

# Granular 12-Tone Harmony via Mixed-Radix Balanced Gray Codes: Interval Sets and the BalaGray Solver

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## 1. Abstract

Chromatic pitch clusters such as  $\{C, C\sharp, D\}$  produce harsh roughness. Conventional twelve-tone techniques distribute pitch classes evenly in melody but do not prevent chromatic clusters in harmony. We partition the twelve-tone space into *interval sets*—contiguous mixed-radix spans that generate complete chord vocabularies free of chromatic clusters—and traverse those chords via *Gray codes*, so that successive chords differ by exactly one pitch. We introduce a heuristic crawler, *BalaGray*, to find optimally balanced, minimal-span Gray cycles in arbitrary mixed-radix spaces, and show that the scarcity of such cycles justifies a dedicated solver. We provide open-source code together with exhaustive tables of prime-form interval sets and their balanced Gray sequences.

## 2. Introduction

Chromatic pitch clusters are acoustically rough<sup>1</sup> [1] [2], and we argue that their prevalence in atonal music has contributed to the genre’s relative unpopularity. We propose a numerical system for generating atonal chord vocabularies that intrinsically avoids chromatic clusters. We then introduce various methods of traversing these chord vocabularies.

We begin by partitioning the chromatic scale into three contiguous spans of four tones each, a structure we refer to as set  $\{444\}$ . This set is analogous to a three-digit base-four number, similar to a codon in genetic code, and it has 64 unique permutations, as  $4^3 = 64$ . Set  $\{444\}$  can be visualized as a one-octave piano keyboard split into thirds.

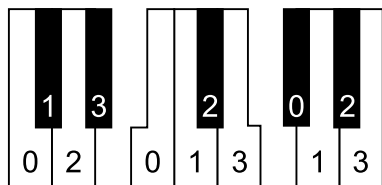


Figure 1: Visualizing set  $\{444\}$  on a piano.

We form trichords by selecting one tone from each span. The permutations of set  $\{444\}$  produce 64 different trichords, summarized in the table below. For a complete list of them, see the Appendix.

Forte	Pc set	Quality	Count
3-4	(0,1,5)		6
3-5	(0,1,6)		6
3-7	(0,2,5)		6
3-8	(0,2,6)		12
3-9	(0,2,7)	suspended	6
3-10	(0,3,6)	diminished	6
3-11A	(0,3,7)	minor	9
3-11B	(0,4,7)	major	9
3-12	(0,4,8)	augmented	4
Total			64

Table 1: Summary of trichords produced by set  $\{444\}$ .

<sup>1</sup> Roughness is the harsh quality of narrow harmonic intervals caused by interference.

Set {444} yields a musically balanced vocabulary. Every common triad type is adequately represented, and major and minor occur in equal proportion. But Forte 3-1 (the chromatic cluster) is notably absent, banished by our method. Even if we select D $\sharp$  from the lower span and G $\sharp$  from the upper span, the narrowest possible configuration, no choice of middle-span note can form a chromatic cluster.

Due to its symmetry, set {444} uses each of the twelve tones the same number of times, and thereby emphasizes all tones equally in conformance with classical twelve-tone practice [3]. For symmetric sets, a simple formula predicts how many times each note will appear: the chord size times the permutation count divided by the total span, in this case  $3 \times 64 \div 12 = 16$ . In the next section, we generalize the concept that set {444} exemplifies to include *asymmetric* sets.

### 3. Interval sets

An interval set is a collection of contiguous, non-overlapping subsets of the chromatic scale. An interval set *partitions* the chromatic scale into disjoint ranges. An interval set can be expressed as a mixed-radix number, each radix of which corresponds to a musical interval in semitones. For example radix 2 is a whole step, radix 3 is a minor third, radix 4 is a major third, and so on.

An interval set is identified by a hexadecimal number called a *set code*, usually enclosed in braces. Each of the code's digits specifies the size of one of the set's intervals in semitones. Each set code digit is also the *span* of one of the number's radices, or places. A place's value varies from zero up to its span minus one. For example, the set code {246} identifies a three-place interval set in which the first place has a span of 2 (values 0–1), the second place has a span of 4 (values 0–3), and the third place has a span of 6 (values 0–5). Set {246} has a total span of 12 ( $2 + 4 + 6$ ), and its number of permutations is the product of the spans, 48 ( $2 \times 4 \times 6$ ).

In musical terminology, set {246} consists of three intervals: a whole step, a major third, and a tritone. Each interval spans two or more semitones, and packed tightly, these spans add up to an octave. Selecting one tone from each of the three spans produces a trichord. Assuming the spans start on C, the trichord's first tone can be C or C $\sharp$ , its second tone can be D, D $\sharp$ , E, or F, and its third tone can be F $\sharp$ , G, G $\sharp$ , A, A $\sharp$ , or B.

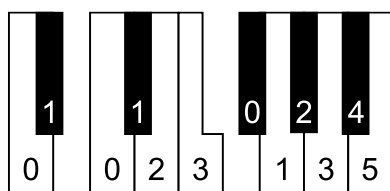


Figure 2: Set {246} is asymmetric.

The following additional constraints apply to interval sets:

1. A set must have at least two places.
2. The minimum span of a place is two.
3. A set's total span can't exceed twelve.

The first constraint arises because it takes at least two notes to form an interval or chord. The second constraint arises because the smallest radix is binary. The third constraint arises because the application is *musical set theory*, which operates on twelve *pitch classes*, commonly known as the twelve tones of the chromatic scale [4]. These constraints intersect to preclude chromatic clusters.

An asymmetric interval set has alternate forms due to rotation (phase shift) or reversal. All forms of a set are equivalent and represented by a single canonical set, the *prime form*, obtained by sorting the set code's digits into ascending order from left to right. For example, sets 246, 462, 624, 642, 426, and 264 are equivalent, and their prime form is 246. The non-prime forms are called *aliases*. The methodology of prime forms is borrowed from musical set theory.

Given the preceding constraints, we present an exhaustive table of the 65 prime form interval sets, having the following columns:

- **Set:** The set's unique prime form identifier, consisting of the set's sorted radices in hexadecimal. Set codes require hexadecimal because one set has a radix of ten: set 2A.
- **Size:** The number of spans in the set.
- **Span:** The set's total span in semitones; the sum of its individual spans.
- **Perms:** The number of permutations the set has; the product of its radices.

Set	Size	Span	Perms	Set	Size	Span	Perms	Set	Size	Span	Perms
22	2	4	4	48	2	12	32	255	3	12	50
23	2	5	6	57	2	12	35	336	3	12	54
24	2	6	8	66	2	12	36	345	3	12	60
33	2	6	9	222	3	6	8	444	3	12	64
25	2	7	10	223	3	7	12	2222	4	8	16
34	2	7	12	224	3	8	16	2223	4	9	24
26	2	8	12	233	3	8	18	2224	4	10	32
35	2	8	15	225	3	9	20	2233	4	10	36
44	2	8	16	234	3	9	24	2225	4	11	40
27	2	9	14	333	3	9	27	2234	4	11	48
36	2	9	18	226	3	10	24	2333	4	11	54
45	2	9	20	235	3	10	30	2226	4	12	48
28	2	10	16	244	3	10	32	2235	4	12	60
37	2	10	21	334	3	10	36	2244	4	12	64
46	2	10	24	227	3	11	28	2334	4	12	72
55	2	10	25	236	3	11	36	3333	4	12	81
29	2	11	18	245	3	11	40	22222	5	10	32
38	2	11	24	335	3	11	45	22223	5	11	48
47	2	11	28	344	3	11	48	22224	5	12	64
56	2	11	30	228	3	12	32	22233	5	12	72
2A	2	12	20	237	3	12	42	222222	6	12	64
39	2	12	27	246	3	12	48				

Table 2: The prime form interval sets, sorted by size, span and permutations.

Interval sets fall into two orthogonal binaries:

- A set is *complete* if its spans cover all twelve pitch classes, otherwise it's *incomplete*.
- A set is *symmetric* if all its spans have the same size, otherwise it's *asymmetric*.

Only four sets satisfy both criteria—complete *and* symmetric—namely 66, 444, 3333, and 222222. The incomplete symmetric sets are 22, 33, 44, 55, 222, 333, 2222, and 22222. Incomplete sets also introduce the additional consideration of span spacing, which we discuss in section 6.

## 4. Ordering

We now consider the *ordering* of an interval set's permutations. Twelve-tone music emphasizes *completism*, the practice of visiting every possible item exactly once before any item is revisited. Schoenberg called this *aggregate completion* [5], and in combinatorics, it's known as a Hamiltonian path [6]. In the spirit of *tone rows*, we only consider complete orderings with no repetitions.

For any given set, the number of possible orderings is the factorial of the permutation count. For example the 64 permutations of {444} have  $1.27 \times 10^{89}$  orderings, a gargantuan number. We begin with the most obvious ordering, *counting order*, produced by repeatedly adding one to the least significant digit and carrying as needed. Below is {444} in counting order.

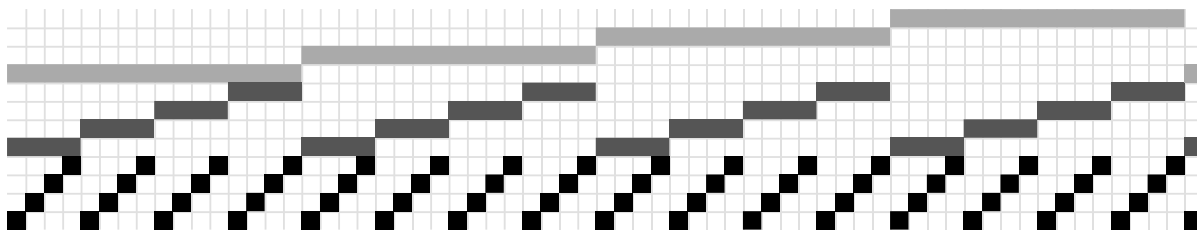


Figure 3: Set {444} in counting order, incrementing by one. [▶ audio](#)

Counting by one has a characteristic problem: the bottom note changes every time, while the middle note changes less often and the top note rarely changes. Another problem is that sometimes two or even all three voices change simultaneously. To mitigate these problems, we can count by a larger number, but we must beware of increments that evenly divide the radix. Counting {444} by two would skip half the permutations, for example. Counting by three yields all permutations, as shown below.

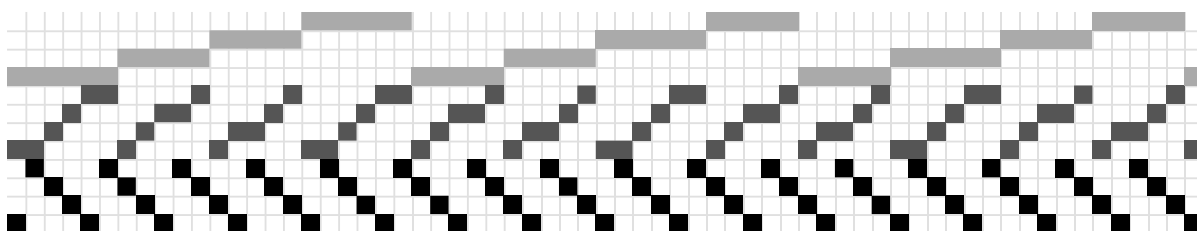


Figure 4: Set {444} in counting order, incrementing by three. [▶ audio](#)

Counting by three helps but fails to solve the first problem. The top voice is still less active than the other two. The second problem has worsened considerably, in that multi-voice transitions are now much more frequent than before. We address these problems more thoroughly in the next section.

## 5. Granularity

We confront two distinct but related problems: some voices changing more often than others, and multiple voices changing simultaneously. Both problems are types of *lumpiness* that frustrate expectations [7]. We strive to avoid lumpiness, and achieve its opposite, which is *granularity*.<sup>2</sup> To clarify the problems and their solutions, we use a more compact interval set, and tackle the multi-voice changes first.

Set {34} has two digits. The first digit is base three, and the second digit is base four. The first digit can have three values (0, 1, 2) and the second digit can have four values (0, 1, 2, 3). The number of permutations is the product of the radices, in this case  $3 \times 4 = 12$ . The twelve permutations are shown below in counting order. Note that three multi-digit transitions occur, between the fourth and fifth, eighth and ninth, and first and last states.

Radix	Sequence											
3	0	0	0	0	1	1	1	1	2	2	2	2
4	0	1	2	3	0	1	2	3	0	1	2	3

Table 3: Set {34} in counting order, exhibiting lumpiness.

### 5.1 Gray code

Fortunately a solution to multi-digit transitions exists, called Gray code, named after physicist and researcher Frank Gray. In a Gray ordering, only one digit changes between successive numerals. Gray's original invention applied to binary numbers, but the concept is extensible to mixed-radix numbers. A Gray ordering of set {34} is shown below. Note that the wraparound from the last to the first numeral is also a single-digit transition, as it must be in a fully Gray ordering.

<sup>2</sup> Aesthetic preference for change via the smallest possible gradations could be called Granularism.

Radix	Sequence											
3	0	0	0	0	1	1	1	1	2	2	2	2
4	0	1	2	3	3	0	1	2	2	3	1	0

Table 4: Set {34} in Gray order; only one digit transitions at a time.

The simplest way to generate such orderings is via *reflected* Gray code [8]. It's similar to counting by one, but enforces single-digit transitions by periodically flipping subsets of the sequence in a nested fashion. The code is easy to calculate due to its predictable structure, but it doesn't distribute the transitions uniformly across all digits. Below is a reflected Gray ordering of {444}.

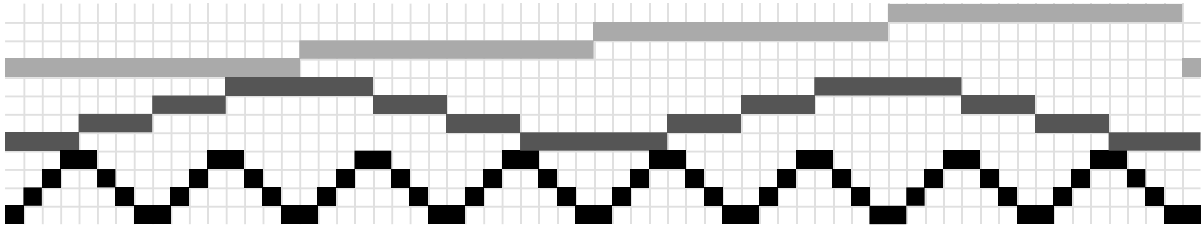


Figure 5: Set {444} in reflected Gray order. [▶ audio](#)

## 5.2 Balance

Having solved the problem of multiple voices changing simultaneously, we proceed to the problem of some voices changing more often than others. Reconsider the Gray ordering of set {34} in Table 4. The upper digit transitions three times, whereas the lower digit transitions nine times. This is an *imbalance*, defined as a non-zero difference between the maximum and minimum per-digit transition counts. The above ordering has an imbalance of  $9 - 3 = 6$ .

The solution is *balanced* Gray code [9]. In a well-balanced ordering, each digit transitions the same number of times, or as close to that as possible. The ordering below is Gray and optimally balanced. Both digits transition six times, so the imbalance is  $6 - 6 = 0$ . Other equally optimal solutions exist.

Radix	Sequence											
3	0	1	2	2	0	1	1	1	2	2	0	0
4	0	0	0	1	1	1	2	3	3	2	2	3

Table 5: Set {34} in balanced Gray order; each digit transitions six times.

We now apply our solution to a larger set. Below is a balanced Gray ordering of {444}. The bottom voice transitions 22 times, with the others transition 21 times, yielding a slight imbalance of one. This is the best we can do when the permutation count isn't even divisible by the number of places [10].

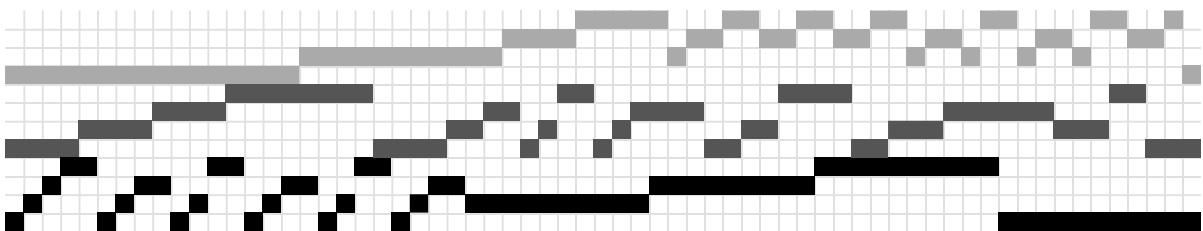


Figure 6: Set {444} in balanced Gray order. [▶ audio](#)

## 5.3 Span length

The above ordering of {444} exhibits a new problem: some spans are much longer than others, or more informally, voices get stuck. In the worst case, the top voice remains static for sixteen steps. This is yet another type of lumpiness, and again we use a smaller set to clarify the problem and its solution. Below is a balanced Gray ordering of set {36} in which both voices transition nine times. The maximum span length is five, because in the top voice, the initial zero and the final four zeros constitute a single span, due to wraparound.

Radix	Sequence																
3	0	1	2	2	0	1	1	2	2	1	1	2	2	1	0	0	0
6	0	0	0	1	1	1	2	2	3	3	4	4	5	5	5	2	3

Table 6: Set {36} in balanced Gray order with a maximum span length of five.

To handle the stuck voice problem, we add another constraint to our solver: given two Gray orderings with equally optimal balance, we take the ordering with the *shortest maximum span length*. The result is shown below. The maximum span length has been reduced to three, the best we can do in this case.

Radix	Sequence																	
3	0	1	2	2	0	1	1	2	2	2	0	0	0	1	1	1	2	0
6	0	0	0	1	1	1	2	2	3	4	4	2	3	3	4	5	5	5

Table 7: Set {36} in balanced Gray order with a maximum span length of three.

Now we return to set {444} and add our new span length constraint to it. In the result below, the maximum span length has been reduced from sixteen to seven, a significant improvement. The ordering is visibly and audibly more granular compared to Figure 6.

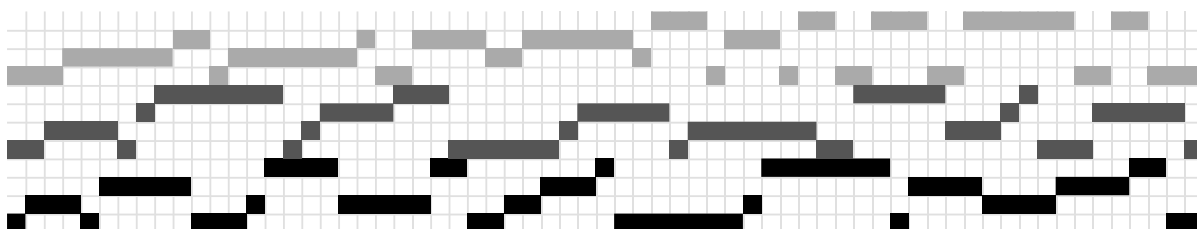


Figure 7: Set {444} in balanced Gray order with shorter spans. [audio](#)

## 6. Incomplete partitions

Complete partitions such as {444} generate 12-tone music innately, but they're a minority: only 20 of the 65 prime form interval sets are complete. An incomplete partition can also generate 12-tone music provided the tones it excludes are reintroduced. The excluded tones could be used as a bass line, for example, or as a melody, or some combination of both.

Incomplete partitions also introduce a new variable: *span spacing*. Until now we've assumed that an interval set is *tightly packed*, meaning all of its spans are adjacent and any unused tones are grouped together at the end. For example, set {234} spans nine semitones in total, leaving three pitches unused.

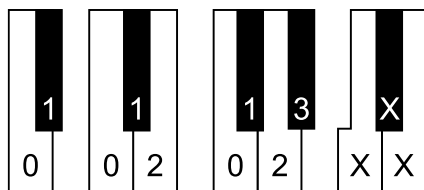


Figure 8: Set {234} tightly packed.

Alternatively, we can *spread* the spans, interleaving the unused tones between them as shown below.

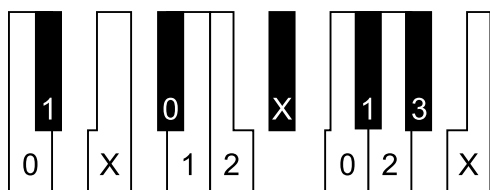


Figure 9: Set {234} spread apart.

Spreading the spans may significantly change the resulting distribution of pitch class sets, as the following table shows. Compared to the packed set, the spread set has more triads and less clusters, and will therefore sound more consonant.

{234} packed				{234} spread			
Forte	Pc set	Quality	Count	Forte	Pc set	Quality	Count
3-3	(0,1,4)		2	3-7	(0,2,5)		2
3-4	(0,1,5)		2	3-8	(0,2,6)		4
3-5	(0,1,6)		2	3-9	(0,2,7)	suspended	3
3-6	(0,2,4)		1	3-10	(0,3,6)	diminished	3
3-7	(0,2,5)		4	3-11A	(0,3,7)	minor	5
3-8	(0,2,6)		4	3-11B	(0,4,7)	major	5
3-9	(0,2,7)	suspended	2	3-12	(0,4,8)	augmented	2
3-10	(0,3,6)	diminished	2	Total			24
3-11A	(0,3,7)	major	2				
3-11B	(0,4,7)	minor	2				
3-12	(0,4,8)	augmented	1				
Total			24				

Table 8: Set {234} trichord summaries, contrasting packed versus spread spans.

Below is set {234} in balanced Gray order, with optimally short spans, and spread as discussed above, so that one unused pitch appears between each of its spans. This is its most consonant spacing.

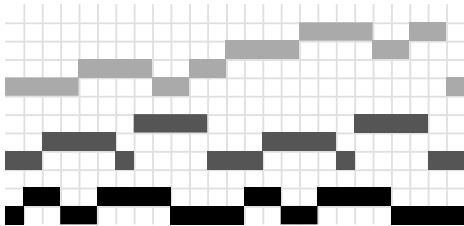


Figure 10: Set {234} in balanced Gray order, with short spans, and spread. [▶ audio](#)

Other configurations are possible, for example one could pack the first two spans together and space only the third span apart. The order of the radixes in a set may also affect its consonance. For example sets {234} and {324} generate subtly different distributions of pitch class sets.

## 7. The BalaGray solver

As permutation count increases, the number of possible orderings increases factorially, and optimal solutions become exponentially scarcer [11]. Hence the most expedient way to satisfy our constraints was by creating a custom solver. Our solver is called BalaGray, it's written in standard C++, and it's freely available. BalaGray is a depth-first heuristic search that, for any mixed-radix interval set,

1. finds a closed Gray cycle with a single-digit transition at each step,
2. distributes the transitions as evenly as possible across all digits, and
3. minimizes the longest span between successive transitions of any digit.

BalaGray does this while visiting only a tiny fraction of the orderings, allowing a laptop to solve sets as large as {3333}, which has 81 permutations. The solver's performance is enhanced by several optimizations, discussed below.

### 7.1 Indices instead of numerals

A unique zero-based index is assigned to each permutation of the set being iterated. These indices can be manipulated more efficiently than their corresponding mixed-radix numerals. The solver operates on indices throughout the iteration, and only maps them back to numerals when outputting the result.

## 7.2 Gray successor table

The solver pre-calculates a table containing one row for each permutation of the set being iterated. Each row lists the possible Gray successors of that permutation. For example if we’re iterating set {23}, the Gray successors of 00 are 01, 02, and 10. The number of Gray successors in a mixed-radix system having radices  $r_1, r_2, \dots, r_n$  is given by:

$$N_{\text{succ}} = \sum_{i=1}^n (r_i - 1)$$

That is, subtract 1 from each radix, and then take the sum of the resulting differences. The Gray successor table expedites crawling the tree of potential solutions. For efficiency, the successors are stored as indices rather than numerals.

## 7.3 Wraparound prediction

While crawling a branch, BalaGray maintains a *visited mask*—a bit-vector with one bit for each permutation—so it can efficiently test whether a permutation has already been used on the branch.

Before the crawl begins, BalaGray also builds a *wrap mask*: a similar bit-vector that indicates which permutations are Gray successors of the initial state. At each node, the intersection of the wrap and visited masks is computed. If their intersection equals the wrap mask, every wraparound successor has been consumed, so the current branch can no longer close into a Gray cycle and is abandoned.

## 7.4 Tree equivalence

The crawler can start from any permutation of a given interval set and obtain equally optimal results. Consequently, the crawler always starts from permutation zero, and thereby skips an entire level of searching. If a different starting value is desired for aesthetic reasons, the crawl result can be rotated, or its digit values can be swapped.

## 7.5 Heuristic branch pruning

For interval sets with more than 20 permutations, exhaustive crawling is impractically slow, even though we only traverse single-digit changes. In such cases, we reduce the number of possibilities by abandoning branches that become too unbalanced [12]. At each node, the *imbalance*—the difference between the maximum and minimum per-digit transition counts—is computed. If the imbalance exceeds a threshold, the current branch is *pruned*, and crawling resumes from the node’s parent. The threshold must be empirically tuned, but for most sets, a value of three is optimal.

Our results are summarized in the table below. The first column indicates whether the set’s number of permutations is evenly divisible by its digit count, which is a prerequisite for perfect balance. Roughly a third of the interval sets are exhaustively crawled, but pruning is necessary for the rest. An optimally balanced Gray ordering of each interval set is provided in a supplemental [table](#).

Evenly Divisible	Perfect Balance	Set Count	Set Codes
Yes	Proved Yes	37	22, 24, 34, 26, 44, 36, 45, 28, 46, 38, 47, 56, 2A, 48, 66, 223, 233, 234, 333, 226, 235, 334, 236, 335, 344, 237, 246, 336, 345, 2222, 2223, 2224, 2225, 2234, 2226, 2244, 2334
	Proved No	4	23, 25, 27, 29
	Unproved	2	2233, 2235
No	Impossible	22	33, 35, 37, 55, 39, 57, 222, 224, 225, 244, 227, 245, 228, 255, 444, 2333, 3333, 22222, 22223, 22224, 22233, 222222
Total		65	

Table 9: Interval sets with perfectly balanced Gray orderings.

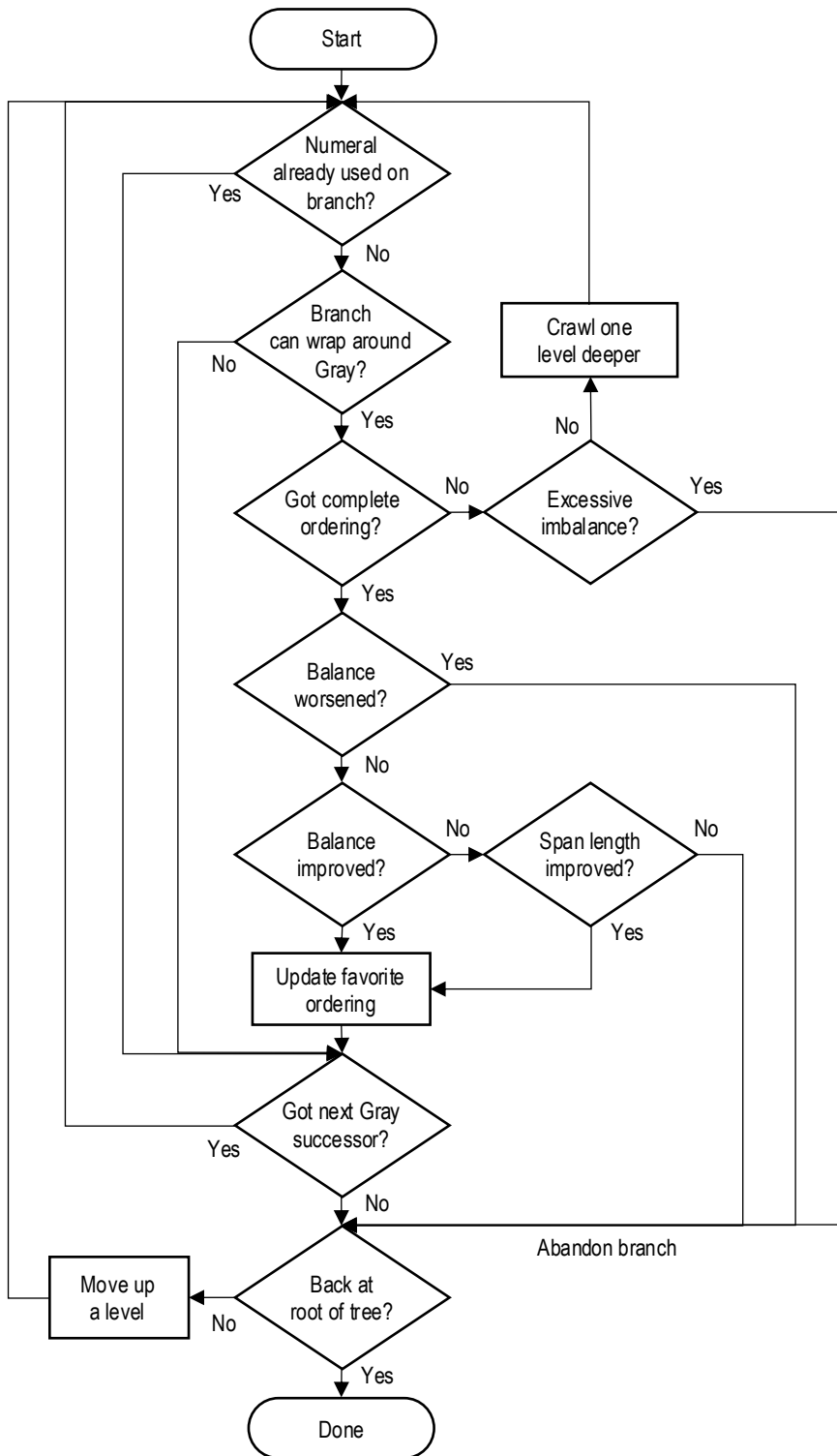


Figure 11: The BalaGray solver.

## 8. Conclusion

We sought to generate atonal chord progressions that maximize granularity while excluding chromatic clusters. By defining and enumerating interval sets, we constructed a comprehensive system of chord generators. We then found granular orderings for all of the prime-form interval sets, using the BalaGray solver. These orderings feature well-balanced Gray transitions and short spans, providing ready-to-use tools for composition.

## 9. References

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## Appendix

### Trichords produced by set {444} in counting order

#	Pc set	Forte	Quality	#	Pc set	Forte	Quality
1	(0,4,8)	3-12	Caug	33	(0,4,A)	3-8	C7
2	(1,4,8)	3-11	Db-	34	(1,4,A)	3-10	Bbdim
3	(2,4,8)	3-8	E7	35	(2,4,A)	3-8	Bb
4	(3,4,8)	3-4	E	36	(3,4,A)	3-5	Gb
5	(0,5,8)	3-11	F-	37	(0,5,A)	3-9	Fsus
6	(1,5,8)	3-11	Db	38	(1,5,A)	3-11	Bb-
7	(2,5,8)	3-10	Ddim	39	(2,5,A)	3-11	Bb
8	(3,5,8)	3-7	F-	40	(3,5,A)	3-9	Bbsus
9	(0,6,8)	3-8	Ab7	41	(0,6,A)	3-8	Gb
10	(1,6,8)	3-9	Dbsus	42	(1,6,A)	3-11	Gb
11	(2,6,8)	3-8	D	43	(2,6,A)	3-12	Daug
12	(3,6,8)	3-7	Eb-	44	(3,6,A)	3-11	Eb-
13	(0,7,8)	3-4	Ab	45	(0,7,A)	3-7	G-
14	(1,7,8)	3-5	Eb7	46	(1,7,A)	3-10	Gdim
15	(2,7,8)	3-5	Bb	47	(2,7,A)	3-11	G-
16	(3,7,8)	3-4	Ab	48	(3,7,A)	3-11	Eb
17	(0,4,9)	3-11	A-	49	(0,4,B)	3-4	C
18	(1,4,9)	3-11	A	49	(1,4,B)	3-7	Db-
19	(2,4,9)	3-9	Asus	51	(2,4,B)	3-7	B-
20	(3,4,9)	3-5	B7	52	(3,4,B)	3-4	E
21	(0,5,9)	3-11	F	53	(0,5,B)	3-5	G7
22	(1,5,9)	3-12	Dbaug	54	(1,5,B)	3-8	Db7
23	(2,5,9)	3-11	D-	55	(2,5,B)	3-10	Bdim
24	(3,5,9)	3-8	F7	56	(3,5,B)	3-8	B
25	(0,6,9)	3-10	Gbdim	57	(0,6,B)	3-5	D
26	(1,6,9)	3-11	Gb-	58	(1,6,B)	3-9	Gbsus
27	(2,6,9)	3-11	D	59	(2,6,B)	3-11	B-
28	(3,6,9)	3-10	Ebdim	60	(3,6,B)	3-11	B
29	(0,7,9)	3-7	A-	61	(0,7,B)	3-4	C
30	(1,7,9)	3-8	A7	62	(1,7,B)	3-8	G
31	(2,7,9)	3-9	Dsus	63	(2,7,B)	3-11	G
32	(3,7,9)	3-8	Eb	64	(3,7,B)	3-12	Ebaug

## Supplements

Optimally balanced Gray orderings for all prime form interval sets ([HTML](#), [CSV](#), [PLM](#)).

BalaGray [source code](#), free and open-source.