# E1 222 Stochastic Models and Applications
## Problem Sheet 3–2

1. Consider a communication system. Let $Y$ denote the bit sent by transmiter. ($Y$ is a binary random variable). The receiver makes a measurement, $X$, and based on its value decides what is sent. The decision at the receiver can be represented by a function $h : \Re \to \{0,\ 1\}$. For any specific $h$, let $R_0$ represent the set of all $x \in \Re$ for which $h(x) = 0$ and let $R_1$ represent the set of $x \in \Re$ for which $h(x) = 1$. An error occurs if a wrong decision is made. Argue that the event of error occurring is: $[h(X) = 0, Y = 1] \cup [h(X) = 1, Y = 0]$. Show that probability of error is
$$\int_{R_0} p_1 f_{X|Y}(x|1)dx \ + \ \int_{R_1} p_0 f_{X|Y}(x|0)dx$$
where $p_i = P[Y = i]$. Now consider a $h$ given by
$$h(x) = 1 \ \text{ if } \ f_{Y|X}(1|x) \geq f_{Y|X}(0|x)$$
(Otherwise $h(x) = 0$). Show that this $h$ would achieve minimum probability of error.

2. Let $A, B$ be two events. Let $I_A$ and $I_B$ denote the indicator random variables of these events. Show that $I_A$ and $I_B$ are independent iff $A$ and $B$ are independent.

3. Let $X, Y$ be independent random variables having Poisson distribution with parameters $\lambda_1$ and $\lambda_2$. Show that $X+Y$ is Poisson with parameter $\lambda_1 + \lambda_2$.

4. Let $X$ and $Y$ be independent random variables each having an exponential distribution with the same value of parameter $\lambda$. Show that (i). $Z = X + Y$ has gamma density with parameters $\alpha = 2$ and $\lambda$, (ii). $Z = \min(X, Y)$ is exponential with parameter $2\lambda$.

5. Let $X$ have gamma density with parameters $\alpha_1$ and $\lambda$ and let $Y$ have gamma desity with parameters $\alpha_2$ and $\lambda$. Suppose $X, Y$ are independent. Show that $X + Y$ has gamma density with parameters $\alpha_1 + \alpha_2$ and $\lambda$.

6. Let $X$ and $Y$ be independent Gaussian random variables with $EX = \mu_1$, $EY = \mu_2$, $\mathrm{Var}(X) = \sigma_1^2$, and $\mathrm{Var}(Y) = \sigma_2^2$. Show that $X + Y$ has gaussian density with mean $\mu_1 + \mu_2$ and variance $\sigma_1^2 + \sigma_2^2$.

7. Let $X, Y$ be independent random variables each having normal density with mean zero and variance unity. Find the joint density of $aX + bY$ and $bX - aY$, where $a^2 + b^2 > 0$.