally a Mixture Model is the weighted sum of a number of probability density functions (pdfs) where the weights are determined by a distribution,  $\pi$ 

$$p(x) = \pi_1 f_1(x) + \pi_2 f_2(x) + ... + \pi_K f_K(x)$$
 where  $\sum_{i=1}^K \pi_i = 1$  
$$p(x) = \sum_{i=1}^K \pi_i f_i(x)$$

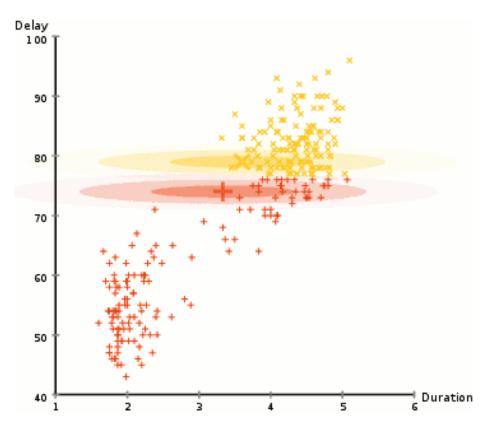
#### Gaussian Mixture Models

• GMM: the weighted sum of a number of Gaussians where the weights are determined by a distribution,  $\pi$ 

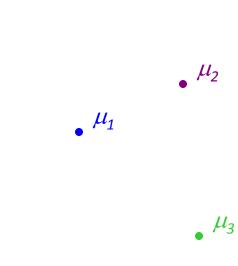
$$p(x) = \pi_1 N(x | \mu_1, \Sigma_1) + \pi_2 N(x | \mu_2, \Sigma_2) + ... + \pi_K N(x | \mu_K, \Sigma_K)$$
 where  $\sum_{i=1}^K \pi_i = 1$  
$$p(x) = \sum_{i=1}^K \pi_i N(x | \mu_i, \Sigma_i)$$

#### Gaussian Mixture Models

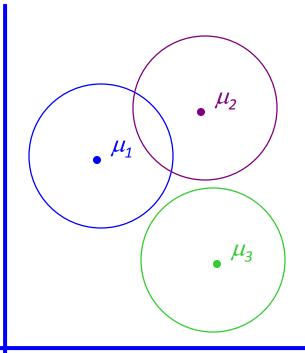
- Rather than identifying clusters by "nearest" centroids
- Fit a Set of K Gaussians to the unlabeled data
- Maximum Likelihood over a mixture model



- There are K components. The i'th component is called  $\omega_i$
- Component  $\omega_i$  has an associated mean vector  $\mu_i$



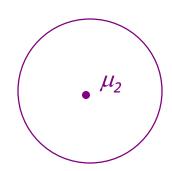
- There are K components. The i'th component is called  $\omega_i$
- Component  $\omega_i$  has an associated mean vector  $\mu_i$
- Each component generates data from a Gaussian model with mean  $\mu_i$  and covariance matrix  $\sigma^2 I$



- There are K components. The i'th component is called  $\omega_i$
- Component  $\omega_i$  has an associated mean vector  $\mu_i$
- Each component generates data from a Gaussian model with mean  $\mu_i$  and covariance matrix  $\sigma^2 I$

Assume that each data point is generated according to the following recipe:

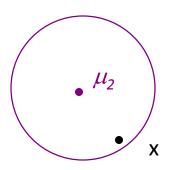
1. Pick a component at random. Choose component i with probability  $P(\omega_i)$ .



- There are K components. The i'th component is called  $\omega_i$
- Component  $\omega_i$  has an associated mean vector  $\mu_i$
- Each component generates data from a Gaussian model with mean  $\mu_i$  and covariance matrix  $\sigma^2 I$

Assume that each data point is generated according to the following recipe:

- 1. Pick a component at random. Choose component i with probability  $P(\omega_i)$ .
- 2. Data point  $\sim N(\mu_i, \sigma^2 I)$

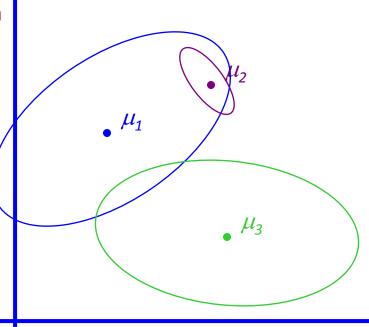


### The General GMM Assumption

- There are K components. The i'th component is called  $\omega_i$
- Component  $\omega_i$  has an associated mean vector  $\mu_i$
- Each component generates data from a Gaussian model with mean  $\mu_i$  and covariance matrix  $\Sigma_i$

Assume that each data point is generated according to the following recipe:

- 1. Pick a component at random. Choose component i with probability  $P(\omega_i)$ .
- 2. Data point  $\sim N(\mu_i, \Sigma_i)$



#### The EM Algorithm

 Expectation-maximization (EM) is a method for finding maximum likelihood (or maximum a posteriori) estimate of parameter(s) in statistical model, where the model depends on unobserved latent variables.

Latent variables are the key properties for EM.

#### The EM Algorithm

- EM is an iterative method which alternates between performing an Expectation (E) step and a Maximization (M) step
  - E-step computes the expectation of the log-likelihood evaluated using the current estimated distributions for the latent variables based on the parameters inferred from previous step
  - M-step computes parameters maximizing the expected log-likelihood from the E-step. These parameter-estimates are then used to determine the distribution of the latent variables in the next E-step.

become constants here.

#### **Simple Example**

Let events be "grades in a class"

```
w_1 = Gets \text{ an } A P(A) = \frac{1}{2}

w_2 = Gets \text{ a} B P(B) = \mu

w_3 = Gets \text{ a} C P(C) = 2\mu

w_4 = Gets \text{ a} D P(D) = \frac{1}{2} - 3\mu

(Note 0 \le \mu \le 1/6)
```

Assume we want to estimate  $\mu$  from data. In a given class, there were

a A's b B's c C's d D's

What's the maximum likelihood estimate of  $\mu$  given a, b, c, d?

#### **Trivial Statistics**

$$P(A) = \frac{1}{2}$$
  $P(B) = \mu$   $P(C) = 2\mu$   $P(D) = \frac{1}{2} - 3\mu$ 

P( a, b, c, d | 
$$\mu$$
) = K( $\frac{1}{2}$ )<sup>a</sup>( $\mu$ )<sup>b</sup>( $2\mu$ )<sup>c</sup>( $\frac{1}{2}$ - $3\mu$ )<sup>d</sup>

$$\log P(a, b, c, d \mid \mu) = \log K + a \log \frac{1}{2} + b \log \mu + c \log 2\mu + d \log (\frac{1}{2} - 3\mu)$$

FOR MAX LIKE 
$$\mu$$
, SET  $\frac{\partial Log P}{\partial \mu} = 0$ 

$$\frac{\partial \text{LogP}}{\partial \mu} = \frac{b}{\mu} + \frac{2c}{2\mu} - \frac{3d}{1/2 - 3\mu} = 0$$

$$K = \frac{(a+b+c+d)!}{a!b!c!d!}$$

Gives max like 
$$\mu = \frac{b+c}{6(b+c+d)}$$

So if class got

Α	В	С	D
14	6	9	10

Max like 
$$\mu = \frac{1}{10}$$

#### Same Problem with Latent Information

Someone tells us that

Number of High grades (A's + B's) = h

Number of C's = c

Number of D's = d

What is the max likelihood estimate of  $\mu$  now?

**REMEMBER** 

$$P(A) = \frac{1}{2}$$

$$P(B) = \mu$$

$$P(C) = 2\mu$$

$$P(D) = \frac{1}{2} - 3\mu$$

$$\log P(h, c, d \mid \mu, b) = \log K(h-b,b,c,d) + (h-b) \log \frac{1}{2} + b \log \mu + c \log 2\mu + d \log (\frac{1}{2}-3\mu)$$

latent variable

#### Same Problem with Latent Information

Someone tells us that

Number of High grades (A's + B's) = h

Number of C's = c

Number of D's = d

What is the max likelihood estimate of  $\mu$  now?

We can answer this question circularly:

**REMEMBER** 

 $P(A) = \frac{1}{2}$ 

 $P(B) = \mu$ 

 $P(C) = 2\mu$ 

 $P(D) = \frac{1}{2} - 3\mu$ 

#### **EXPECTATION**

If we know the value of  $\mu$  we could compute the expected value of b

Since the ratio a:b should be the same as the ratio ½ :  $\mu$ 

$$E_{\mu}(b) = \frac{\mu}{\frac{1}{2} + \mu} h$$

#### **MAXIMIZATION**

If we know the expected values of b we could compute the maximum likelihood value of  $\mu$ 

$$\mu = \frac{E_{\mu}(b) + c}{6(E_{\mu}(b) + c + d)}$$

Already computed as in slide #19

#### **EM for This Problem**

We begin with a guess for  $\mu$  We iterate between **EXPECTATION** and **MAXIMIZATION** to improve our estimates of b and  $\mu$ .

**REMEMBER** 

 $P(A) = \frac{1}{2}$ 

 $P(B) = \mu$ 

 $P(C) = 2\mu$ 

 $P(D) = \frac{1}{2} - 3\mu$ 

Define  $\mu(t)$  the estimate of  $\mu$  on the t'th iteration

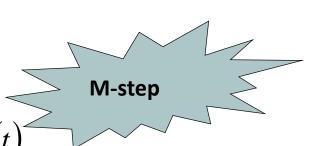
b(t) the estimate of b on t'th iteration

 $\mu(0) = initial guess$ 

$$b(t) = \frac{\mu(t)h}{\frac{1}{2} + \mu(t)} = E[b | \mu(t)]$$

$$\mu(t+1) = \frac{b(t)+c}{6(b(t)+c+d)}$$

= max like est of  $\mu$  given b(t)



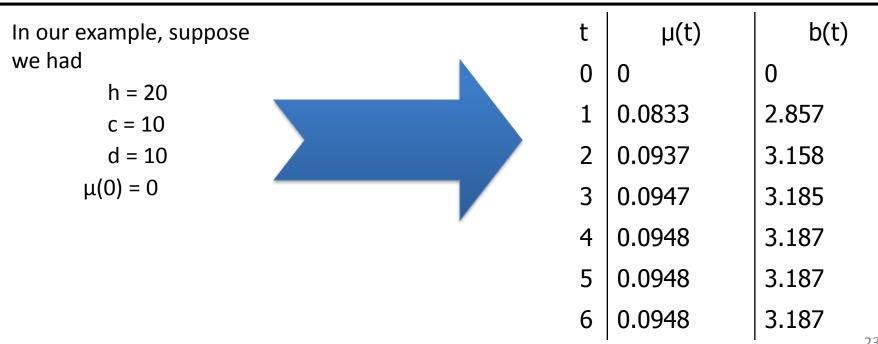
Continue iterating until converged.

Good news: Converging to local optimum is assured.

Bad news: We have "local" optimum.

#### **EM Convergence**

- Convergence proof based on fact that  $Prob(data \mid \mu)$  must increase or remain same between each iteration [NOT OBVIOUS, BUT NOT STUDY HERE]
- But it can never exceed 1
- So it must therefore converge



#### **Back to Learning of GMM**

#### Remember:

We have unlabeled data  $x_1 x_2 \dots x_N$ 

We know there are *K* components

We know  $P(\omega_1) P(\omega_2) P(\omega_3) \dots P(\omega_k)$ ,  $\sigma$ 

We don't know  $\mu_1 \mu_2 ... \mu_k$ 

Hidden variables  $z_k^n$  indicating which component k the datum n is sampled from

#### **Compute Likelihood**

We define:

The define: 
$$\pi_i = P(\omega_i) \quad \text{where} \quad \sum_i \pi_i = 1$$
 
$$z_i = p(\omega_i|x) = \frac{P(\omega_i)p(x|\omega_i)}{\sum_{j=1}^K P(\omega_j)p(x|\omega_j)}$$
 
$$z_k^n = p(\omega_k|x_n)$$

Identify a likelihood function

$$p(x_1,\ldots,x_N|\pi,\mu)=\prod_{n=1}^N p(x_n|\pi,\mu)$$
  $x_n$ 's were drawn independently 
$$=\prod_{n=1}^N \sum_{k=1}^K p(x_n|\omega_k,\mu_k)P(\omega_k)$$

#### **Maximum Likelihood over a GMM**

Identify a log-likelihood function

$$\ln p(x_1, \dots, x_n | \pi, \mu) = \sum_{n=1}^N \ln \left[ \sum_{k=1}^K p(x_n | \omega_k, \mu_k) P(\omega_k) \right]$$

Compute and set partials to 0

$$\frac{\partial \ln p(x_1, \dots, x_n | \pi, \mu)}{\partial \mu_k} = \sum_{n=1}^{N} \frac{1}{p(x_n | \pi, \mu)} \frac{\partial \sum_{k=1}^{K} N(x_n | \mu_k) P(\omega_k)}{\partial \mu_k}$$

$$= \sum_{n=1}^{K} \frac{P(\omega_k)}{p(x_n | \pi, \mu)} \frac{\partial N(x_n | \mu_k)}{\partial \mu_k}$$

$$= \sum_{n=1}^{N} \frac{P(\omega_k) N(x_n | \mu_k)}{p(x_n | \pi, \mu)} \frac{\partial \ln f(x)}{\partial \mu_k}$$

$$= \sum_{n=1}^{N} \frac{P(\omega_k) N(x_n | \mu_k)}{p(x_n | \pi, \mu)} \frac{\partial \ln N(x_n | \mu_k)}{\partial \mu_k}$$

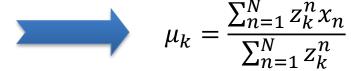
see next slide

#### Maximum Likelihood over a GMM

$$\frac{\partial \ln p(x_1, \dots, x_n | \pi, \mu)}{\partial \mu_k} = \sum_{n=1}^{N} p(\omega_k | x_n) \frac{\partial \ln \exp\left(-\frac{1}{2\sigma^2} (x_n - \mu_k)^2\right)}{\partial \mu_k}$$

$$= \sum_{n=1}^{N} z_k^n \left( -\frac{1}{2\sigma^2} \frac{\partial (x_n - \mu_k)^2}{\partial \mu_k} \right)$$

$$= \sum_{k=1}^{N} z_k^n \frac{x_n - \mu_k}{\sigma^2} = 0$$
 set partials to 0



#### **EM for General GMMs**

We don't know  $P(\omega_1), P(\omega_2), \dots, P(\omega_K), \mu_1, \mu_2, \dots, \mu_K, \Sigma_1, \Sigma_2, \dots, \Sigma_K$ 

Similarly, after compute the log likelihood and take partials to 0, we have

$$\mu_k = \frac{\sum_{n=1}^N z_k^n x_n}{\sum_{n=1}^N z_k^n}$$

$$\Sigma_k = \frac{\sum_{n=1}^{N} z_k^n (x_n - \mu_k) (x_n - \mu_k)^T}{\sum_{n=1}^{N} z_k^n}$$

$$\pi_k = \frac{\sum_{n=1}^N z_k^n}{N}$$

#### **Summary: EM for GMMs**

- Initialize the parameters
  - Evaluate the log likelihood

• Expectation-step: Compute the expectation

- Maximization-step: Re-estimate Parameters
  - Evaluate the log likelihood
  - Check for convergence

#### **EM for GMMs**

 E-step: Compute "expected" classes of all data points for each class

$$z_k^n = \frac{\pi_k N(x_n | \mu_k, \Sigma_k)}{\sum_{j=1}^K \pi_j N(x_n | \mu_j, \Sigma_j)}$$

where 
$$\pi_k = p(\omega_k)$$

#### **EM for GMMs**

M-Step: Re-estimate Parameters

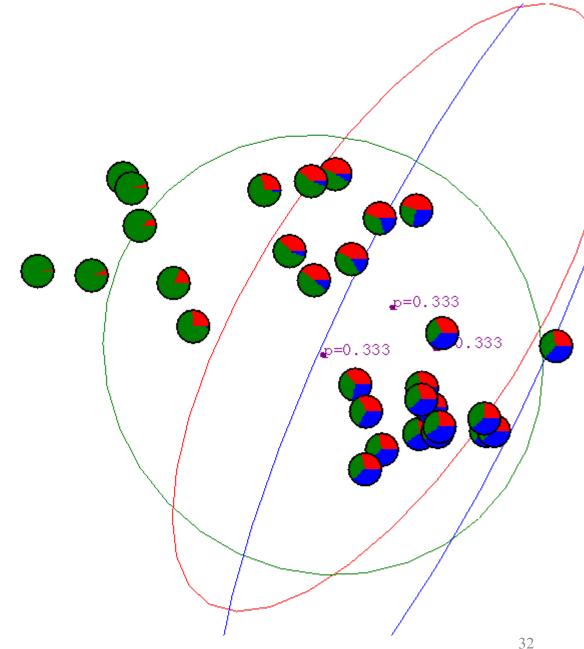
$$\mu_k^{new} = \frac{\sum_{n=1}^{N} z_k^n x_n}{\sum_{n=1}^{N} z_k^n}$$

$$\Sigma_k^{new} = \frac{\sum_{n=1}^{N} z_k^n (x_n - \mu_k^{new}) (x_n - \mu_k^{new})^T}{\sum_{n=1}^{N} z_k^n}$$

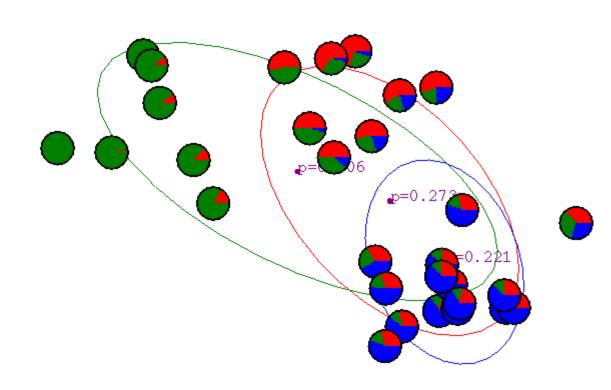
$$\pi_k^{new} = p(\omega_k)^{new} = \frac{\sum_{n=1}^N z_k^n}{N}$$

Latent variables become constants here.

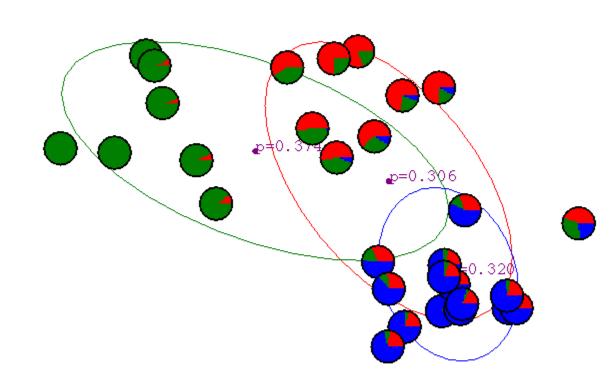
## Gaussian **Mixture** Model Example: Start



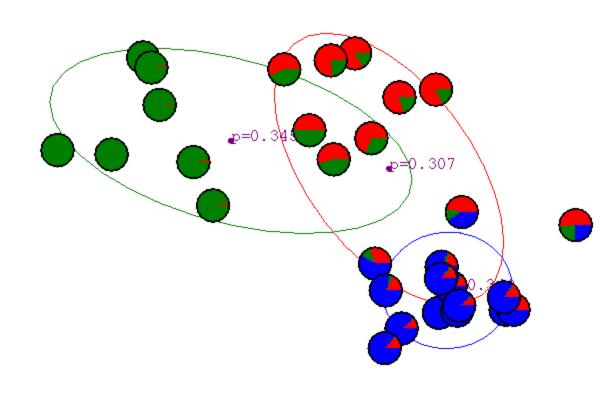
## After 1st iteration



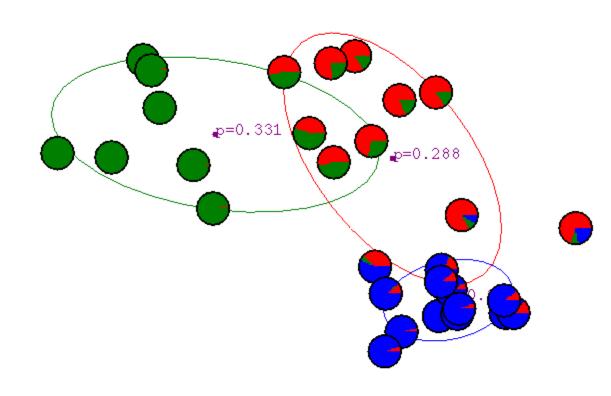
## After 2nd iteration



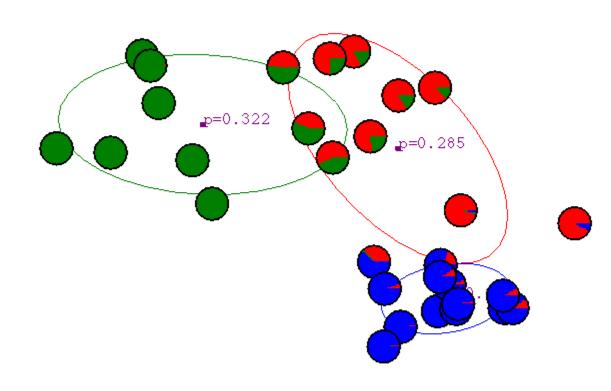
# After 3rd iteration



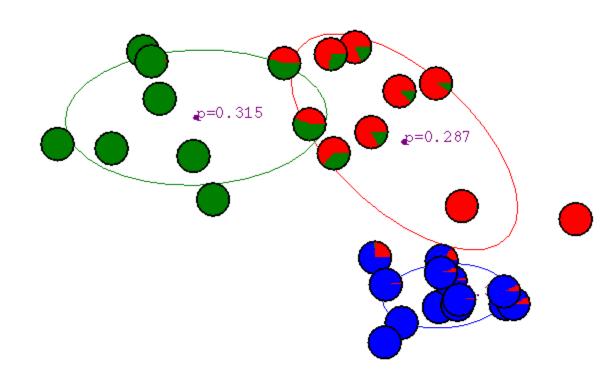
## After 4th iteration



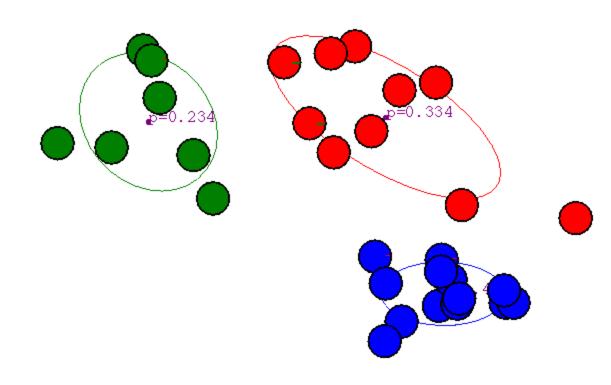
# After 5th iteration



# After 6th iteration



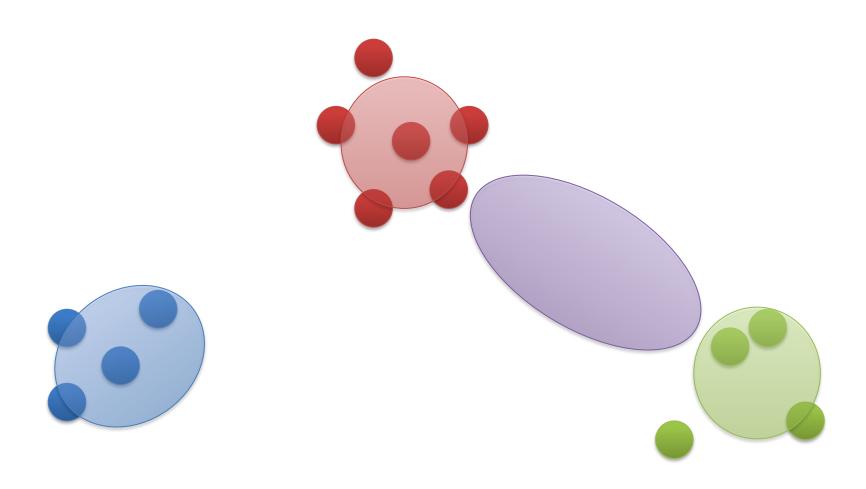
# After 20th iteration



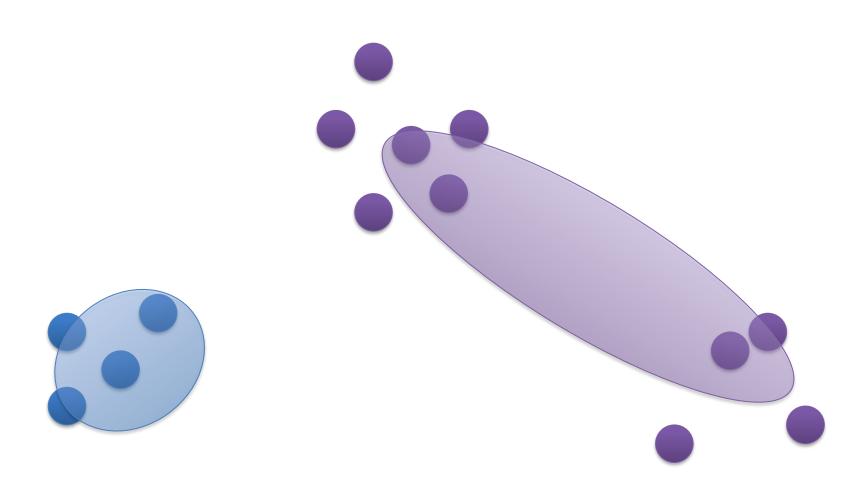
## **Relationship to K-means**

- K-means makes hard decisions.
  - Each data point gets assigned to a single cluster.
- GMM makes soft decisions.
  - Each data point yields a posterior
- Potential problem:
  - Incorrect number of Mixture Components

## **Incorrect Number of Gaussians**



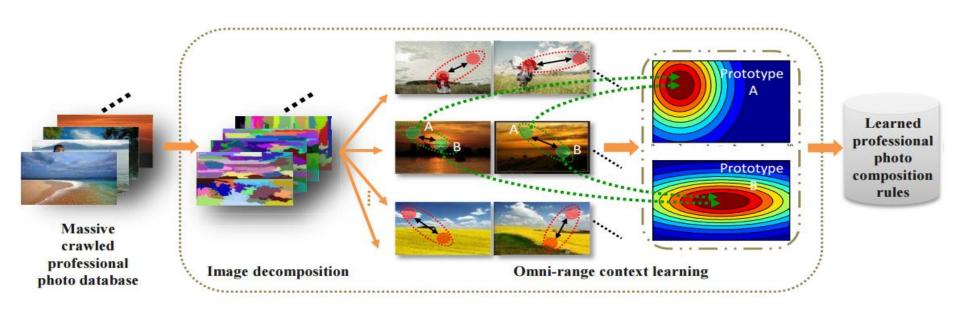
## **Incorrect Number of Gaussians**

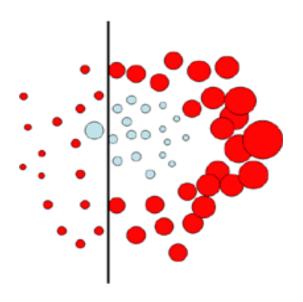


## **GMM for Classification**

- Train universal GMM, and then adapt it for individual class, and finally do classification
  - Widely used in speech recognition
- Note that we can initiate GMM by using K-means

# **Another Application**





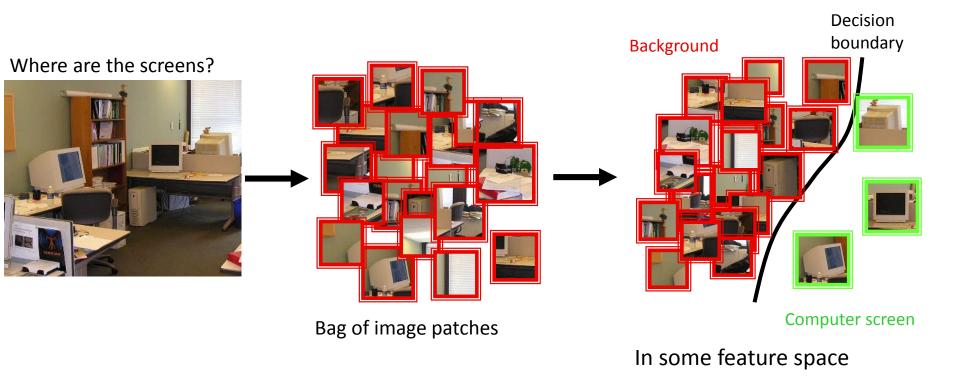
# **Discriminative: Boosting**

## **Example Task**

Object detection and recognition is formulated as a classification problem.

The image is partitioned into a set of overlapping windows

... and a decision is taken at each window about if it contains a target object or not.



## **Formulation**

Formulation: binary classification















Features x =

 $X_1$   $X_2$   $X_3$   $\cdots$   $X_N$ 

 $X_{N+1}$   $X_{N+2}$  ...  $X_{N+M}$ 

Labels y = -1 +1 -1

Training data: each image patch is labeled as containing the object or background

Test data

Classification function

 $\hat{y} = F(x)$  where F(x) belongs to some family of functions

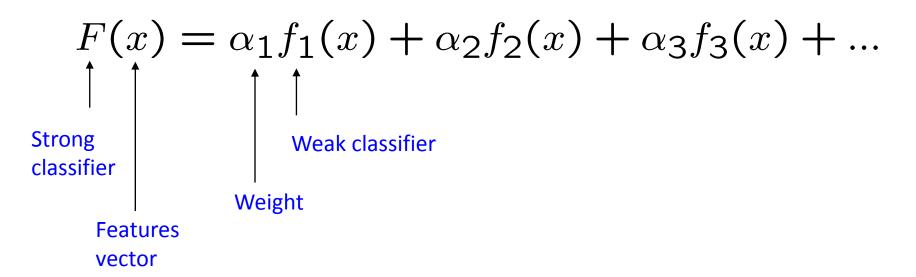
Minimize misclassification error

## Why Boosting?

- A simple algorithm for learning robust classifiers
  - Freund & Shapire, 1995
  - Friedman, Hastie, Tibshhirani, 1998
- Provides efficient algorithm for sparse visual feature selection
  - Tieu & Viola, 2000
  - Viola & Jones, 2003
- Easy to implement, not requires external optimization tools

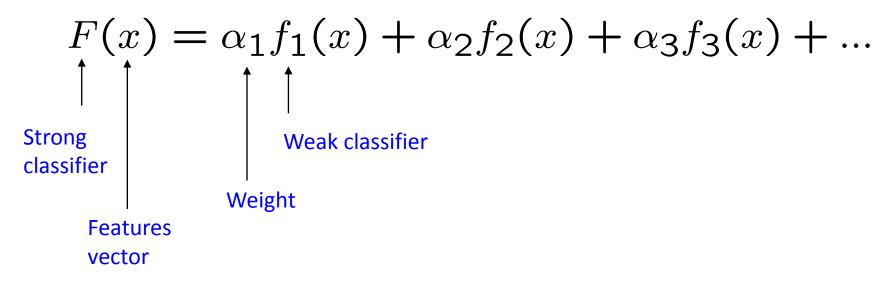
## **Boosting**

Defines a classifier using an additive model:



## **Boosting**

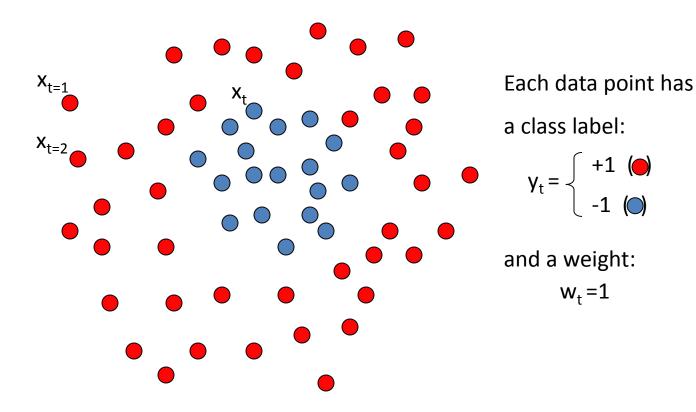
Defines a classifier using an additive model:



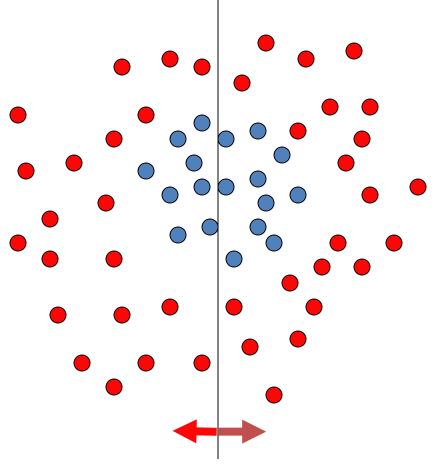
We need to define a family of weak classifiers

 $f_k(x)$  from a family of weak classifiers

It is a sequential procedure:



Weak learners from the family of lines



Each data point has

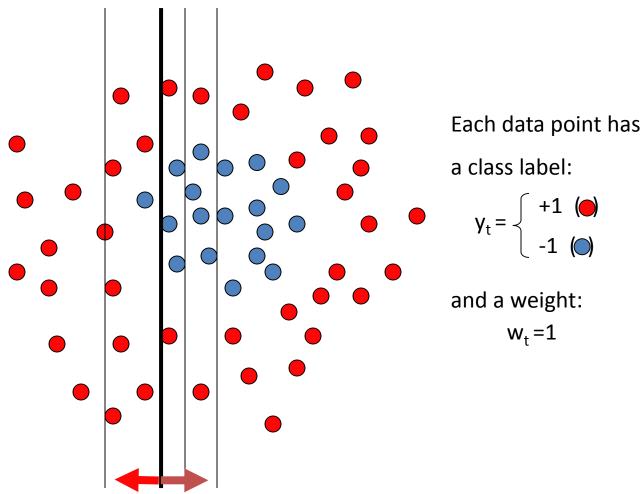
a class label:

$$y_t = \begin{cases} +1 & \bullet \\ -1 & \bullet \end{cases}$$

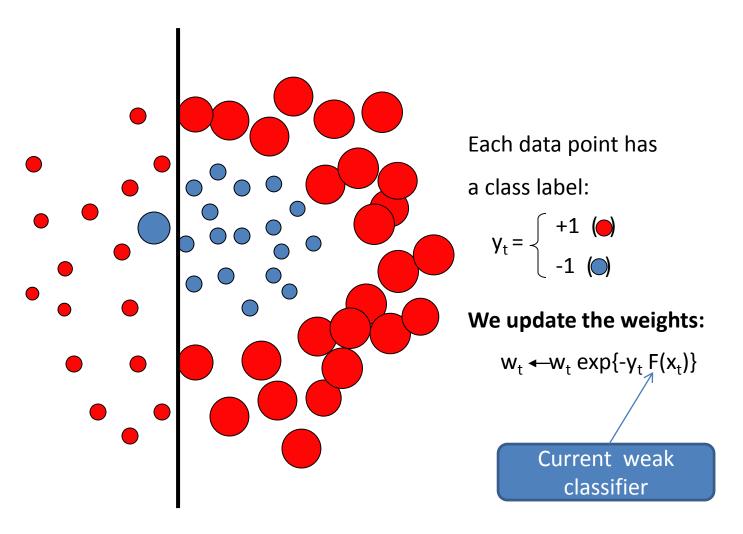
and a weight:

$$w_t = 1$$

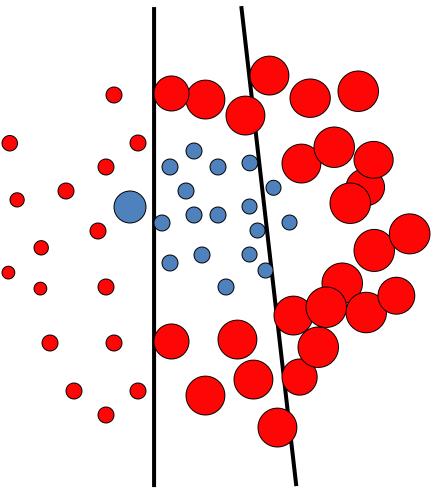
Weak classifier  $h \Rightarrow p(error) = 0.5$  it is at chance



This one seems to be the best



We set a new problem for which the current classifier performs at chance again



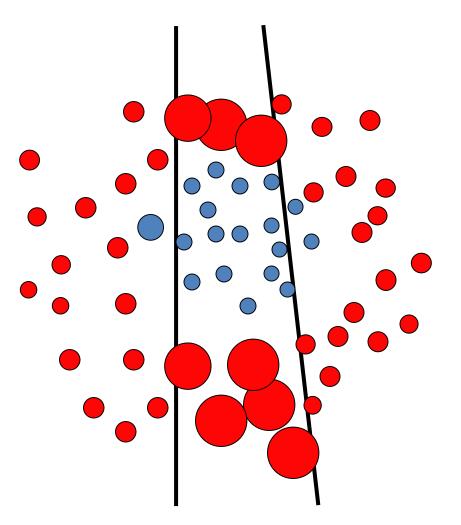
Each data point has

a class label:

$$y_t = \begin{cases} +1 & \bullet \\ -1 & \bullet \end{cases}$$

We update the weights:

$$w_t \leftarrow w_t \exp\{-y_t F(x_t)\}$$



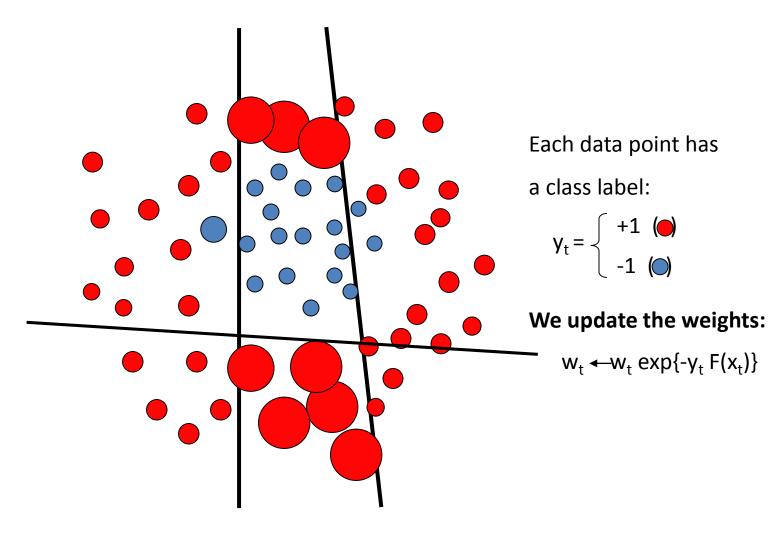
Each data point has

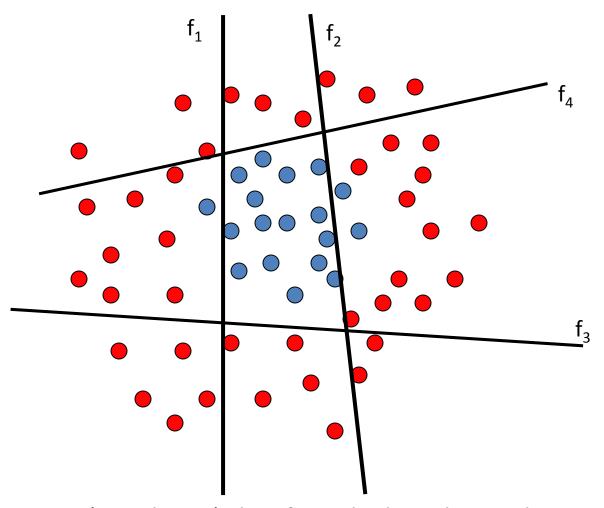
a class label:

$$y_t = \begin{cases} +1 & \bullet \\ -1 & \bullet \end{cases}$$

We update the weights:

$$w_t \leftarrow w_t \exp\{-y_t F(x_t)\}$$





The strong (non-linear) classifier is built as the combination of all the weak (linear) classifiers.

## **Boosting**

 For different cost function and minimization algorithm, the result is a different flavor of Boosting

- We shall introduce gentleBoosting
  - It is simple to implement and numerically stable.

## **Boosting**

Boosting fits the additive model

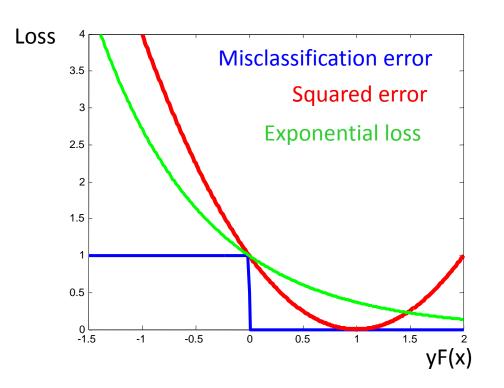
$$F(x) = f_1(x) + f_2(x) + f_3(x) + \dots$$

by minimizing the exponential loss

$$J(F) = \sum_{t=1}^{N} e^{-y_t F(x_t)}$$
Training samples

The exponential loss is a differentiable upper bound to the misclassification error.

## **Exponential Loss**



#### Squared error

$$J = \sum_{t=1}^{N} [y_t - F(x_t)]^2$$

#### **Exponential loss**

$$J = \sum_{t=1}^{N} e^{-y_t F(x_t)}$$

## **Boosting**

Sequential procedure. At each step m we add

$$F(x) \leftarrow F(x) + f_m(x)$$

to minimize the residual loss

$$(\phi_m) = \arg\min_{\phi} \sum_{t=1}^N J\left(y_t, F(x_t) + f(x_t; \phi)\right)$$
 Parameters of the weak classifier Desired output input weak classifier



## gentleBoosting

• At each iteration:

We chose  $f_m(x)$  that minimizes the cost:

$$J(F + f_m) = \sum_{t=1}^{N} e^{-y_t(F(x_t) + f_m(x_t))}$$

Instead of doing exact optimization, gentle Boosting minimizes the approximation of the error:

$$J(F) \propto \sum_{t=1}^N e^{-y_t F(x_t)} (y_t - f_m(x_t))^2$$
 At each iterations we just need to solve a weighted least squares problem Weights at this iteration



## **Weak Classifiers**

 The input is a set of weighted training samples (x,y,w)

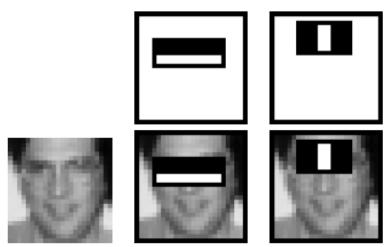
 Regression stumps: simple but commonly used in object detection.

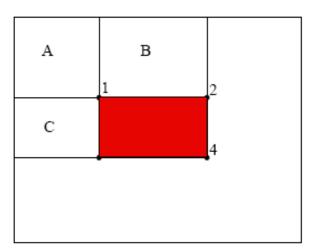
$$f_m(x) = a[x_k < \theta] + b[x_k \ge \theta]$$
 Four parameters:  $\phi = [a, b, \theta, k]$  
$$= \begin{bmatrix} a = E_w(y \mid x < \theta) \\ \theta \end{bmatrix}$$

### **Features -> Weak Detectors**

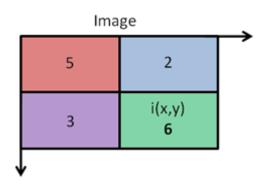
### Haar filters and integral image

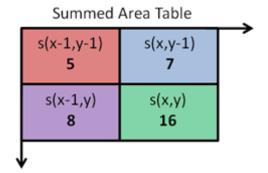
Viola and Jones, ICCV 2001

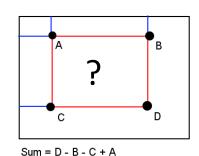




The average intensity in the block is computed with four sums independently of the block size.

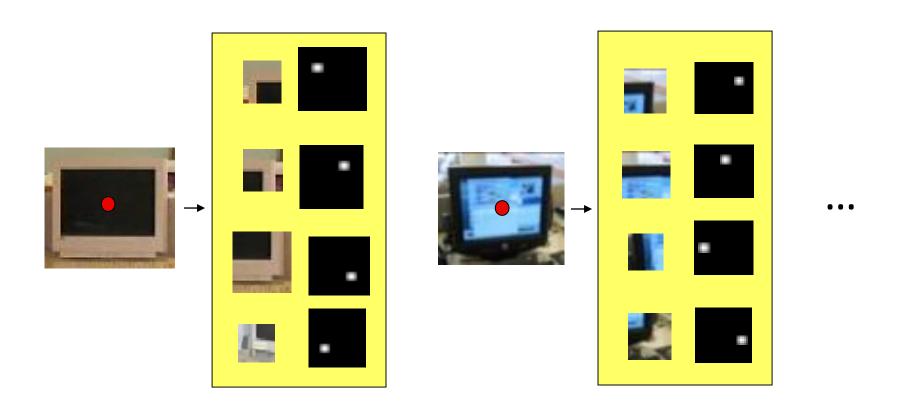


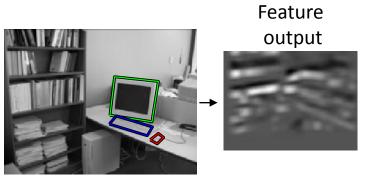


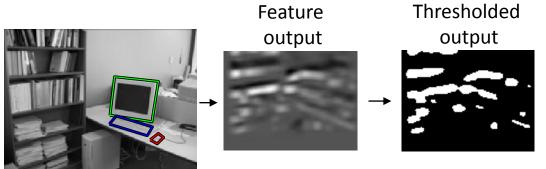


## **Features -> Weak Detectors**

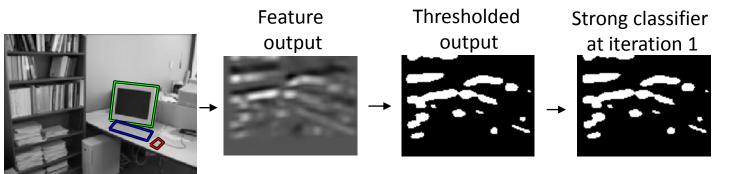
For screen detection, we may collect a set of part templates from a set of training objects to build feature set.

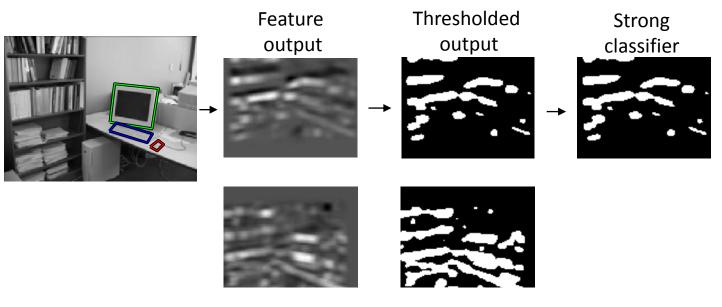




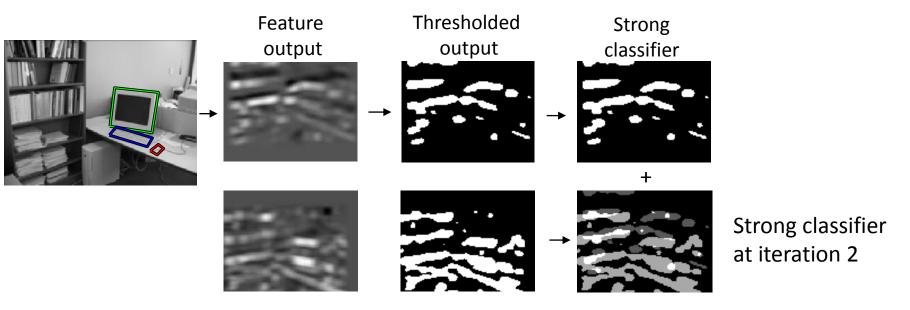


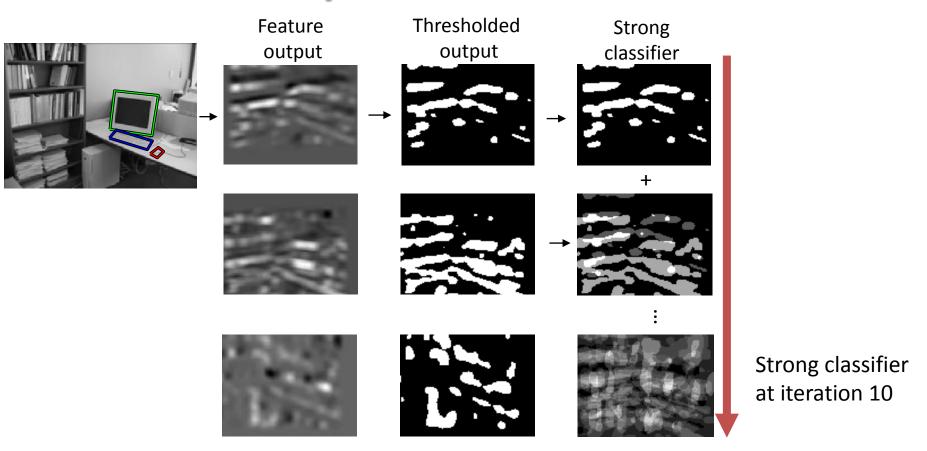
Weak 'detector' Produces many false alarms.

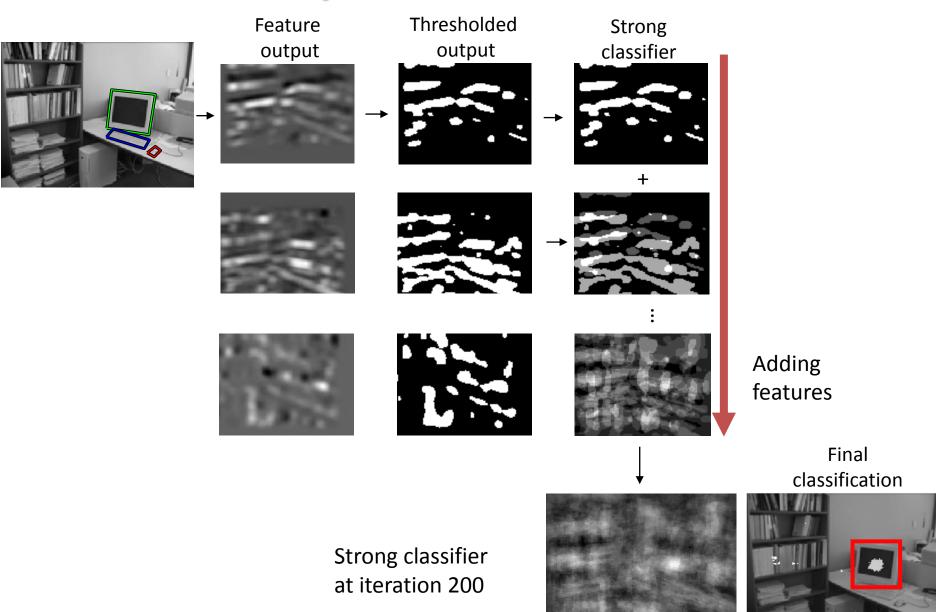




Second weak 'detector'
Produces a different set of false alarms.

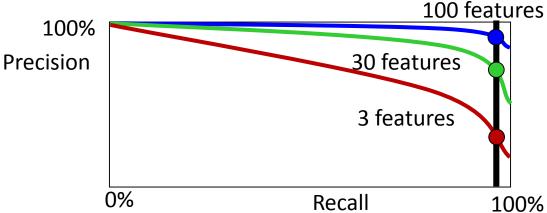






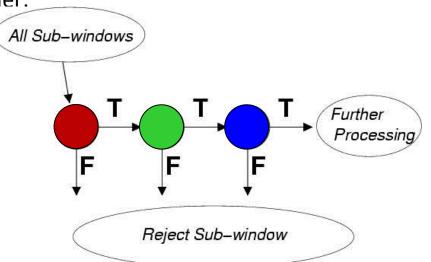
### **Cascade of classifiers**

What is the motivation: some negative samples may be rejected based on few features!



We want the complexity of the 3 features classifier with the performance of the 100

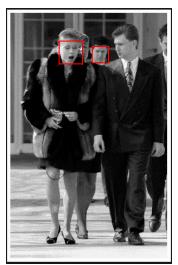
features classifier:

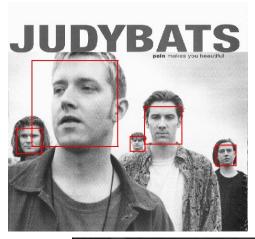


Select a threshold with high recall for each stage.

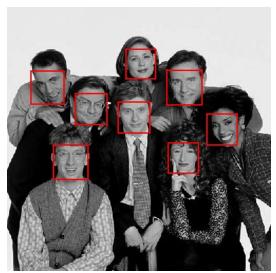
We increase precision using the cascade

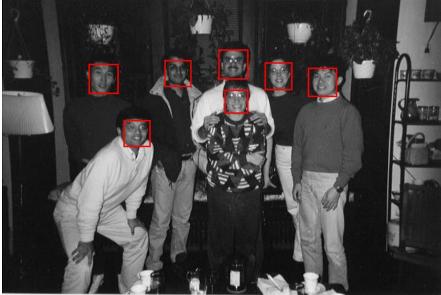
## **Output of Face Detector on Test Images**







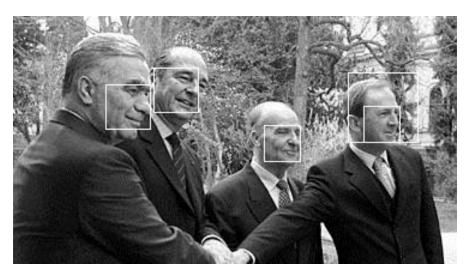




## **Other detection tasks**

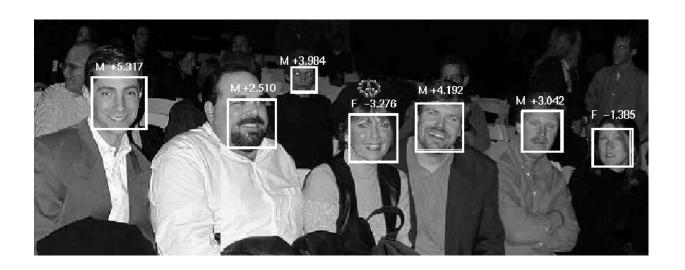


Facial Feature Localization



Profile Detection

Male vs. female



## **Profile Detection**

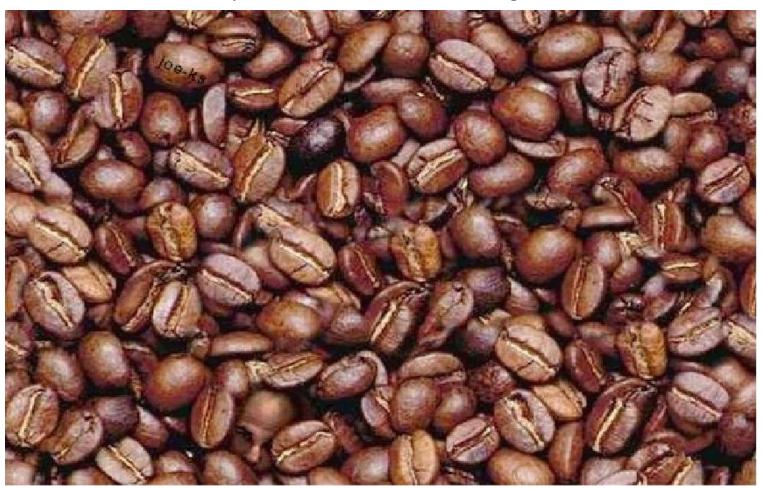






# "Head in the coffee beans problem"

Can you find the head in this image?



## **Weakness of Boosting**

- Features are extracted at fixed positions, and thus not deformable (not perfect for deformable objects)
- No mechanism for handling occlusion
- Extension to "deformable model" + "and/or model"?







## Papers to Read and Study

Friedman, Hastie, Tibshirani.

Additive Logistic Regression: a Statistical View of Boosting (1998). Pdf

Robert E. Schapire.

The boosting approach to machine learning: An overview.

In D. D. Denison, M. H. Hansen, C. Holmes, B. Mallick, B. Yu, editors, *Nonlinear Estimation and Classification*. Springer, 2003.

Postscript or gzipped postscript.

Ron Meir and Gunnar Rätsch.

An introduction to boosting and leveraging.

In Advanced Lectures on Machine Learning (LNAI2600), 2003. Pdf.