E1 222 Stochastic Models and Applications
Problem Sheet 3–4

1. Let X, Y be jointly Gaussian with $EX = EY = 0$, $\text{Var}(X) = \text{Var}(Y) = 1$. Let ρ denote the correlation coefficient of X, Y. Write down the joint density of X, Y.

2. Let X, Y have joint density

$$f_{XY}(x, y) = \frac{\sqrt{3}}{2\pi} e^{-0.5(x^2+4y^2-2xy)}, \; -\infty < x, y < \infty$$

Find the marginal densities of X, Y and the conditional density $f_{X|Y}$. (Hint: You do not need to do any integration)

3. Let X, Y be continuous random variables with the following joint density

$$\begin{aligned} f_{XY}(x, y) &= \frac{1}{2}\left\{\frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2(1-\rho)^2}(x^2 - 2\rho xy + y^2)\right] + \right. \\ &\qquad \left. \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2(1-\rho)^2}(x^2 + 2\rho xy + y^2)\right]\right\} \end{aligned}$$

Find f_X, f_Y and EXY. Is $EXY = EX\ EY$? Are X, Y jointly normal?

4. Let X, Y be jointly normal with $EX = EY = 0$, $\text{Var}(X) = \text{Var}(Y) = 1$ and correlation coefficient ρ. Show that $Z = X/Y$ has Cauchy distribution. A Cauchy distribution with parameters μ and θ is given by

$$f(x) = \frac{\mu}{\pi} \frac{1}{(x-\theta)^2 + \mu^2}$$

5. Let X_1, X_2, X_3, X_4 be iid Gaussian random variables with mean zero and variance one. Show that the density function of $Y = X_1X_2 + X_3X_4$ is $f(y) = 0.5e^{-|y|}$, $-\infty < y < \infty$. (Hint: Try finding mgf of Y).