Functional and logic programming
- **written exam -**

Important:

1. Subjects are graded as follows: of - 1p; A – 1.5p; B - 2.5p; C - 2.5p; D - 2.5p.
2. Prolog problems will be resolved using SWI Prolog. The following are required: (1) explanation of the code and of the reasoning behind it; (2) recursive model that solves the problem, for all the predicates used; (3) specification of every predicate (parameters and their meaning, flow model, type of the predicate - deterministic/non-deterministic).
3. Lisp problems will be resolved using Common Lisp. The following are required: (1) explanation of the code and of the reasoning behind it; (2) recursive model that solves the problem, for each function used; (3) specification of every function (parameters and their meaning).

A. Given the following PROLOG predicate definition **f(integer, integer),** with the flow model (i, o):

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f(1, 1):-!.
f(K,X):-K1 is K-1, f(K1,Y), Y>1, !, K2 is K1-1, X is K2.
f(K,X):-K1 is K-1, f(K1,Y), Y>0.5, !, X is Y.
f(K,X):-K1 is K-1, f(K1,Y), X is Y-1.
```

Rewrite the definition in order to avoid the recursive call **f(J,V)** in all clauses. Do NOT redefine the predicate. Justify your answer.

B. Given a nonlinear list that contains numerical and non-numerical atoms, write a Lisp program that verifies if the following three lists are equal: the list of all atoms on levels multiple of 3 (3, 6, etc.), the list of all atoms on levels of the form *3k+1* (1, 4, 7, etc.) and the list of all atoms on levels of the form *3k+2* (2,5,8, etc.). For example, for the list (A 1 (A 1(A 1(B 777 (B (B 777 C) 777 C) C) D) D) D) the result will be true.

C. Write a PROLOG program that generates the list of all subsets with N elements, using the elements of a list, such that the sum of elements from a subset is an even number. Write the mathematical models and flow models for the predicates used. For example, for the list L=[1, 3, 4, 2] and N=2 ⇒ [[1,3], [2,4]].

D. An n-ary tree is represented in Lisp as (node subtree1 subtree2 ...).. Write a function to return the list of nodes on even levels, in increasing level order (0, 2, …). The root level is assumed zero. **A MAP function shall be used.**
Example for the tree (a (b (g)) (c (d (e (h))) (f))) => (a g d f h)