TUTORIAL-2

Naman Gupta

February 2025

Reinforcement Learning(E1:227), CSA, Indian Institute of Science

1 SSP

1.1 Terminologies etc

READ THEM

1) Trajectory : sequence of state and reward pair that you observe while going through MDP.

2) A1 : Probability of going from any nonterminal state to any non terminal state is nonzero.

1.2 Problems

TRY TO SOLVE AS MANY AS YOU CAN

Q1) Consider a MDP with two states only. One state is terminal/dead. The probability of returning to the nonterminal state from nonterminal state is $0 \leq p < 1$, and the reward you receive during this transition is r. The reward for entering into the terminal state from nonterminal state is R. Find the expected total reward received starting from the nonterminal state . Can you say something about exptected total reward received starting from nonterminal state? If $0 \leq p \leq 1$(Hint for the later part : Think in terms of boundness of expected total rewards).

Q2) Given an improper policy for a finite state MDP. What is the probability of the set of trajectories for which the total reward is unbounded?

Q3) Consider an MDP and the policy chosen is improper, the number of states is finite, and rewards are nonrandom positive. Discount factor is ONE. Prove that the total reward is unbounded for each trajectory. Assume that there are some transitions with zero rewards and Assumption A1 is there, Does there exist trajectories for which the total reward is bounded?

Q4) Give an example of a MDP such that even if the policy is improper, the expected total reward is bounded [Hint: consider MDP with number of states countably infinite,and for the simplicity assume that there is only one starting state].

Q5) Consider an finite state MDP, the discount factor is less than one. Then, prove that the total reward is bounded for any trajectory.

Q6) Give an example of a MDP(with no terminal state) such that, rewards are positive for each transition, the discount factor is also less than one. But still, the cost is unbounded[Hint: the number of states is countably infinite].

2 Contraction mapping

2.1 Theory

IMPORTANT TO KNOW MATERIAL

Read the definition of the Cauchy sequence and the convergence of a sequence from Rudin or online.

Read the definition of the complete metric space from Rudin or online.

2.2 Problems

DO AS MANY AS POSSIBLE

Q7) Does the given sequence converge $a_n = \{1/n\}, n \in N$? and if yes, then where?

Q8) Which one of them is a complete metric space : (a) {0,1} , (b) (0,1] (metric is Euclidean distance)?

Q9) Give an example of a contraction function/mapping such that no fixed point exists. [hint : consider (0,1] as domain and codomain].

Q10) Prove the contraction mapping theorem. and also prove the uniqueness of the fixed point.

3 Fixed point theorem(SUPPLEMENTARY, NOT NEEDED FROM THE COURSE POINT OF VIEW)

3.1 Useful information

NOT NEEDED AS SUCH BUT STILL

Read the definitions of closed set, open set, and compact set from Rudin or anywhere. You need to know the definitions, nothing else is required.

Theorem: The compact subsets of R^n are closed and bounded and vice versa.

3.2 Problems

TRY TO SOLVE AS MANY AS POSSIBLE

Hints for the remaining questions:
1) You do not need anything that I have mentioned above
2) The sum of two continuous functions is continuous.
3) Consider a function whose domain is $[a, b] \cup [c, d]$ where $b < c$. To check the continuity at a, c check only right continuity, and for b, d check only left. Use this fact for Q13.
4) If a continuous function is positive then it cannot become negative without becoming zero and vice-versa.

Q11) Consider a continuous function $F : [0, 1] \to [0, 1]$. Prove the fixed point theorem.(Hint : Use hint-4)

Q12) Suppose the domain and codomain is $(0, 1]$, Is the fixed point theorem still valid? If not, then give an example of a continuous function that does not have a fixed point.

Q13) Suppose the domain and codomain is $[n, n + 1]$ where 'n' is an even positive number, Is the fixed point theorem still valid? If not, then give an example of a continuous function that does not have a fixed point.