Computational Linear Algebra – Assignment 3

(points: 35)

Problem 1 (points: $3+3+2=8$)

Let $\mathcal{P}$ be the collection of polynomials defined on the real line. Is $\mathcal{P}$ a vector space? Argue that $\mathcal{P}$ cannot be finite-dimensional.

Next, let $\mathcal{P}_0$ be those polynomials $p \in \mathcal{P}$ such that $p(0) = 1$. Is $\mathcal{P}_0$ a subspace of $\mathcal{P}$?

Problem 2 (points: $2+3=5$)

Let $\mathbb{V} = \mathbb{R}^3$ and $\mathbb{U} = \{(x_1, x_2, x_3) \in \mathbb{V} : x_1 + x_2 + x_3 = 0\}$. Verify that $\mathbb{U}$ is a subspace of $\mathbb{V}$? What would be a complementary subspace of $\mathbb{U}$, i.e, a subspace $\mathbb{W} \subset \mathbb{V}$ such that $\mathbb{V} = \mathbb{U} \oplus \mathbb{W}$?

Problem 3 (points: $3+2+4=9$)

Consider the space $\mathcal{P}_n$ of real polynomials of degree at most n. Show that the functions $p_0(t) = 1, p_1(t) = t, \ldots, p_n(t) = t^n$ form a basis for $\mathcal{P}_n$.

Now, consider the set

$$\mathcal{Q} = \Big\{ p \in \mathcal{P}_n : \int_0^1 p(t)dt = 0 \Big\}.$$

Is $\mathcal{Q}$ a subspace of $\mathcal{P}_n$? Find the dimension of $\mathcal{Q}$, and in particular give a basis for $\mathcal{Q}$.

Problem 4 (points: $3+2+2+2=9$)

Let $\mathcal{M}_n$ be the set of real $n \times n$ matrices. Is $\mathcal{M}_n$ a vector space? What is its dimension? Next, consider the set of matrices in $\mathbf{A} \in \mathcal{M}_n$ such that $\mathbf{A}^\top = -\mathbf{A}$. Show that this forms a subspace of $\mathcal{M}_n$ and determine its dimension.

Problem 5 (points: 5)

Let $p_0, p_1, \ldots, p_n$ be polynomials of degree at most n. If $p_0(1) = \cdots = p_n(1) = 0$, then show that $p_0, p_1, \ldots, p_n$ cannot be linearly independent.

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