b) $x = c_1 t + c_2 t^{-2}$

Type: second order linear homogeneous diff with variable coeficient

c) $t^2 x'' + 2t x' - 2x = 0$, $x(1) = 0$, $x'(1) = 1$

$x(1) = c_1 + c_2 = 0$
$x'(1) = c_1 - 2c_2 = 1$ $\Rightarrow \begin{cases} c_1 + c_2 = 0 \quad | \cdot (-1) \\ c_1 - 2c_2 = 1 \end{cases}$

$\Rightarrow \begin{cases} -c_1 - c_2 = 0 \\ c_1 - 2c_2 = 1 \quad (+) \end{cases}$

$/ \quad -3c_2 = 1 \Rightarrow c_2 = -\frac{1}{3}$

$c_1 + \left(-\frac{1}{3}\right) = 0 \Rightarrow c_1 = \frac{1}{3}$

$\Rightarrow x = \frac{1}{3} t - \frac{1}{3} t^{-2}$

1.7.35 $\quad x' + \frac{1}{t^2} x = 0$, $t \in (-\infty, 0)$

a) $x = e^{\frac{1}{t}}$

$x' = \left(\frac{1}{t}\right)' \cdot e^{\frac{1}{t}} = -\frac{1}{t^2} \cdot e^{\frac{1}{t}}$

$\Rightarrow -\frac{1}{t^2} \cdot e^{\frac{1}{t}} + \frac{1}{t^2} \cdot e^{\frac{1}{t}} = 0$

So, $x = e^{\frac{1}{t}}$ is a solution for $x' + \frac{1}{t^2} x = 0$

$x = \frac{1}{e^t}$ solution for the LHDE $\Rightarrow x = c \cdot e^{\frac{1}{t}}$

b) $x' + \frac{1}{t^2} x = 0$, $x(-1) = 1$

$\Rightarrow x = c \cdot e^{\frac{1}{t}}$

$\Rightarrow x(-1) = c \cdot e^{-1} = c \cdot \frac{1}{e} = 1 \Rightarrow c = e$

$x = e \cdot e^{\frac{1}{t}} = e^{\frac{1}{t}+1}$

c) $x' + \frac{1}{t^2} x = 1 + \frac{1}{t}$, $t \in (-\infty, 0)$

$x_p(t) = t$ is a particular sol.

$\Rightarrow x_p'(t) = 1$

$x_p'(t) + \frac{1}{t^2} x_p(t) = 1 + \frac{1}{t}$

$\Rightarrow 1 + \frac{1}{t^{\cancel{2}}} \cdot \cancel{t} = 1 + \frac{1}{t}$

$\Rightarrow 1 + \frac{1}{t} = 1 + \frac{1}{t}$ (T)

$x = x_p + x_h$

$x_p = t$ $\quad x_h = c \cdot e^{\frac{1}{t}}$

$\Rightarrow x = t + c \cdot e^{\frac{1}{t}}$

1.7.34. $L(x) = x'' + 25x$

i) $L(x) = 0$, $x(0) = 0$, $x'(0) = 1$

the charact. eq. : $r^2 + 25 = 0 \Rightarrow r^2 = -25$

$\Rightarrow r = \pm 5i \vdash \cos 5t, \sin 5t$

$\Rightarrow x = c_1 \cos 5t + c_2 \sin 5t$

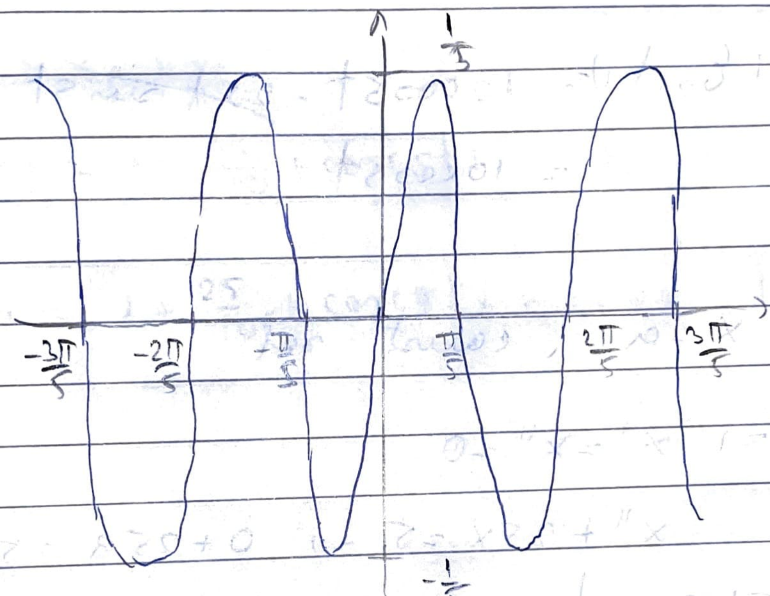
$x(0) = 0 \Rightarrow c_1 \cdot 1 + c_2 \cdot 0 = 0 \Rightarrow c_1 = 0$

$x' = -5c_1 \sin 5t + 5c_2 \cos 5t$.

$x'(0) = 1 \Rightarrow -5c_1 \cdot 0 + 5 \cdot c_2 \cdot 1 = 1$

$\Rightarrow 5 \cdot c_2 = 1 \Rightarrow c_2 = \frac{1}{5}$

$\Rightarrow x = \frac{1}{5} \sin 5t$



It is bounded $-\frac{1}{5}$ to $\frac{1}{5}$

It is oscillatory with const. amplitude

iii) $\varphi_1(t) = t\cos(5t)$ $\quad \varphi_2(t) = t\sin(5t)$

$x = 5 \Rightarrow x' = 0 \Rightarrow x'' = 0$

$L(5) = 0 + 25 \cdot 5 = 125$

$\varphi_1'(t) = \cos 5t - 5t \sin(5t)$

$\varphi_1''(t) = -5 \sin 5t - 5(\sin 5t + 5t \cos 5t)$

$= -10 \sin 5t - 25t \cos 5t$

$\Rightarrow L(\varphi_1(t)) = -10\sin 5t - \cancel{25t}\cos 5t + \cancel{25t}\cos 5t$

$= -10\sin 5t$

$\varphi_2'(t) = \sin 5t + 5t\cos 5t$

$\varphi_2''(t) = 5\cos 5t + 5(\cos 5t - 5t\sin 5t)$

$= 10\cos 5t - 25t\cdot\sin 5t$

$L(\varphi_2(t)) = 10\cos 5t - \cancel{25t}\sin 5t + \cancel{25t}\sin 5t$

$= 10\cos 5t$

iii) $x = a$, const. sol

$\Rightarrow x' = x'' = 0$

$x'' + 25x = 5 \Rightarrow 0 + 25a = 5$

$\Rightarrow a = \frac{1}{5} \Rightarrow x = a = \frac{1}{5}$

iv) gen. sol. for $L(x) = 25 - 25\sin(5t)$

$x'' + 25x = 25 - 25\sin(5t)$

$= 5\cdot 5 + \frac{25}{10}\cdot(-10\sin(5t))$

$= 5f_1 + \frac{25}{10}f_2$

$\Rightarrow f = 5f_1 + \frac{25}{10}f_2$

Using superpos. principle: $x_p = 5x_{p1} + \frac{25}{10}x_{p2}$

We already know that:

$x_{p1} = 5$ which is a part. sol. for $L(x) = 5$

$x_{p2} = t\cos(5t)$ which is a part. sol. for $L(x) = -10\sin(5t)$

$x_p = 5 \cdot \frac{1}{5} + \frac{25}{10} + \cos(5t)$

$= 1 + \frac{25}{10} + \cos(5t)$

$x = x_p + x_h$

$x_p = 1 + \frac{25}{10} + \cos(5t)$ $\quad x_h = c_1\cos 5t + c_2\sin 5t$

$\Rightarrow x = 1 + \frac{25}{10} + \cos(5t) + c_1\cos 5t + c_2 \cdot \sin 5t$