

E1 222 Stochastic Models and Applications
Problem Sheet 4.2

1. Let X_1, X_2, \dots be independent normally distributed random variables having mean zero and variance σ^2 .
 - (a). What is the mean and variance of X_1^2 ?
 - (b). How should $P[X_1^2 + X_2^2 + \dots + X_n^2 \leq x]$ be approximated in terms of standard normal distribution?
 - (c). Suppose $\sigma^2 = 1$. Find (approximately) $P[80 \leq X_1^2 + \dots + X_{100}^2 \leq 120]$.
 - (d). Find c such that (approximately) $P[100 - c \leq X_1^2 + \dots + X_{100}^2 \leq 100 + c] = 0.95$.
2. Candidates A and B are contesting an election and 55% of the electorate favour B . What is the (approximate) probability that in a sample of size 100 atleast one-half of the people sampled favour A .
3. A fair coin is tossed until 100 heads appear. Find (approximately) the probability that atleast 230 tosses will be necessary.
4. Let $\{X_n, n \geq 0\}$ be a two state Markov Chain. Let $P(\cdot, \cdot)$ be the transition probability function and let $\pi_0(\cdot)$ be the initial state probabilities. Find (a). $\text{Prob}[X_1 = 0 | X_0 = 0, X_2 = 0]$; (b). $\text{Prob}[X_1 = X_2]$; (c). $\text{Prob}[X_3 = 0 | X_4 = 0]$.