A First Simulation Example on Designing and Assessing a Regression Function

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Load the packages

```
library(mvtnorm)
library(tidyverse)
```

First Simulation Example on Designing and Assessing a Regression Function

binormal distribution : mean = 0 and rho = 0.8

```
set.seed(8725)
sigma = matrix(c(1,0.8, 0.8,1),2,2)    # covariance matrix
sim10 <- rmvnorm(n=10, mean = c(0,0),sigma = sigma)
sim10 <- data.frame(X = sim10[,1], Y = sim10[,2])
sim10
```

```
##              X          Y
## 1  -1.0841175 -0.4781037
## 2   0.5094787  0.7089661
## 3  -1.0807710 -1.5924509
## 4   0.5471389  0.6857206
## 5   1.1352392  1.4700571
## 6  -1.3675838 -1.2846516
## 7   0.3524412  0.5583856
## 8   0.1494487 -0.4885702
## 9   1.4501004  1.2701760
## 10  0.3195384 -1.8624451
```

Fit the data to a linear model

```
model1 <- lm(Y~X,data = sim10)
summary(model1)
```

```
##
## Call:
## lm(formula = Y ~ X, data = sim10)
##
## Residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -1.9751 -0.2642  0.2773  0.4163  0.7354
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.1892     0.2671  -0.709   0.4987
## X             0.9448     0.2912   3.245   0.0118 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8401 on 8 degrees of freedom
## Multiple R-squared:  0.5682, Adjusted R-squared:  0.5143
## F-statistic: 10.53 on 1 and 8 DF,  p-value: 0.01179
```

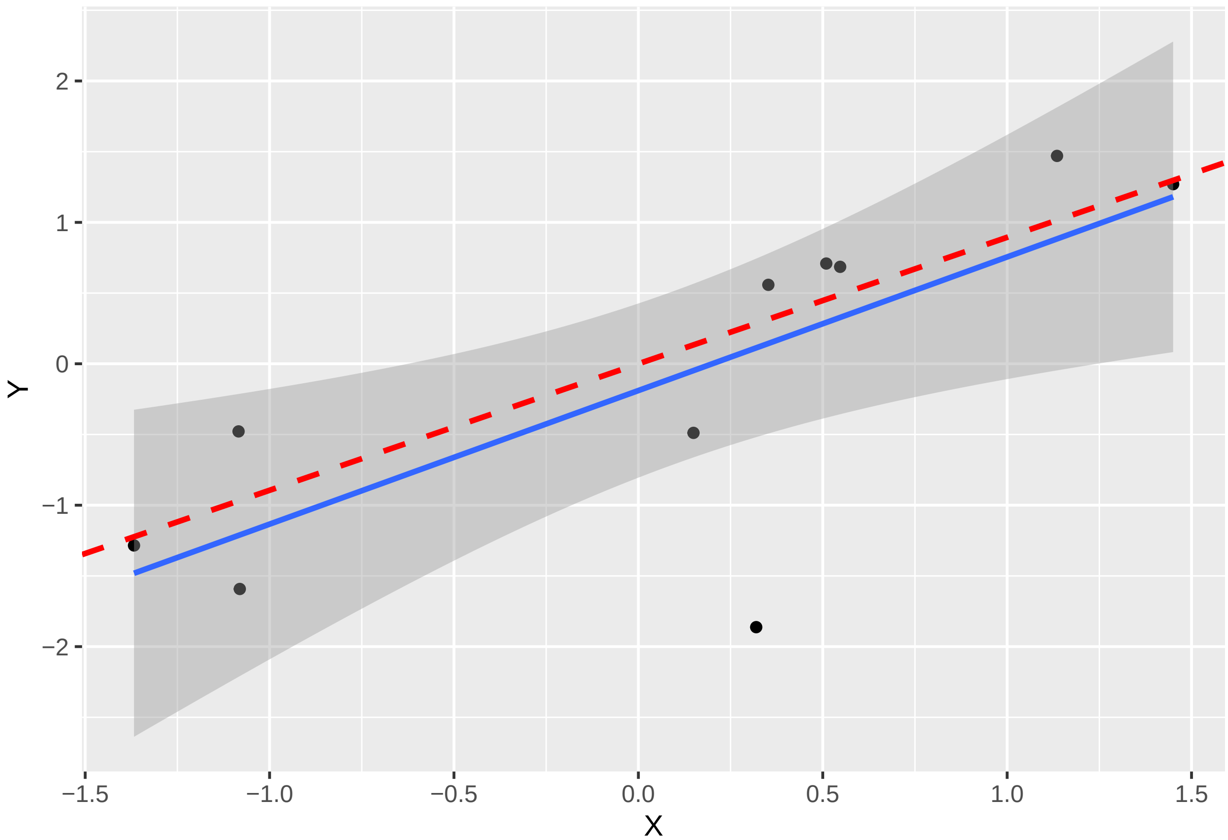
Mean Squared Error (MSE)

```r
MSE <- sum(model1$residuals^2)/10
MSE
```

```
## [1] 0.5646718
```

Plot the linear model, data and best regression function

```r
p1 <- ggplot(data = sim10,aes(x = X, y = Y))+
  geom_point()+
  stat_smooth(method = lm)+
  geom_abline(intercept = 0, slope = sqrt(0.8), color="red",
              linetype="dashed", size=1)
p1
```



Generate the large data (1000 observations)

```
set.seed(8725)
sigma = matrix(c(1,0.8, 0.8,1),2,2)
sim1000 <- rmvnorm(n=1000, mean = c(0,0),sigma = sigma)
sim1000 <- data.frame(X = sim1000[,1], Y = sim1000[,2])
sim1000
```

```
##                X             Y
## 1   -1.084117490 -0.4781036725
## 2    0.509478738  0.7089660761
## 3   -1.080770972 -1.5924509202
## 4    0.547138899  0.6857205810
## 5    1.135239198  1.4700570799
## 6   -1.367583774 -1.2846516340
## 7    0.352441175  0.5583855798
## 8    0.149448680 -0.4885701990
## 9    1.450100366  1.2701760499
## 10   0.319538359 -1.8624451047
## 11   0.704338313  0.3630831230
## 12  -1.198137838 -1.7410700665
## 13  -0.075257783 -0.2848967451
## 14  -1.280428226 -2.1119171086
## 15   1.057342002 -0.3714832624
## 16   0.918981582  1.4873301476
## 17   0.158159753  0.3449520181
```

```
## 18   -0.702192123 -0.5152454412
## 19    0.783171828 -0.3016003124
## 20   -0.200699883  0.4631900726
## 21    0.020624180  0.9599737333
## 22   -1.532790606 -0.7757525370
## 23   -1.393812866 -0.5096054761
## 24    1.421154501  0.6051814175
## 25    1.268667241 -0.2122785712
## 26    1.403520525  2.3309283876
## 27    0.959065287  0.1954050754
## 28   -0.704940279 -0.5077383884
## 29   -1.515211867 -1.3330210554
## 30    0.486164854  0.6154623078
## 31    0.283567895  0.2682433407
## 32    0.899616176 -0.3191040481
## 33    1.330829370  1.1057098175
## 34    0.836168851  0.0377949446
## 35    0.642927710  1.4879092508
## 36    0.269197464 -0.0370532767
## 37   -0.119703395 -0.3532883243
## 38    0.967724492  0.4155019307
## 39    0.346642718  0.4816896718
## 40   -0.563245798 -1.5010237018
## 41   -1.450346579 -0.5108962671
## 42   -1.050389634 -0.8726119221
## 43   -0.563759775 -1.1282739458
## 44   -0.696144147 -0.8150765704
## 45   -0.720750939  0.0606610309
## 46   -0.075026476  0.4456644491
## 47   -0.094303103 -0.0078163719
## 48   -0.160890621  0.1247445367
## 49    0.056678025 -0.2081023118
## 50   -0.592477042 -1.2043439700
## 51    0.577897284  1.1196814533
## 52    1.033836428  1.2848728304
## 53    1.164014512  1.1061509629
## 54    0.063114327 -0.3545443583
## 55   -0.321019302 -0.3837952752
## 56   -0.991882431 -0.7029455684
## 57   -2.007266467 -1.5359069738
## 58    0.421672810  0.1152624304
## 59   -1.105161653 -0.7320671798
## 60   -0.572673542 -0.8878899804
## 61    0.701484615  0.7026718346
## 62    0.718117617  0.6086776973
## 63   -0.753576211 -0.4083428700
## 64    1.574990138  0.9330051767
## 65   -0.534758726 -0.1586869455
## 66    0.841889635  0.6695660831
## 67    0.854089746  0.5639856567
## 68    1.385841228  1.9610407462
## 69   -1.771746902 -1.6558809028
## 70   -0.907177347  0.0143793767
## 71    0.710385394  0.3725961853
```

```
## 72    1.511672113  2.3509488324
## 73    0.603065588  0.6426170897
## 74    0.649423043  0.7059143954
## 75   -0.980272360  0.0874836007
## 76    0.092703828  0.3369433500
## 77    0.689218449  0.9959692224
## 78   -0.441605734  0.3136346113
## 79    0.300422446  1.1935159461
## 80   -0.999105251 -0.8231785434
## 81    1.688154266  1.1610531732
## 82    1.390362964  1.4807473427
## 83    0.525109762 -0.0704605648
## 84   -0.696484103 -0.2103861073
## 85   -0.144771539 -0.1358711583
## 86    0.706321504  0.2053770830
## 87   -0.109682906  0.0550299828
## 88   -1.779841133 -1.6621384457
## 89   -1.021196496 -0.2660303654
## 90   -0.210121000 -0.0541577320
## 91    0.134572263  0.2684540987
## 92    0.147077403 -0.0702307242
## 93   -1.205174987 -0.4707905128
## 94   -1.601584512 -1.0095918368
## 95   -1.047078710 -1.1067088855
## 96    0.314883812 -0.2526571236
## 97   -0.634134633 -0.5320335633
## 98   -0.121037574 -0.8291092549
## 99   -0.345523574  0.1870797147
## 100   0.823925343  1.5794222173
## 101   0.512375492  0.1473006831
## 102   0.765263695  1.3489887571
## 103  -1.008014089 -0.4596462614
## 104  -1.068398707 -1.7260172914
## 105   0.772050941  0.2408230523
## 106  -0.231874294  0.6818456147
## 107  -0.220600045 -0.4846058077
## 108  -0.666954109  0.6415664885
## 109   0.881499734  1.2337515284
## 110   0.901358427 -0.0599977381
## 111  -1.727084998 -1.4453310826
## 112   0.779257261  0.9962534987
## 113   1.672769086  1.1189819723
## 114   1.438298907  2.0406008201
## 115   0.023625028 -0.4749147328
## 116  -1.693938168 -1.8043138896
## 117   1.801517957  1.9861427410
## 118   2.369553589  2.3681743205
## 119  -0.256466288 -0.6941410800
## 120  -1.096932880 -2.1799259239
## 121   0.720660793 -0.3486597799
## 122   0.703588680  0.2733110865
## 123  -2.073089184 -0.8748485670
## 124  -0.454609323  0.0176947050
## 125  -1.051047385 -1.4244891955
```

```
## 126  -0.389816229 -0.2571363609
## 127  -0.512560196 -0.5409256876
## 128  -0.102913628 -0.6322997524
## 129  -1.894542804 -0.9663216728
## 130   0.054679008  0.9651915255
## 131  -1.467326250 -1.6530663769
## 132   1.098769738  2.1595497112
## 133  -1.482491625 -1.9198257915
## 134   0.139039355  0.7212779476
## 135   0.430160814 -0.0143831104
## 136   0.232032947 -0.7241527497
## 137   0.644113321  0.1043003443
## 138  -2.126014028 -2.6646657893
## 139  -0.211984550  0.1133716097
## 140   0.062494027  0.2133463515
## 141  -0.311329540 -1.4872255832
## 142  -1.316763814 -2.2790878065
## 143   0.535079392 -0.5599560228
## 144   0.216148110  1.6014961058
## 145  -1.087897073 -1.3379720202
## 146   0.756121418  1.0732516831
## 147  -1.096627790 -0.5807174466
## 148   1.147443172  1.0380809514
## 149  -1.291280948 -0.8857693589
## 150   1.022820615  0.8208307921
## 151  -0.644090765 -0.3291649937
## 152  -1.687881640 -1.3102545896
## 153   2.277814824  1.6725279247
## 154   0.656565052 -0.4505102062
## 155   1.570696660  1.6648471713
## 156   0.045089950  0.3090312976
## 157   0.179710430  0.2238681886
## 158  -0.938402438 -1.3197967560
## 159   0.776430619  1.4373902301
## 160   0.405725725  0.3071289435
## 161  -1.454284397  0.1269584093
## 162  -1.597128088 -2.1434976729
## 163  -0.409195141 -0.8851716106
## 164   1.021848003  0.9263445063
## 165   0.140248136 -0.2828427833
## 166  -1.022625905 -0.9235434410
## 167  -0.793431732 -0.3728652202
## 168  -0.496330536  0.0641292647
## 169   1.878331044  2.2421889887
## 170   1.400577137  0.6730835686
## 171   2.115112554  2.3694465023
## 172   0.700164156 -0.0168407816
## 173   0.298467590  0.1599327102
## 174   0.676726280  1.4673724552
## 175  -0.548921867  0.5781312913
## 176   0.110054917  0.6881793459
## 177  -0.387316319 -0.0401570543
## 178  -1.240055899 -1.2496534164
## 179  -0.386154699 -0.4334113171
```

```
## 180  -1.795429929 -1.2548380539
## 181   0.341053090 -0.4203535276
## 182  -0.286944723 -1.0899870306
## 183  -1.706141145 -2.2108076271
## 184   1.089114357  2.0985201773
## 185  -0.156915847 -0.0212309492
## 186   0.300692003  0.0619402181
## 187  -2.173354632 -1.9764591129
## 188  -0.953228908 -0.1613243165
## 189   1.086158760  1.7524757204
## 190  -2.020435105 -1.0729101725
## 191  -1.593542961 -1.2948307768
## 192  -0.910455371 -0.0124808560
## 193   0.812592431 -0.1643552085
## 194  -1.441835996 -1.9710013288
## 195   0.959480934  0.3628538598
## 196   0.855892294  1.0377189099
## 197  -0.237388474  0.1153716452
## 198  -2.356609851 -2.1325330738
## 199  -0.345213083 -0.4421992275
## 200   0.830596332  0.5298833850
## 201  -0.459224845  0.1252964804
## 202   0.827158397  0.2587558176
## 203  -0.044186203  1.1038003150
## 204  -1.048576520 -0.6447126746
## 205  -1.010790611 -1.0403073787
## 206  -0.221005906  0.4445596301
## 207   0.131558190 -0.6048293803
## 208  -0.292871360  0.2904210854
## 209  -0.034501649  0.7641722811
## 210   0.257764543 -0.0278336406
## 211   0.389686043  0.8164816200
## 212  -0.274560336 -0.2426351303
## 213  -2.074969949 -1.3262304821
## 214   0.847883279  0.3692408393
## 215   1.277852760  0.9374955454
## 216  -0.835156732 -1.3213237529
## 217  -1.110417696 -0.8172995910
## 218   0.430385961  0.1816964408
## 219   0.118819155  0.0746504034
## 220   0.785160356  0.3562434100
## 221   0.884779410  0.8492409816
## 222  -0.219212211 -0.0470041091
## 223   0.245590106  0.1070036365
## 224  -2.579831763 -1.5579804721
## 225   0.825036786  0.4536448661
## 226  -0.522903768 -0.0495039653
## 227  -0.858145852 -1.3256748701
## 228   1.616445562  1.8291738874
## 229   2.635226752  1.9205850820
## 230   0.951456341  0.9026420566
## 231  -1.044473102 -0.1429318151
## 232   0.200902159 -1.1455615384
## 233  -0.665784932 -0.6996500623
```

```
## 234  -0.981931302 -0.2864865537
## 235  -1.237577515 -0.5188546036
## 236   0.218063765 -0.1072726376
## 237   0.021344219  0.0739784093
## 238   1.699797998  1.4961360745
## 239   1.417871911  1.9024879158
## 240  -1.125802012 -1.4369559747
## 241  -0.185163796  0.4367650712
## 242  -1.586233578  0.2130995132
## 243  -0.295002110 -0.5145801441
## 244  -1.328588968 -1.0507676043
## 245   0.014727635  0.3839358641
## 246   0.512857012  0.1027969374
## 247  -1.605461382 -1.0484603657
## 248  -0.143756349 -0.6121782938
## 249  -0.602129222 -0.7484843239
## 250  -1.391632129 -1.8126784337
## 251  -0.028385640 -0.3966536297
## 252  -1.926136094 -1.6219447536
## 253   1.339209992  1.0995026914
## 254   1.018352020  0.3406974529
## 255   0.647904581  0.3222894529
## 256   0.351439718 -0.2486449575
## 257   1.085052924  0.9866904008
## 258  -0.592376196 -0.4409147423
## 259  -1.350433765 -1.3835823628
## 260   0.872281410 -0.0006797150
## 261   0.360089865  0.0307130940
## 262   0.714187950  0.5915246937
## 263  -1.886450758 -1.4574743215
## 264  -0.533252210 -0.0644285765
## 265   1.344366181  0.4899349612
## 266  -1.168808473 -1.5105688586
## 267   0.433513647 -0.1062483190
## 268   1.420601800  1.0864900724
## 269  -3.053015010 -2.5215658485
## 270  -0.023223620  0.4016425820
## 271  -0.077887943 -0.0205546257
## 272   0.706648778  0.1291430848
## 273   0.556584545  0.7210094286
## 274  -0.693020982 -1.0187856325
## 275   1.523265069  1.1713882771
## 276  -1.118018978 -1.6322670477
## 277  -0.314700983 -0.7430785924
## 278   0.472434199  1.7901048512
## 279  -0.038627280 -0.6728108622
## 280   0.188278036 -0.1431653447
## 281  -0.298882393  0.2431275384
## 282  -0.411916657 -1.2404249323
## 283   1.045859859  1.0715227535
## 284  -1.571584248 -2.4605529287
## 285   0.835671035  1.1376714169
## 286   0.168275456  0.7086585925
## 287   0.748647264 -0.3233681094
```

```
## 288   1.425261072  1.9189938929
## 289  -0.151945360 -1.1615218971
## 290  -0.507439696 -1.4395964123
## 291   0.200883776  1.7745403328
## 292  -2.492806980 -2.2068683494
## 293   0.047925549 -0.5444762675
## 294  -0.167744737 -0.5565312680
## 295   0.751247194  1.2329857841
## 296   0.611163211 -0.6736135291
## 297   1.827997337  2.0251905909
## 298   1.154425868  0.5877426866
## 299   0.774373597  1.5877315437
## 300   0.894828745  0.3412298705
## 301   0.133522805  0.2024030671
## 302  -1.754302811 -1.5683058609
## 303   0.965275709 -0.2655776951
## 304   1.795020501  0.9089818353
## 305  -0.755825256 -1.1066563052
## 306   1.176435903  0.6522542404
## 307   0.372367179 -0.5780884179
## 308  -1.425637002 -1.0906541000
## 309   0.644472883  1.1632066769
## 310  -1.300727338 -1.3853115096
## 311  -0.483630502 -0.3814827087
## 312   0.025089436  0.6072226926
## 313   0.249056424  0.2052788335
## 314  -0.707168246  0.2770493193
## 315  -0.439382225 -0.9477432877
## 316  -2.088480799 -2.0959295408
## 317  -1.980212334 -1.2172685634
## 318  -0.050351923 -0.0342986881
## 319   0.101634435  0.1295256113
## 320  -1.356681732 -0.9519082004
## 321  -0.798678688 -1.3646991068
## 322   0.604808459  1.5044752932
## 323  -0.749486082  0.3042690045
## 324  -0.070420827  1.2124060543
## 325  -0.066315041 -0.3507662443
## 326  -0.849145570 -1.2989472711
## 327   0.297257302 -0.6054576871
## 328  -0.084206296 -0.3567529472
## 329   0.738359307  0.4440655325
## 330   1.562600993  0.3371282412
## 331  -1.458520045 -1.0325325803
## 332  -0.743424382 -1.0387069381
## 333  -1.290509273 -0.9086116086
## 334   1.005315332  0.4130314363
## 335  -0.570244683  0.3096943301
## 336   1.207532593  1.0212296857
## 337  -0.953096146 -0.0990840554
## 338   0.139014723 -0.0160834326
## 339  -0.831098645 -1.0292340563
## 340   1.236023938  1.2307982922
## 341   0.856057810 -0.0671894527
```

```
## 342  -0.790172036 -1.5219957827
## 343   0.513161030  1.9317306200
## 344   0.423552995  0.2549149177
## 345   0.280455608 -0.2613135988
## 346  -0.739737909 -0.0805822107
## 347  -1.623136476 -2.3210496686
## 348  -0.657577114 -1.3095551230
## 349  -1.024522604 -0.7511614850
## 350  -0.620119454  0.9324728425
## 351  -1.268172725 -1.7923667370
## 352   0.749701906  0.7820725989
## 353  -0.903349148  0.5223556979
## 354  -0.655604477  0.0006776565
## 355  -0.950288090 -1.1523425360
## 356   0.942322671  0.5616510900
## 357  -1.567342091 -1.5589775357
## 358   0.057238771  0.4756358468
## 359   0.316390366  0.2325902332
## 360  -1.552030718 -1.5477213873
## 361  -2.679599584 -2.6030415975
## 362   0.680477011  1.2418120865
## 363   2.201971573  1.9972204210
## 364   0.628223333  0.2746167694
## 365   0.222212934  1.5111137779
## 366   0.521211733  0.6720524137
## 367   0.187381784  0.3353416313
## 368  -0.734869536 -1.1784271340
## 369  -1.115408152 -0.7458906350
## 370  -0.706991877 -0.7077323148
## 371   0.444471935  0.4569005565
## 372  -1.539568445 -0.4584848705
## 373   0.140011127  0.2923186317
## 374  -0.612372685  0.5164282034
## 375   0.565054168  0.1281039859
## 376  -0.446716564 -0.1281100703
## 377  -0.447766325 -0.1771127806
## 378   0.487261377  0.1484286164
## 379  -0.183967412 -0.4186976177
## 380  -0.613981420  0.8169472828
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## 802   1.431557025  1.5720111951
## 803  -1.899188338 -0.7452215779
## 804   0.283612326  0.1858965422
## 805   0.226115234 -0.2488124406
## 806  -0.334536203  0.4476105014
## 807   1.114580365  1.0962848936
## 808   0.672311859  0.2376436784
## 809   1.122742465  0.9800352029
## 810   0.651190037  0.9672933056
## 811  -0.313388285 -0.5980818498
## 812   0.308107138  0.7683255015
## 813  -0.743848740 -0.8649499837
## 814   0.911722692  0.9543660972
## 815  -0.427139815  0.1983157617
## 816  -0.113239840 -0.4031570453
## 817  -1.764104246 -1.5428529042
## 818  -0.159228559 -0.1824117197
## 819  -0.353845325 -0.7330111419
## 820   1.357886955  2.3126745983
## 821   1.114075656  1.2847171341
## 822   0.847259130 -0.4724675217
## 823  -0.039134695 -0.8036817425
## 824   2.261274837  1.3080684927
## 825   2.130268583  1.8932718529
## 826   0.390574385  0.9360755556
## 827   0.327432595  0.3856871982
```

```
## 828  -0.560575727 -0.2247789191
## 829  -0.716643918 -1.8009085288
## 830   1.702239512  1.4309305233
## 831  -0.997189533 -0.6477196137
## 832   0.104411038  0.3667365804
## 833  -0.707433895 -1.1982439041
## 834  -0.516593665 -0.3534067842
## 835  -2.012703738 -1.2518181274
## 836  -0.431681862  0.1505491878
## 837  -0.615423336 -0.1587497672
## 838  -0.033255716  0.1894930281
## 839  -1.498333664 -0.5663437614
## 840  -0.849124423 -0.6713496465
## 841   1.884396741  1.3053538861
## 842  -0.433710996  0.6787928748
## 843   0.535644148 -1.0198121490
## 844  -1.766488333 -2.0302572026
## 845   0.975669571  0.8116890970
## 846   1.361336481  0.3198877789
## 847   0.665534177  0.2217814911
## 848  -1.286549351 -1.3987596415
## 849   0.991106064  1.4607003062
## 850  -0.983042406 -0.2623210252
## 851  -2.062067317 -1.5001833982
## 852  -0.205504213 -0.3555435742
## 853  -0.303357059 -0.3585699036
## 854   0.339543636  1.0866692780
## 855  -0.346587924 -0.9000607324
## 856   0.406631122  0.2761494550
## 857  -0.486485325 -0.4647514426
## 858   1.208290729  0.7962490380
## 859  -1.889081310 -0.5572450346
## 860   0.418881622  0.5628562692
## 861  -1.138572428 -1.7099272725
## 862  -0.498904411 -0.2545709916
## 863   0.543565261  0.4170341967
## 864  -0.765188025 -0.4794042751
## 865  -0.705241496 -0.1099139969
## 866   0.347378001 -0.1963227733
## 867   1.121890370  1.1475924140
## 868  -0.212009617 -0.8892348178
## 869  -2.776865746 -2.8890639886
## 870   1.048118339  1.0579419663
## 871   0.102976592  0.8703877976
## 872  -0.603640673 -0.5807650388
## 873  -0.494236583 -0.7913335917
## 874  -2.171956917 -1.8224080737
## 875   0.490663830 -0.7719314565
## 876   0.691128552  1.5483379253
## 877   1.005053662  1.0043545869
## 878  -0.416923337 -0.2318894137
## 879  -1.536578813 -1.2523868741
## 880  -0.021777152 -0.9737392327
## 881   1.870679084  0.9875171529
```

```
## 882   0.211801195 -0.3509655091
## 883  -1.056594581 -0.0630780378
## 884   1.633725636  1.1860264590
## 885   0.190772322  0.9484342985
## 886   0.077613056 -0.3656657090
## 887   2.066368713  2.5745183742
## 888   0.473484200  1.5540790265
## 889  -1.395380635 -0.5084349917
## 890  -1.153866021 -0.7249230434
## 891   1.047588985  1.5026934244
## 892  -1.765797844 -2.1285240155
## 893  -0.355125622 -0.6979329212
## 894   0.400158846 -0.3174441543
## 895  -0.825900958 -0.6693870188
## 896  -0.248170094  0.2193066247
## 897  -0.709710128 -0.9525074131
## 898   0.778790028  1.0836409239
## 899  -1.277614779 -1.1195319721
## 900   1.703898016  2.4016253555
## 901  -0.438421717 -0.7536382674
## 902  -0.866602417 -1.8487666369
## 903  -1.176593534 -0.4055366516
## 904   0.620406662  0.7463050699
## 905   0.748660894  0.6345586937
## 906  -0.212853201 -0.3795441910
## 907   1.371834998  0.9348743089
## 908  -1.030321605  0.2753089189
## 909   0.804288367  0.6857212230
## 910  -0.221301859  0.6691516038
## 911   1.267432702  0.4825804813
## 912   0.702419134  1.3755730127
## 913  -0.129894318 -0.3297956543
## 914  -0.112361074 -0.0325495395
## 915   0.360241835  0.6334786787
## 916   0.292173447  0.8666992531
## 917   1.170401100  0.2276981240
## 918   0.226275128 -0.0378637688
## 919  -0.321461709 -0.5115294254
## 920  -0.003895724  0.0366563192
## 921  -0.565702062 -0.3668475915
## 922  -1.342267233 -0.7144980257
## 923  -1.333617109 -0.7525911103
## 924  -1.317838721 -0.8782674121
## 925   1.321930910  0.6999645469
## 926   0.157520265 -0.2095824570
## 927   0.114831440 -0.4705054461
## 928   2.099263303  0.6437850223
## 929   1.070168176  2.1640695681
## 930   0.517191123 -0.1151680069
## 931  -0.785461992 -0.7375504606
## 932  -1.720103029 -1.0190958019
## 933   1.117136763  0.6298822278
## 934   0.372106970  0.9465552370
## 935  -1.761980659 -1.0833297986
```

```
## 936   1.670461952  2.1743982130
## 937  -0.688081653  0.4131097905
## 938   0.569566421  0.4078730080
## 939   0.598062636  1.1230553161
## 940  -0.457663035  0.4422720147
## 941  -1.055852766 -0.7818977112
## 942   2.126808205  1.6259680813
## 943  -0.283452153 -0.1926240677
## 944   1.554937361  1.4219725534
## 945   1.583157713  1.2480965335
## 946   1.088878440  0.6490708912
## 947  -1.949305936 -1.5801394162
## 948   0.309261408  0.0368039317
## 949   1.382699922  1.0945767064
## 950   0.971051347  0.7219133440
## 951   1.311518927  1.1717109899
## 952  -0.796552566 -0.4416278853
## 953   1.058608713  0.5906205855
## 954   0.837484468  1.0115571525
## 955  -0.098877134  0.1783796461
## 956   0.053471789 -0.0906917586
## 957  -1.287322991 -1.4880172329
## 958   0.252556059 -0.4310221233
## 959   0.600277551  0.7811859308
## 960   1.606650866  1.4662583590
## 961  -0.291187804 -0.8171843267
## 962  -1.065172201 -0.9375292660
## 963  -0.349279767  0.1259482372
## 964  -0.341415670 -0.6830520626
## 965  -0.501280683 -0.3714177500
## 966   1.721205643  0.0675266449
## 967  -0.445561491 -1.0967647616
## 968   0.219708668  0.9710028837
## 969  -1.312637287 -1.9484519395
## 970  -0.533746262  0.2879206631
## 971  -0.081895408 -0.1317190923
## 972  -0.289174187 -0.5594259656
## 973   0.248072357  0.4765551864
## 974  -0.539504497 -0.3842229273
## 975  -1.159558489 -0.4569987179
## 976  -0.456511926 -0.5275603115
## 977   0.855482898  0.7871707444
## 978   0.657921038  0.9169095256
## 979   1.486437234  0.6452961984
## 980   0.096034310 -0.0216460770
## 981  -0.237378828  0.8230000315
## 982   1.361506695  0.9437980762
## 983   0.358538393  0.9644671849
## 984  -0.017892368 -0.9562738657
## 985   0.852923021 -0.6189998866
## 986  -0.268153027 -0.1733986353
## 987  -0.127674563 -0.1320547392
## 988  -0.209538435  0.0655513034
## 989  -0.028416456  0.5963666068
```

```
## 990   0.990946143  0.5413114689
## 991  -0.589011182 -0.5308646047
## 992  -0.828418859 -1.1176077300
## 993  -0.587309245 -0.4094707654
## 994   1.001466571  0.9020508919
## 995   2.012791266  0.9974000893
## 996   0.897592367  0.1143106615
## 997   1.919049925  1.2812581621
## 998   0.118505274  0.8732252269
## 999  -0.251090874 -0.8490484920
## 1000  0.645329960 -0.0479988614
```

True error rate

```r
model2 <- lm(Y~X,data = sim1000)
summary(model2)
```

```
##
## Call:
## lm(formula = Y ~ X, data = sim1000)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.13447 -0.39809 -0.00929  0.39848  1.66011
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.01300    0.01816   0.716    0.474
## X            0.81063    0.01776  45.643   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5739 on 998 degrees of freedom
## Multiple R-squared:  0.6761, Adjusted R-squared:  0.6758
## F-statistic:  2083 on 1 and 998 DF,  p-value: < 2.2e-16
```

```r
pred <- predict(model1, newdata = sim1000)

test <- data.frame(actual=sim1000$Y, pred = pred)
test$error <- with(test, pred-actual)


errtr <- with(test, mean(error^2))
errtr
```

```
## [1] 0.3904059
```

Perfomance of best regression function: err*

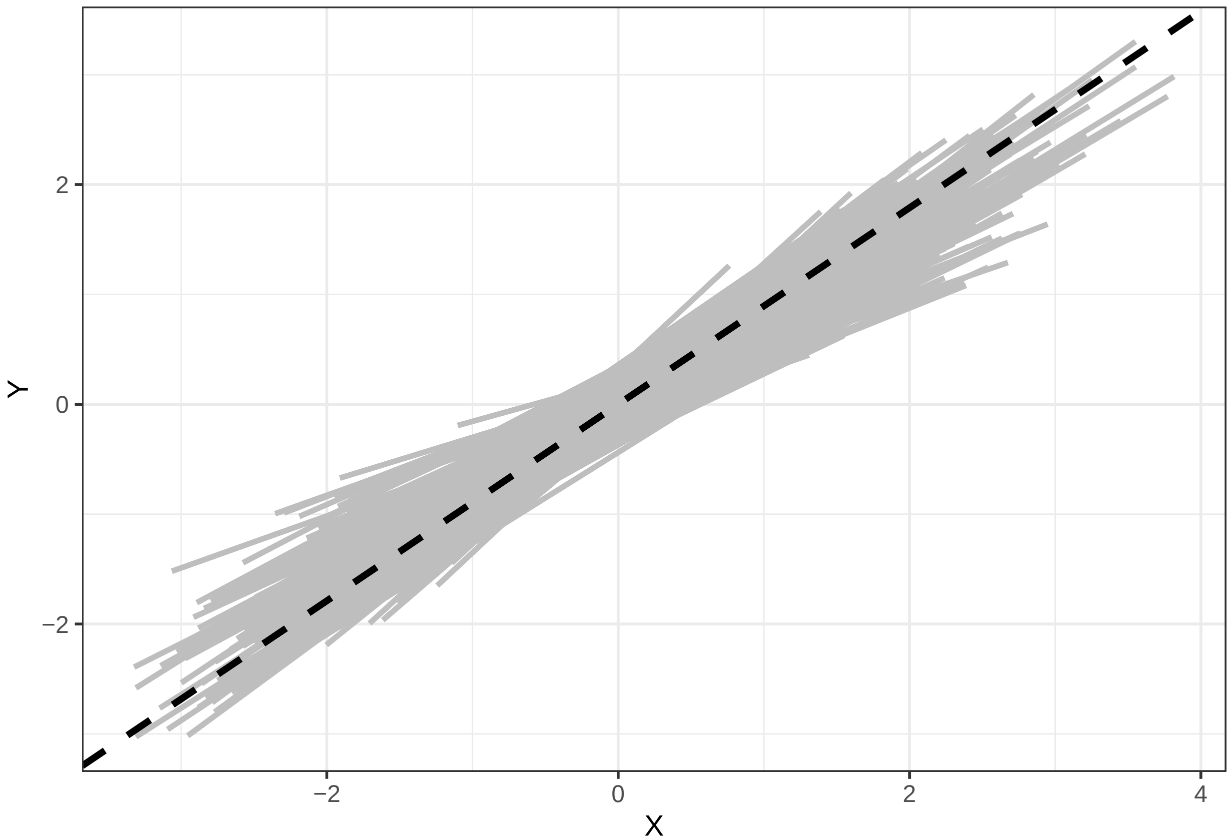
```r
errstar <- sum(model2$residuals^2)/1000
errstar
```

```
## [1] 0.3287304
```

Monte-Carlo (MC) : simulation by repeating the above for 500 training sets of the same size (10 observations) and the same large testing sets (1000 observations)

Plot showing the best regresson function (in bold) and the 500 linear models (in gray).

```
simfunct <- function(v){
  sigma = matrix(c(1,0.8, 0.8,1),2,2)
  sim <- rmvnorm(n=v, mean = c(0,0),sigma = sigma)
  sim <- data.frame(X = sim[,1], Y = sim[,2])
  return(sim)
}
p2 <- ggplot()+
  theme_bw()

Vect_MSE <- NULL
Err <- NULL
for (i in 1:500) {
w <-   simfunct(20)
mod1 <- lm(Y~X,data = w)
MSE1 <- sum(mod1$residuals^2)/10
Vect_MSE <- c(Vect_MSE, MSE1)
p2 <- p2+stat_smooth(method = lm,se = FALSE,data = w,col ="gray",aes(x=X, y=Y))

pred1 <- predict(mod1, newdata = sim1000)

test <- data.frame(actual=sim1000$Y, pred = pred1)
test$error <- with(test, pred1-actual)

errtr <- with(test, mean(error^2))
Err <- c(Err,errtr)
}

p2+geom_abline(intercept = 0, slope = sqrt(0.8), color="black",
                     linetype="dashed", size=1.2)
```



500 MSE values

```
Vect_MSE
```

```
##   [1] 0.4888336 1.1162297 0.9198214 0.7299773 0.5466800 0.8398448 0.7787552
##   [8] 0.7280950 0.4720425 0.6664960 0.7623093 0.4204074 0.5765178 0.4142873
##  [15] 0.6921937 0.6534025 0.5850862 0.4252988 0.3575504 0.4950514 1.0990851
##  [22] 0.5583363 0.5252230 0.4400884 1.0222649 0.5470401 0.8444163 1.0684661
##  [29] 1.0766981 0.4892124 0.7612947 1.0916981 1.1538315 0.8998079 0.4948006
##  [36] 0.7334190 0.5619232 0.5406772 0.3494272 0.7106610 1.0025428 0.4276489
##  [43] 0.7492724 0.6856618 0.4445658 0.4251730 0.5305599 0.5786269 0.4285741
##  [50] 0.9739845 0.6258955 0.3961094 0.3779641 0.5715243 0.7418997 1.0300714
##  [57] 0.3457984 0.4480744 0.8956668 0.4536574 0.8529755 0.3790155 0.8252001
##  [64] 0.6543063 0.5479540 0.5722884 0.6423324 0.5925474 0.3413253 0.7197672
##  [71] 0.7402747 0.4905024 0.9061633 0.6918537 0.6521923 0.4674297 0.5966946
##  [78] 0.4045352 0.6109378 0.5840442 0.4550791 1.0315381 0.6253357 0.6708467
##  [85] 0.5472056 0.4515540 0.7693752 0.5564597 0.3376325 0.6822368 0.5806376
##  [92] 0.6262968 0.4675107 0.4947324 0.6561792 0.6000975 0.6795530 0.5825922
##  [99] 0.9175737 0.4875089 0.9641065 0.6102218 0.6347097 0.7127894 0.9506334
## [106] 0.8916370 0.9194341 0.9686733 0.5882724 0.2575772 0.7099969 0.6425683
## [113] 0.4598153 0.5008298 0.9695116 0.5576804 0.3729730 0.8222172 0.7266429
## [120] 0.4300424 0.2314703 0.5052359 0.5126789 0.6135140 0.4967746 0.5116847
## [127] 0.6575562 0.6274807 0.6084842 0.2844911 0.6597198 0.4357874 0.6219588
## [134] 1.0930685 0.7881179 0.5371920 1.1205367 0.5211763 0.4588964 0.6525475
## [141] 0.6783261 0.7429350 0.4697219 0.8123899 0.8994566 0.5585652 0.4609478
## [148] 0.9357096 0.4892449 0.9139728 0.6602972 0.7246135 0.4800909 0.4667266
```

```
## [155] 0.6332574 0.5158841 0.6081326 0.4236413 0.5162136 1.0113176 0.4765499
## [162] 0.5929937 1.0007866 1.1919409 0.3478944 0.8961478 0.5341193 0.8351860
## [169] 0.6016467 0.6220071 0.8127049 0.5543649 0.7615444 0.4547263 0.4855151
## [176] 0.6094458 0.4433825 0.4725185 0.7233158 0.8334894 1.1271290 0.2894376
## [183] 0.7041478 1.0328859 0.6210605 0.8394832 0.5361571 0.5680911 0.5891098
## [190] 0.4959746 0.6876587 0.5119049 0.6695640 0.9517622 0.7787901 1.1669976
## [197] 0.5137858 0.6542067 0.5613443 0.5973365 0.3632488 0.3392889 1.0442798
## [204] 0.5725773 0.7323862 0.7661204 0.3093553 0.3994996 0.7508004 0.6900500
## [211] 0.8255250 0.5272785 0.7533725 0.4743286 0.8370346 0.6279814 0.3941811
## [218] 0.7685335 1.1230007 0.5654566 0.9725101 0.4799040 0.4922668 0.8043063
## [225] 0.2867780 0.8140899 0.8254599 0.5037118 1.2151711 0.3915388 0.4407652
## [232] 0.6372695 0.4081864 0.8502736 0.8707704 0.9319381 0.7031397 0.5283030
## [239] 1.1809235 0.2939290 0.5513121 0.8799672 0.4870141 0.5183892 0.6854621
## [246] 0.5107752 0.6055199 0.6616653 0.5546307 0.4724400 0.2777769 0.6518602
## [253] 0.7311840 0.5831406 0.6110123 0.6203558 0.4040536 0.7997645 0.7583146
## [260] 0.8548892 0.6648058 0.7161213 0.8492215 0.8312374 0.5820727 0.6597412
## [267] 0.5512969 0.8858589 0.3509122 0.5933791 0.8166448 0.9487631 0.2626715
## [274] 0.6503238 0.3983828 0.4825773 0.4534303 1.0722179 0.7011551 0.9511852
## [281] 0.9910366 0.9488263 0.4647706 0.2089774 0.5846017 0.6117646 0.4269349
## [288] 0.4967912 0.7190509 0.8095872 0.6562887 0.5443398 0.4641901 0.5762443
## [295] 1.0831368 0.8005651 0.4715798 0.8477125 0.3091080 1.2574341 0.5457677
## [302] 0.5643651 0.7766581 0.5476165 0.6595366 0.7372175 0.3706491 0.3145945
## [309] 0.4652562 0.6302915 1.0005391 0.6598489 0.6001261 0.5364217 0.4735265
## [316] 0.7014533 0.4976991 0.5047261 0.4644197 0.6158461 0.5699519 0.8126519
## [323] 0.5786906 0.7172186 0.5808440 0.6143508 1.1919054 0.5840939 0.9261538
## [330] 0.2642137 0.6587688 0.4140470 0.7556838 0.6905473 0.2947181 0.4887895
## [337] 0.6900299 0.8043774 0.8586362 0.6262589 0.4361041 0.7369490 0.9880012
## [344] 0.6544615 0.6197539 0.5905880 0.3534367 0.4407402 0.8366888 0.9402036
## [351] 0.6254605 0.4683338 0.5809625 0.5496078 0.4957112 0.7109149 0.3888964
## [358] 0.8932043 0.3142919 0.5220584 0.7594910 0.7555940 0.4163443 0.6969583
## [365] 0.3268752 0.4638232 0.8645210 0.4709476 0.3914409 0.6229291 0.6121674
## [372] 0.6552146 0.8517021 0.5281565 0.5808843 1.2780437 0.3010645 0.8204333
## [379] 0.4694648 0.4194196 0.6654838 0.7008384 0.4664583 0.7461591 0.4300184
## [386] 0.8212269 0.5118746 0.7396136 0.6533243 0.4366410 0.8047255 0.7962574
## [393] 0.8109491 0.5736768 0.6869414 0.7149414 0.5156399 0.5228788 0.3553293
## [400] 0.4928347 0.6240276 0.3961613 0.4430038 0.6018769 0.5937410 0.3250041
## [407] 0.7645617 0.8218857 0.3814678 0.7090728 0.8578599 0.5604284 0.4928421
## [414] 1.1474552 0.7011868 0.2192342 0.6201945 0.5464264 0.4010913 1.3214779
## [421] 0.5862107 0.5146550 0.4016997 0.9629412 0.5864053 0.3505804 1.1205060
## [428] 0.8561293 0.3295659 0.5906992 1.1229850 0.6483666 0.7886958 0.6156244
## [435] 0.9434228 0.7802980 0.5409571 0.9100848 0.5099421 0.7531933 0.5977470
## [442] 0.4949001 0.7402273 0.5323457 0.8079680 1.0833170 0.4245283 0.5013963
## [449] 0.7802281 0.4784753 0.5492251 0.5275970 0.4656695 0.8527461 0.5455697
## [456] 0.3863432 0.5218853 0.8613990 1.0491465 0.6326350 0.6346402 0.5957202
## [463] 1.0336953 1.2431234 1.1020840 0.6920093 0.6437188 0.4632806 0.6127176
## [470] 0.4462193 0.7940934 0.7769500 0.7592279 0.5766425 0.5078372 0.4799658
## [477] 0.4479424 0.9649070 0.6814942 0.5568222 0.5369749 0.3381950 0.6893624
## [484] 0.5640614 0.9521901 0.6164426 0.7933836 1.0088264 0.5665134 0.7407041
## [491] 0.7144348 0.7231879 0.3412316 0.7322458 0.7307519 0.5688446 0.4937473
## [498] 0.6476666 0.4343396 0.4319162
```

Mean of true errors rate

```
meanv <- mean(Err)
meanv
```

```
## [1] 0.3662152
```

Variance of true errors rate

```
varerr <-  sum((Err-meanv)^2)/(499)
varerr
```

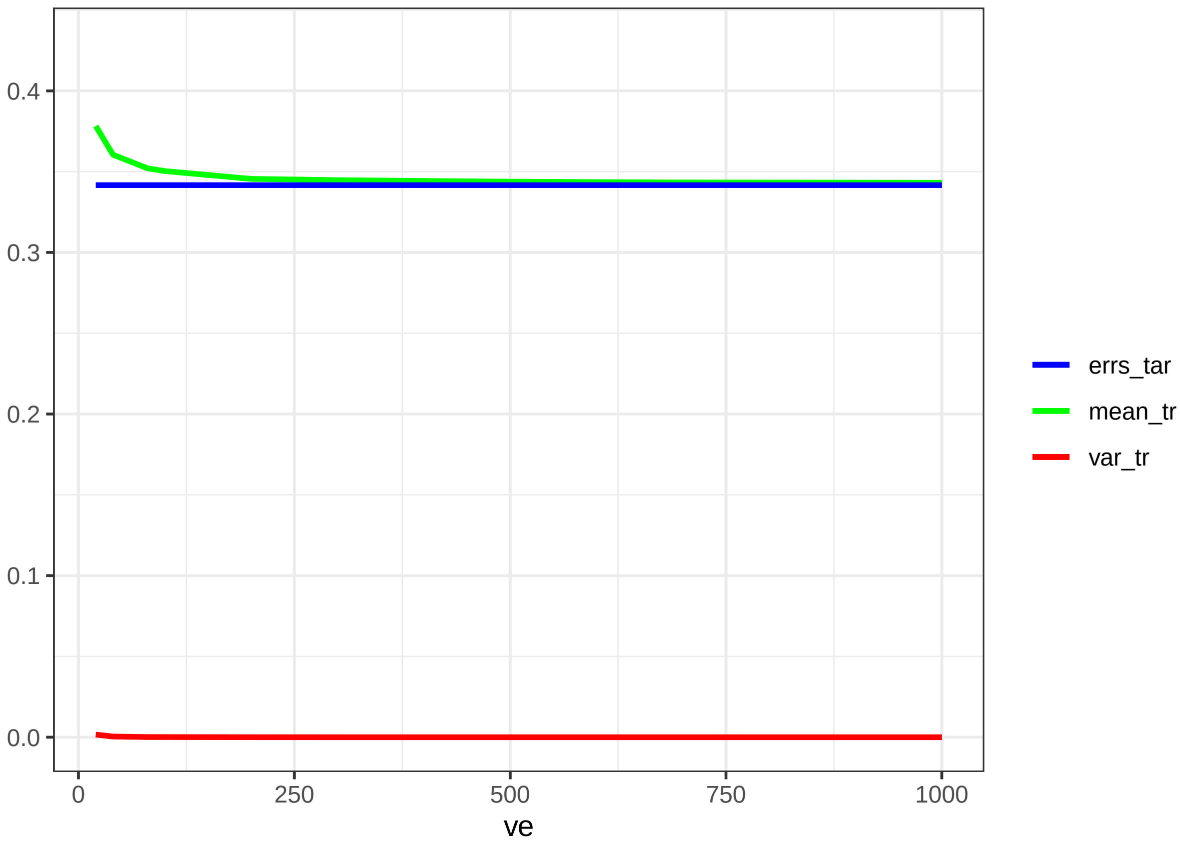
```
## [1] 0.001510281
```

Repetition of the obove in the following training-set sizes: 20, 40, 80, 100, 200, 300, 400, 500, 700, 1000

Plot of the performance of best regression function, the mean and variance of true errors rate.

```
simulfunct <- function(vect){
  sigma = matrix(c(1,0.8, 0.8,1),2,2)
  sim1000 <- rmvnorm(n=1000, mean = c(0,0),sigma = sigma)
  sim1000 <- data.frame(X = sim1000[,1], Y = sim1000[,2])

  vect_errstar <- NULL
  vect_mean <- NULL
  vect_var <- NULL
  for (ntr in vect) {
    model <- lm(Y~X,data = sim1000)
    errstar <- sum(model$residuals^2)/1000
    vect_errstar <- c(vect_errstar,errstar)
    Vect_MSE <- NULL
    Err <- NULL
    meanv <- NULL
    for (i in 1:500) {
      sim <- rmvnorm(n=ntr, mean = c(0,0),sigma = sigma)
      sim <- data.frame(X = sim[,1], Y = sim[,2])
      mod1 <- lm(Y~X,data = sim)
      pred1 <- predict(mod1, newdata = sim1000)
      test <- data.frame(actual=sim1000$Y, pred = pred1)
      test$error <- with(test, pred1-actual)
      errtr <- with(test, mean(error^2))
      Err <- c(Err,errtr)
    }
    meanv <- mean(Err)
    vect_mean <- c(vect_mean, meanv)


    varerr <-  sum((Err-meanv)^2)/(499)
    vect_var <- c(vect_var, varerr)
  }
```

```r
  dt <- data.frame(errs_tar=vect_errstar, mean_tr= vect_mean, var_tr= vect_var )
  return(dt)
}

ve <- c(20, 40, 80, 100, 200, 300, 400, 500, 700, 1000)

dat <- simulfunct(ve)
dat$ve <- ve
p <- ggplot(data=dat,aes(x = ve))+
  geom_line(aes(y=mean_tr,colour = "mean_tr"),size = 1)+
  geom_line(aes(y=var_tr, colour = "var_tr"), size = 1)+
  geom_line(aes(y = errs_tar, colour = "errs_tar"),size = 1)+
  theme_bw()+
  scale_colour_manual("",
                      values = c("mean_tr"="green", "var_tr"="red",
                                 "errs_tar"="blue"))+
  ylim(0,0.43)+
  ylab("")
p
```



Observations

We observe that as we increase the training size, the variance of true error rate remains constant while the mean decrease and when the training set approximates the testing set the mean of true error rate approximate the performance of the best regression function.

We can conclude that the size of the training set has a large effect on the performance of the regression model when the training size is too small compared to the testing set.