

1.(15 pts)Let $A = \begin{bmatrix} -3+4i & 4+3i \\ 2-i & 1+2i \end{bmatrix}$, calculate $\|A\|_1$, $\|A\|_2$ and $\|A\|_\infty$.

2.(15 pts)Given $u = [1, 3]^T$.Calculate $\oint_{|\xi|=100} (1+3\xi+2\xi^2)(\xi I_2 - uu^T)^{-1} d\xi$

3.(15 pts)Construct a matrix $A \in \mathbb{R}^{n \times n}$ such that A^2 is symmetric,while A not.

4.(15 pts)Given linear transformation $T:v \rightarrow v(v \in \mathbb{R}^n)$ which satisfies $T^{n-1} \neq 0$ and $T^n = 0$,prove that $\exists x \in v$ such that $x, T(x), \dots, T^{n-1}(x)$ are linearly independent.

5.(15 pts) A and B are nonsingular matrices.Prove that $B^{-1} - A^{-1} = A^{-1}(A - B)A^{-1} + A^{-1}(A - B)B^{-1}(A - B)A^{-1}$

6.(15 pts)Given $A, B \in \mathbb{C}^{n \times n}$. A is negative finite and B is oblique Hermitite.Prove that there exists $X \in \mathbb{C}^{n \times n}$,let X^*AX and X^*BX be diagonal.

7.(15 pts)Prove that $\text{adj}(A)$ can be represented as the polynomial of A .

8.(15 pts)Known $A \in \mathbb{C}^{n \times n}$.Prove that: $\rho(A) < 1 \Leftrightarrow \exists$ Hermite Q ,satisfies $Q - A^*QA \succ 0$.