Statistical Modeling of Earth's Plasmasphere

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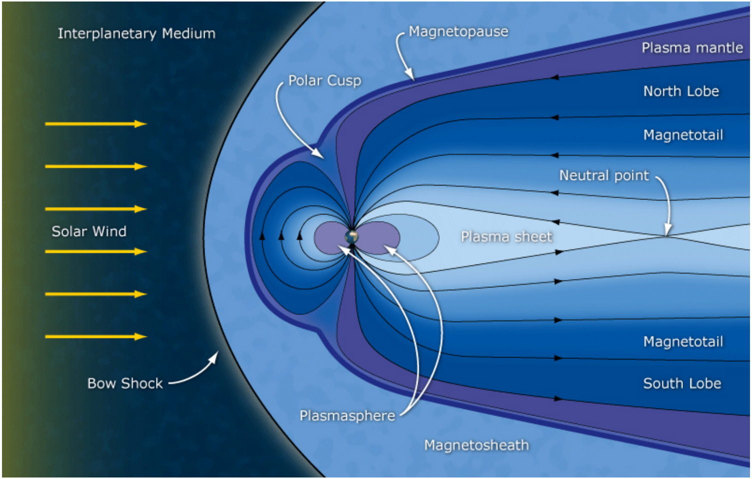
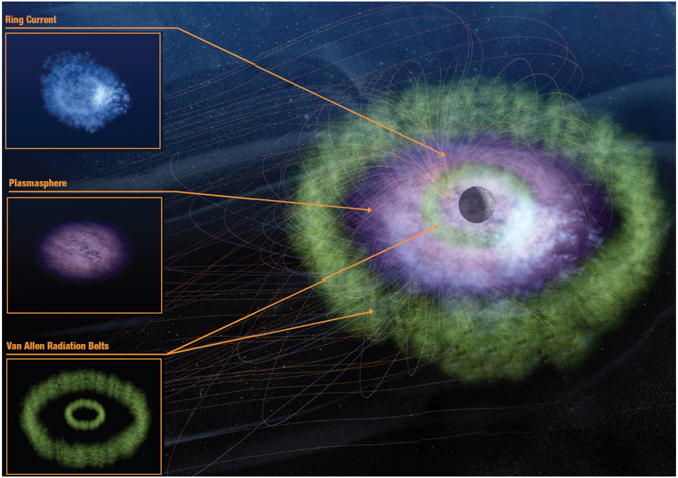
July 21, 2016

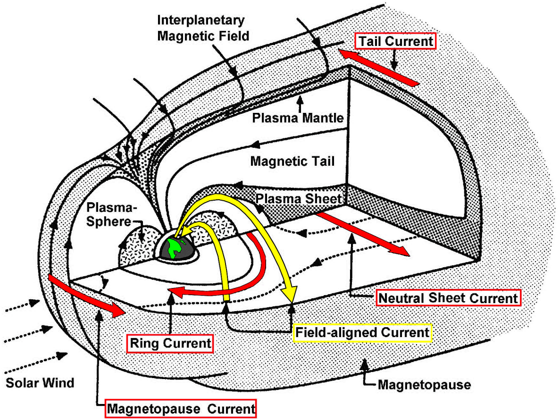
Introduction



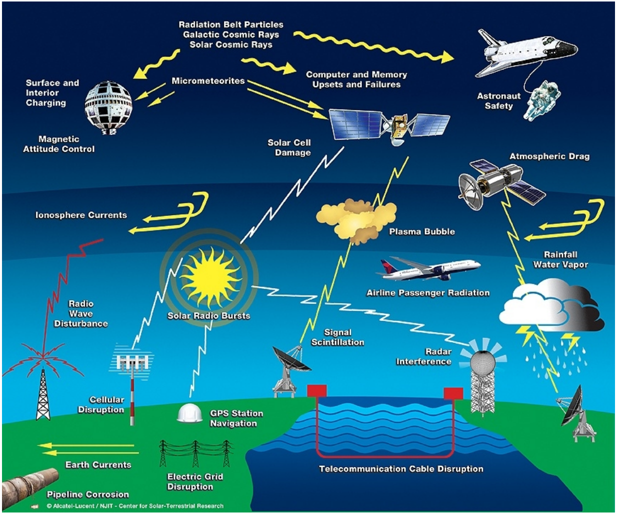
Overview of the magnetosphere and plasmasphere [Russel(2007)].



Overview of inner magnetosphere. Adapted from [NASA()].



Currents in/around the magnetosphere. Adapted from [Maus(2010)].



Impacts of Space Weather [Lanzerotti]

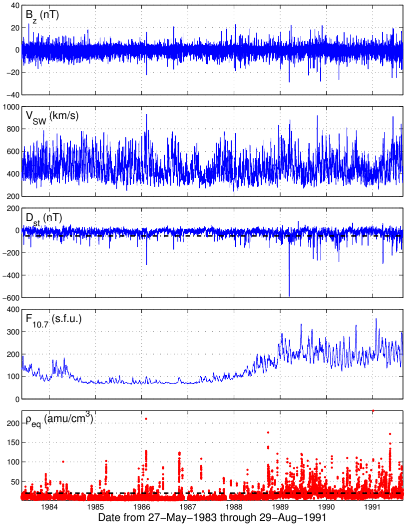
Data

Data used come from three sources:

- Denton (2007) for ρ_{eq}, MLT, and AE
- King (2005) for $F_{10.7}$
- Kondrashov (2014) for B_z, V_{SW}, Kp, ρ_{sw}, and D_{st}

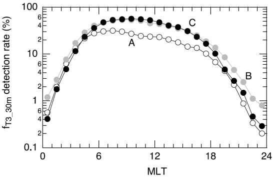
Data coverage:

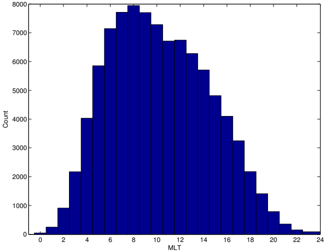
- ∼ Denton (2007): 10 minute, non-uniform, non-complete, from 1980-1991, GOES 2, 5, 6, and 7
- ∼ King (2005): 1 hour uniform, non-complete, from 1972-2013
- ∼ Kondrashov (2014): 1 hour uniform, complete, from 1972-2013



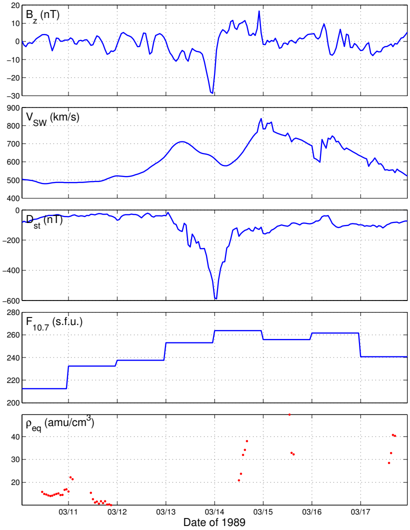
Data coverage with dashed lines indicating default event thresholds.

ρ_{eq} is derived from toroidal harmonic frequencies in plasmatrough. Harmonics are not always detectable.

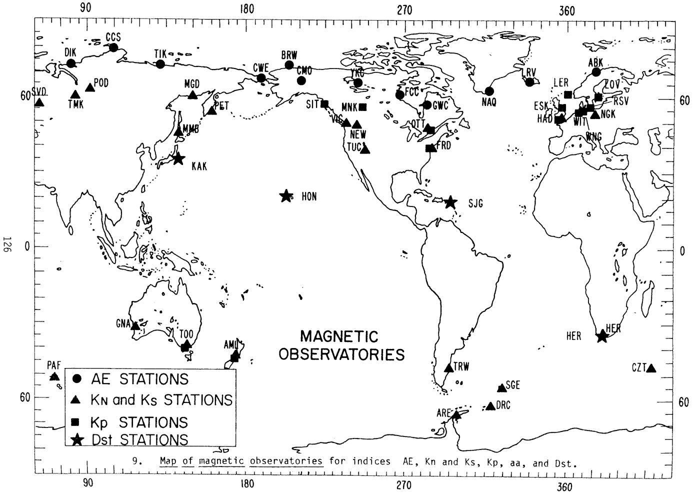




Left: Detection rate of f_{T3_30m} for magnetic latitudes of 5, 9, and 11 degrees (curves A, B, and C respectively) [Takahashi (2010)]. Right: MLT of all available data.

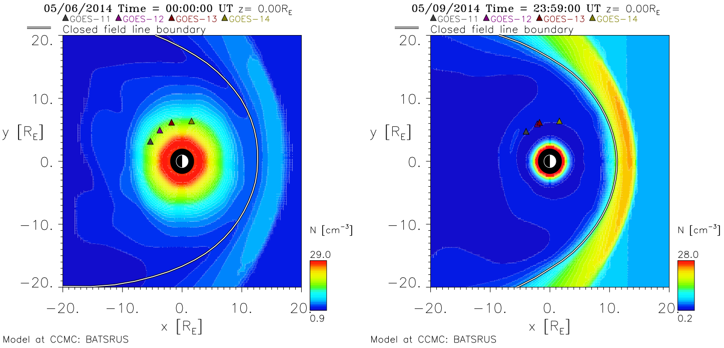


Data from GOES 6 around March 1989 geomagnetic storm.

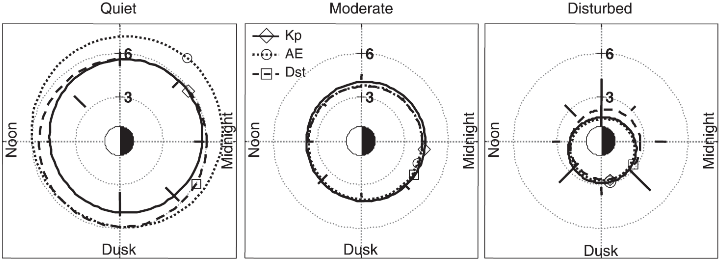


9. Map of magnetic observatories for indices AE, Kn and Ks, Kp, aa, and Dst.

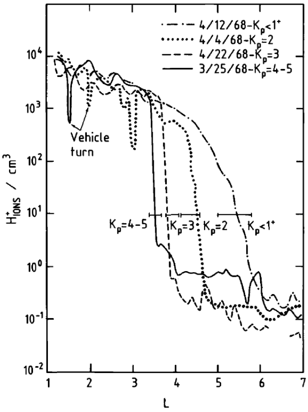
Map of ground stations used to measure the K_p, AE, and D_{st} indices



Model of magnetopause/plasmasphere before and after geomagnetic activity, showing location of GOES satellites in geosynchronous orbit [CCMC()].



Model of plasmapause location as it varies with geomagnetic activity where the symbols indicate the local time of maximum plasmapause location [O'Brien and Moldwin(2003)].



Plasmapause position varying with K_p as represented by several particular plasmapause crossings made on outbound passes between local times of midnight and 0400 [Lemaire(1998)].

Methodology

Three main methods of analysis used:

1. Linear/Auto-Regressive with exogenous inputs (ARX)
2. Nonlinear Neural Network
3. Epoch

Linear and ARX

Start with Box-Jenkins model:

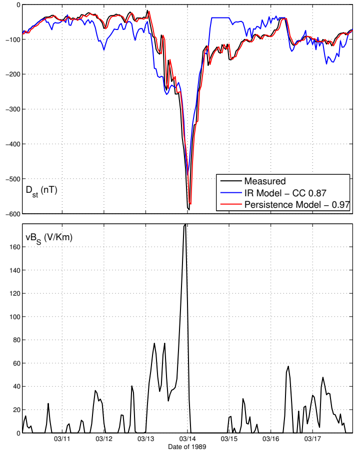
$$x(t) = \sum_{j=1}^{m} b_j \cdot f(t - j\Delta t) + c + \varepsilon_t$$

Modified to be an autoregressive model with exogenous inputs (ARX):

$$\hat{x}(t) = \sum_{i=1}^{l} a_i \cdot x(t - i\Delta t) + \sum_{j=1}^{m} b_j \cdot f(t - j\Delta t) + c + \varepsilon_t$$

Resultant matrix to solve:

$$\begin{pmatrix} x_0 & \dots & x_{\tau-1} & f_0 & \dots & f_{\tau-1} & 1 \\ x_1 & & x_\tau & f_\tau & & f_\tau & 1 \\ \dots & & & & & & \\ x_{N-\tau} & \dots & x_{N-1} & f_{N-\tau} & \dots & f_{N-1} & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ \dots \\ a_{\tau-1} \\ b_0 \\ \dots \\ b_{\tau-1} \\ c \end{pmatrix} = \begin{pmatrix} x_\tau \\ x_{\tau+1} \\ \dots \\ x_N \end{pmatrix}$$



Top: D_{st} (black), persistence (red), 12-hour impulse response model (blue). Bottom: vB_S input.

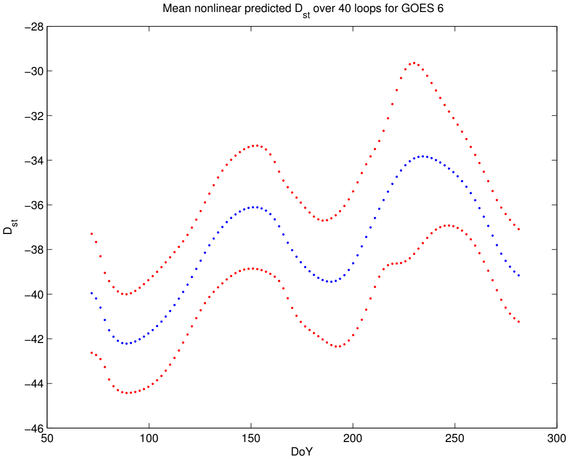
Neural Network

Neural Network model benefits:

1. Can model nonlinear effects
2.

Neural Network model disadvantages:

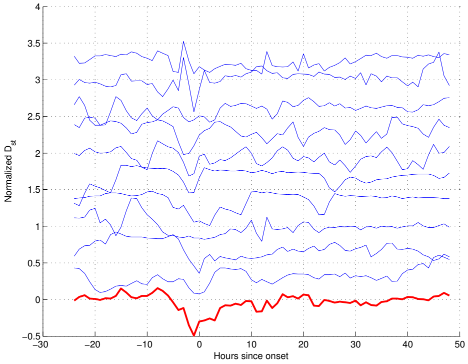
1. Susceptible to overfitting
2. More complex to create and analyze
3. No closed-form optimum solution



D_{st} predicted by nonlinear model of day of year.

Epoch

Epoch Analysis averages multiple events to find patterns.



Sample epoch of first ten events and median

Results

Linear and ARX

	GOES 2	GOES 5	GOES 6	GOES 7
DoY	-0.08 ± 0.08	$+0.14 \pm 0.13$	-0.06 ± 0.06	$+0.09 \pm 0.10$
MLT	-0.10 ± 0.21	-0.07 ± 0.12	$+0.01 \pm 0.23$	-0.06 ± 0.05
B_z	$+0.16 \pm 0.21$	-0.13 ± 0.15	$+0.08 \pm 0.14$	-0.07 ± 0.06
V_{sw}	-0.04 ± 0.10	$+0.27 \pm 0.09$	$+0.06 \pm 0.11$	-0.06 ± 0.06
D_{st}	$+0.26 \pm 0.17$	$+0.66 \pm 0.08$	$+0.06 \pm 0.13$	$+0.23 \pm 0.14$
ρ_{sw}	$+0.35 \pm 0.24$	$+0.63 \pm 0.31$	$+0.12 \pm 0.19$	$+0.36 \pm 0.17$
$F_{10.7}$	$+0.43 \pm 0.08$	$+0.12 \pm 0.12$	$+0.51 \pm 0.06$	$+0.40 \pm 0.06$
$B_z + V_{sw}$	$+0.11 \pm 0.17$	$+0.20 \pm 0.17$	$+0.12 \pm 0.10$	-0.12 ± 0.06
$D_{st} + F_{10.7}$	$+0.44 \pm 0.09$	$+0.71 \pm 0.08$	$+0.54 \pm 0.07$	$+0.47 \pm 0.06$
All	-0.03 ± 0.19	$+0.34 \pm 0.27$	$+0.61 \pm 0.11$	$+0.40 \pm 0.12$

Table of linear model test-set correlations showing the median of 100 random samples. Each sample trained on half of the data (via randomly selected rows of the least squares matrix) and tested on the other half.

Linear and ARX

	GOES 2	GOES 5	GOES 6	GOES 7
DoY	-0.08 ± 0.08	$+0.14 \pm 0.13$	-0.06 ± 0.06	$+0.09 \pm 0.10$
MLT	-0.10 ± 0.21	-0.07 ± 0.12	$+0.01 \pm 0.23$	-0.06 ± 0.05
B_z	$+0.16 \pm 0.21$	-0.13 ± 0.15	$+0.08 \pm 0.14$	-0.07 ± 0.06
V_{sw}	-0.04 ± 0.10	$+0.27 \pm 0.09$	$+0.06 \pm 0.11$	-0.06 ± 0.06
D_{st}	$+0.26 \pm 0.17$	$+0.66 \pm 0.08$	$+0.06 \pm 0.13$	$+0.23 \pm 0.14$
ρ_{sw}	$+0.35 \pm 0.24$	$+0.63 \pm 0.31$	$+0.12 \pm 0.19$	$+0.36 \pm 0.17$
$F_{10.7}$	$+0.43 \pm 0.08$	$+0.12 \pm 0.12$	$+0.51 \pm 0.06$	$+0.40 \pm 0.06$
$B_z + V_{sw}$	$+0.11 \pm 0.17$	$+0.20 \pm 0.17$	$+0.12 \pm 0.10$	-0.12 ± 0.06
$D_{st} + F_{10.7}$	$+0.44 \pm 0.09$	$+0.71 \pm 0.08$	$+0.54 \pm 0.07$	$+0.47 \pm 0.06$
All	-0.03 ± 0.19	$+0.34 \pm 0.27$	$+0.61 \pm 0.11$	$+0.40 \pm 0.12$

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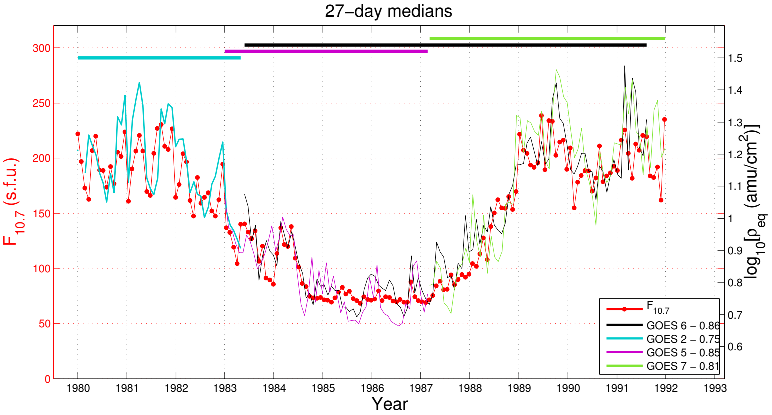


Comparing $F_{10.7\_27d}$ and $\log_{10}(\rho_{eq\_27d})$ using all available satellites.

Neural Network

	GOES 2	GOES 5	GOES 6	GOES 7
DoY	$+0.05 \pm 0.31$	$+0.31 \pm 0.30$	$+0.32 \pm 0.22$	$+0.12 \pm 0.17$
MLT	$+0.29 \pm 0.41$	$+0.15 \pm 0.34$	$+0.40 \pm 0.32$	$+0.17 \pm 0.21$
B_z	$+0.24 \pm 0.23$	$+0.21 \pm 0.28$	$+0.17 \pm 0.19$	-0.00 ± 0.20
V_{sw}	$+0.20 \pm 0.25$	$+0.36 \pm 0.19$	$+0.19 \pm 0.24$	$+0.06 \pm 0.18$
D_{st}	$+0.08 \pm 0.27$	$+0.18 \pm 0.25$	$+0.02 \pm 0.17$	$+0.18 \pm 0.24$
ρ_{sw}	$+0.02 \pm 0.29$	$+0.25 \pm 0.42$	$+0.20 \pm 0.22$	$+0.12 \pm 0.29$
$F_{10.7}$	$+0.26 \pm 0.27$	$+0.32 \pm 0.29$	$+0.48 \pm 0.25$	$+0.36 \pm 0.15$
$B_z + V_{sw}$	$+0.11 \pm 0.25$	$+0.20 \pm 0.38$	$+0.15 \pm 0.21$	$+0.02 \pm 0.17$
$D_{st} + F_{10.7}$	$+0.17 \pm 0.25$	$+0.21 \pm 0.32$	$+0.47 \pm 0.15$	$+0.35 \pm 0.17$
All	$+0.21 \pm 0.41$	$+0.67 \pm 0.40$	$+0.60 \pm 0.35$	$+0.17 \pm 0.33$

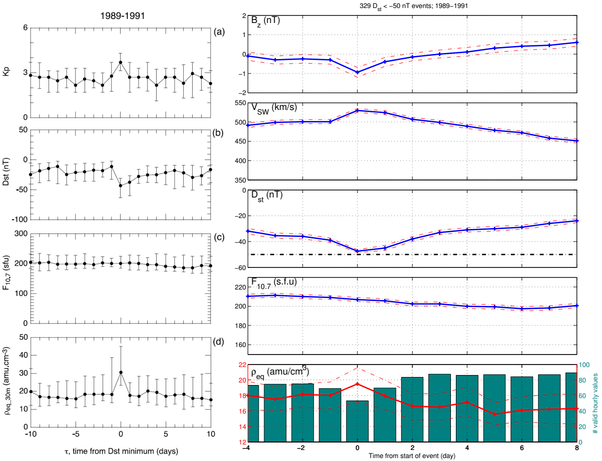
Table of nonlinear model test correlations showing the median of 100 random samples. Each sample trained on half of the data (via randomly selected rows of the least squares matrix) and tested on the other half.

Epoch

Epoch Analysis performed on two types of events:

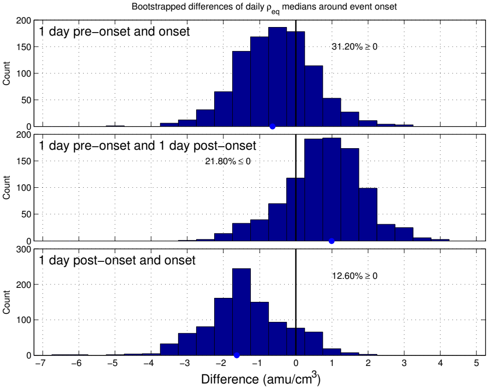
- ~ $\rho_{eq} > 20$ amu/cm^3
- ~ $D_{st} <$ -40 nT

Threshold crossings for events considered at an hourly timescale.
First wanted to verify results of Takahashi (2010), looking at D_{st} events from 1989-1991 hitting a minimum between 06-12 MLT.



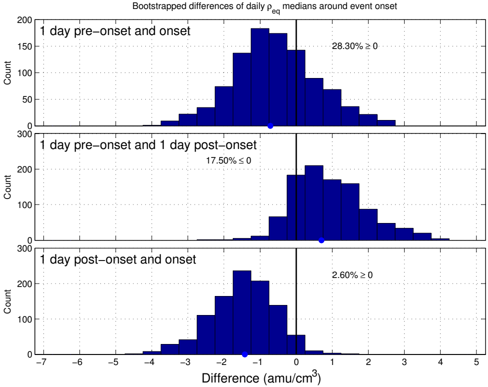
Epoch analysis for D_{st} events on an daily timescale using only the years of 1989-1991 from Takahashi (2010).

To verify whether events were significantly different between day of onset and the surrounding days, a bootstrap test of differences was performed:

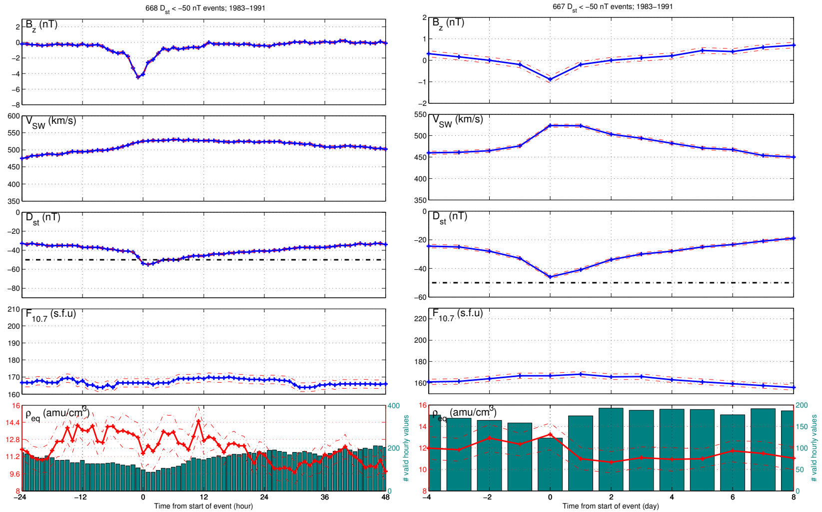


Bootstrap differences between median daily value of events using only the years of 1989-1991.

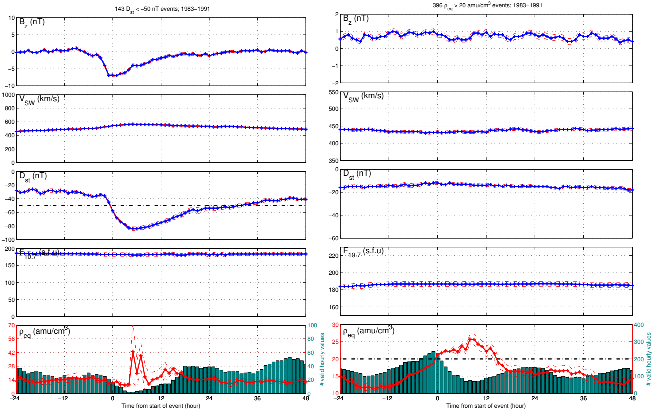
Using all of GOES 6's 1983-1991 range instead:



Bootstrap differences between median daily value of events using the years of 1983-1991.

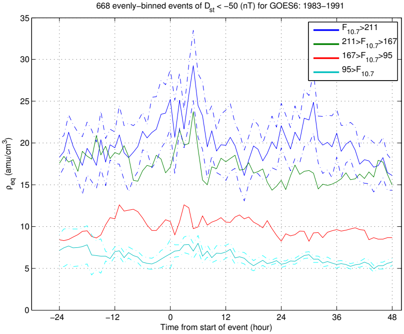


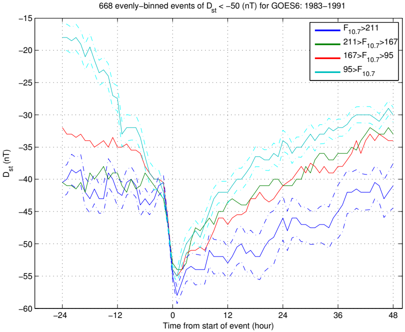
D_{st} events on an hourly (left) and daily (right) timescale.



Left: D_{st} events lasting longer than 12 hours. Right: ρ_{eq} events on hourly timescale.

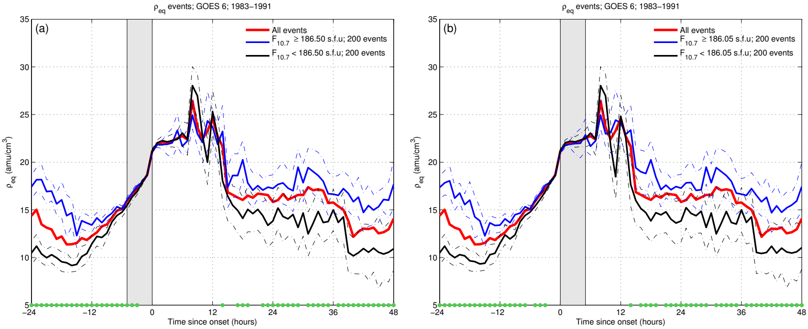
Want to investigate nonlinear dependencies by binning events:





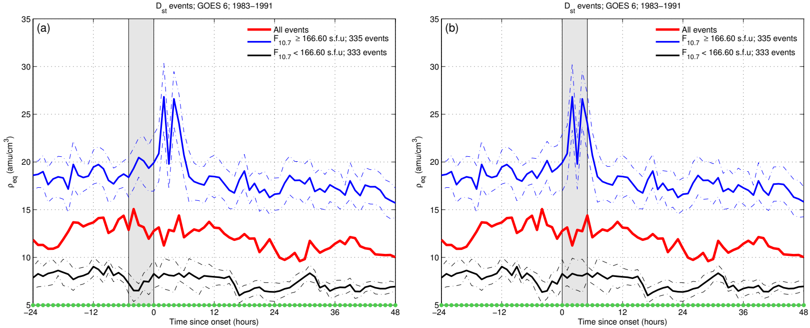
ρ_{eq} (left) and D_{st} (right) of D_{st} events binned by median $F_{10.7}$ values.

Verifying that distribution of ρ_{eq} is significantly different per $F_{10.7}$ bin before and after ρ_{eq} event onset.



ρ_{eq} events binned by median $F_{10.7}$ before (left) and after (right) event onset.

Verifying that distribution of ρ_{eq} is also significantly different per $F_{10.7}$ bin before and after D_{st} event onset.



D_{st} events binned by median $F_{10.7}$ before (left) and after (right) event onset.

Classification and Forecasting

Two main questions:

- Can event onsets be distinguished from the hours or days before onset?
- Can event onsets be forecasted using the hours or days before?

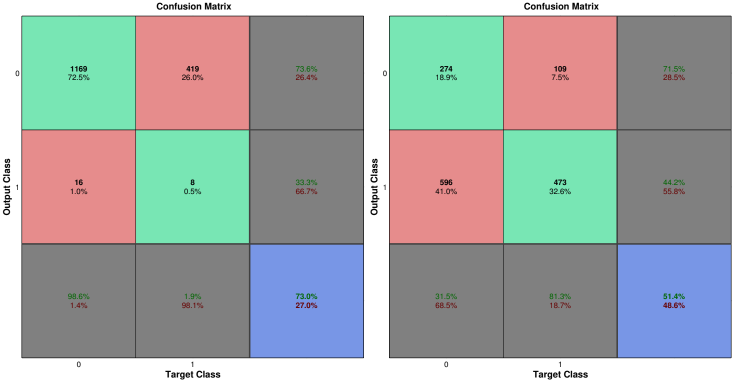
Utilizing MATLAB's `patternnet` neural network toolbox, classifying onsets as "1" and all other times as "0". Weighting "1"s more heavily based on timescale.

Is an onset distinguishable using just the variables provided?

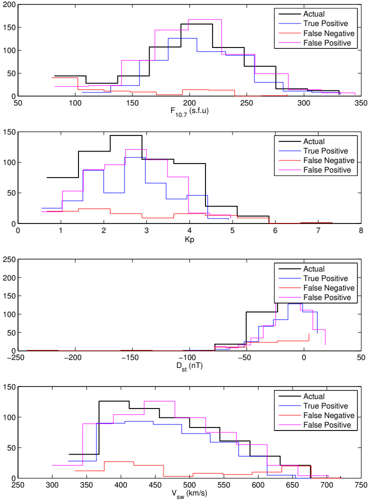
0	0	0	1
$B_{z_{\tau-3}}$	$B_{z_{\tau-2}}$	$B_{z_{\tau-1}}$	$B_{z_{\tau}}$
$F_{10.7_{\tau-3}}$	$F_{10.7_{\tau-2}}$	$F_{10.7_{\tau-1}}$	$F_{10.7_{\tau}}$
$K_{p_{\tau-3}}$	$K_{p_{\tau-2}}$	$K_{p_{\tau-1}}$	$K_{p_{\tau}}$
$\rho_{eq_{\tau-3}}$	$\rho_{eq_{\tau-2}}$	$\rho_{eq_{\tau-1}}$	$\rho_{eq_{\tau}}$

0	0	0	1
$B_{z_{\tau-3}}$	$B_{z_{\tau-2}}$	$B_{z_{\tau-1}}$	$B_{z_{\tau}}$
$F_{10.7_{\tau-3}}$	$F_{10.7_{\tau-2}}$	$F_{10.7_{\tau-1}}$	$F_{10.7_{\tau}}$
$K_{p_{\tau-3}}$	$K_{p_{\tau-2}}$	$K_{p_{\tau-1}}$	$K_{p_{\tau}}$
$\rho_{eq_{\tau-3}}$	$\rho_{eq_{\tau-2}}$	$\rho_{eq_{\tau-1}}$	$\rho_{eq_{\tau}}$

Diagram of classification method.

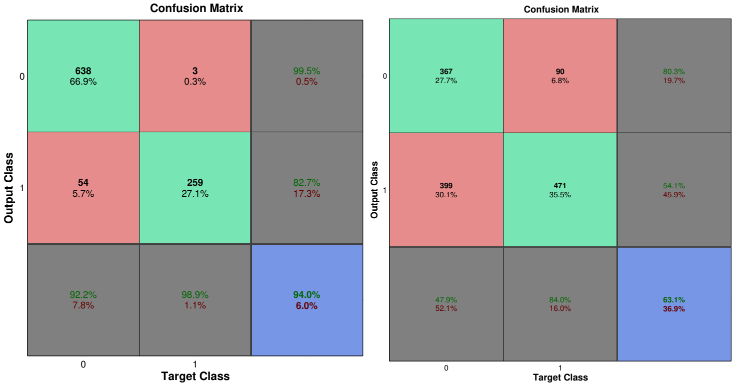


Classification confusion matrix for hourly (left) and daily (right) ρ_{eq} onset.



Histogram of onset conditions for daily-averaged ρ_{eq} events binned by correctness of prediction.

Including ρ_{eq} as a variable should make classification near perfect:

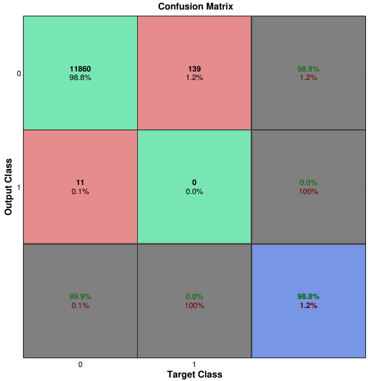


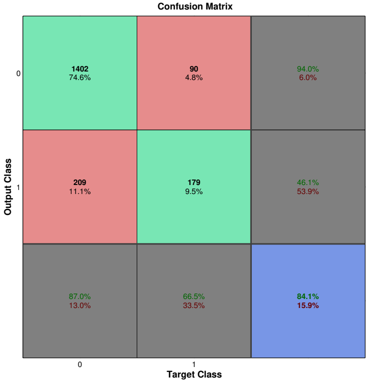
Classification confusion matrix for hourly (left) and daily (right) ρ_{eq} onset events, including ρ_{eq} as an input variable.

Can an onset be forecasted using the previous four hours or days?

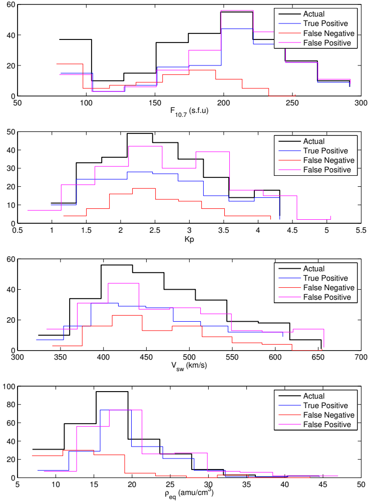
0	0	0	0	0	0	0	1
$B_{z_{\tau-7}}$	$B_{z_{\tau-6}}$	$B_{z_{\tau-5}}$	$B_{z_{\tau-4}}$	$B_{z_{\tau-3}}$	$B_{z_{\tau-2}}$	$B_{z_{\tau-1}}$	$B_{z_{\tau}}$
$F_{10.7_{\tau-7}}$	$F_{10.7_{\tau-6}}$	$F_{10.7_{\tau-5}}$	$F_{10.7_{\tau-4}}$	$F_{10.7_{\tau-3}}$	$F_{10.7_{\tau-2}}$	$F_{10.7_{\tau-1}}$	$F_{10.7_{\tau}}$
$K_{p_{\tau-7}}$	$K_{p_{\tau-6}}$	$K_{p_{\tau-5}}$	$K_{p_{\tau-4}}$	$K_{p_{\tau-3}}$	$K_{p_{\tau-2}}$	$K_{p_{\tau-1}}$	$K_{p_{\tau}}$
$\rho_{eq_{\tau-7}}$	$\rho_{eq_{\tau-6}}$	$\rho_{eq_{\tau-5}}$	$\rho_{eq_{\tau-4}}$	$\rho_{eq_{\tau-3}}$	$\rho_{eq_{\tau-2}}$	$\rho_{eq_{\tau-1}}$	$\rho_{eq_{\tau}}$

Diagram of prediction method.





Prediction confusion matrix for hourly (left) and daily (right) ρ_{eq} onset events.



Histogram of onset conditions for daily ρ_{eq} events including ρ_{eq} as an input, binned by correctness of prediction.

Conclusions

Conclusions

- Drop in D_{st} does not significantly influence ρ_{eq} on an hourly timescale, but does over the following day.
- ρ_{eq} increases do not correlate, on average, with significant changes in D_{st}, $F_{10.7}$, or B_z.
- D_{st} most affects ρ_{eq} during periods of high solar activity (via $F_{10.7}$)
- Classifying or forecasting the day of an event onset are possible but both prone to false positives

Future work

Many things could be done to improve:

- ∼ Exploring remaining signal after all collinear variables removed
- ∼ Adding model complexity and extra information
- ∼ Use new satellites and/or new Alfvn wave detection methods
- ∼ Extend time period of analysis

Questions?

- Do I need to explain F10.7? How about geomagnetic indices, and to what depth?
- References page? Fine to fill out by hand?
- Too many tables? Fine to drop extra satellites to fit L, ARX, and NL on one slide?

Allen, J.
Some Commonly Used Magnetic Activity Indices: Their Derivation, Meaning, and Use, 1982.
URL https://www.ngdc.noaa.gov/stp/solar/magindices.html.

CCMC.
Community coordinated modeling center.
URL http://ccmc.gsfc.nasa.gov/.

Lemaire, J.
The Earth's Plasmasphere.
Cambridge University Press, Cambridge, U.K. New York, 1998.
ISBN 9780521430913.

Maus, S.
Magnetospheric magnetic field, August 2010.
URL http://geomag.org/info/magnetosphere.html.

NASA.
Earths magnetosphere.
URL https://www.nasa.gov/mission_pages/sunearth/science/inner-mag-mos.html.

O'Brien, T. P. and Moldwin, M. B.
Empirical plasmapause models from magnetic indices.
Geophys. Res. Lett., 30:1152, February 2003.
doi: 10.1029/2002GL016007.

Russel, C.
Schematic of Earth's Night-Side Magnetosphere, June 2007.
URL http:
//www.esa.int/spaceinimages/Images/2007/12/Schematic_of_Earth_s_night-side_magnetosphere.