

E1 222 Stochastic Models and Applications

Problem Sheet 2-2

1. Let X be a continuous random variable with uniform density over $(-1, 1)$. Find the density (or mass function) of the random variables:
(a.) $U = (X + 1)/2$, (b.) $U = \frac{X}{1+X}$, (c.) $U = e^X$, (d.) $U = g(X)$ where $g(x) = -1$ if $x > 0$, $g(x) = 0$ if $x = 0$, and $g(x) = 1$ if $x < 0$.
2. Let X be a random variable, g be some density function and ϕ a differentiable strictly increasing function on $(-\infty, \infty)$. Suppose that

$$P[X \leq x] = \int_{-\infty}^{\phi(x)} g(z) dz.$$

Show that the density of $Y = \phi(X)$ is $g(y)$.

3. Let X be a discrete random variable having geometric distribution with parameter p . Let $U = \min(X, 5)$ and $V = \max(X, 10)$. Find mass function of U , V and $E[U]$.
4. Let X be a random variable uniformly distributed over $\{0, 1, \dots, N\}$. Find $E[X]$.
5. Let X be a rv with density function

$$f(x) = 4x^3, \quad \text{if } 0 \leq x \leq 1.$$

Let $Y = 2X^2 + 3X + 5$. Find $E[Y]$.

6. Ram wants to buy a used Maruthi car. His colleague is selling his used 2008 model Maruthi for Rs 70000. The market rate for this brand of cars is in fact Rs 80000, if it is in a good shape. Ram does not know much about cars. A mechanic says that he can test it and ascertain its condition. For this he charges Rs 1000. If the car is in bad shape, let us assume it takes Rs.15000 to get it repaired. Ram thinks that the probability of his colleagues car being in good shape is 70%. Calculate the expected net gain from buying his colleagues car without getting it tested by the mechanic. The mechanic and the tests promised by him are not very reliable.

$$P[\text{car passes test} | \text{it is in good shape}] = 0.8$$

$$P[\text{car passes test}|\text{it is in bad shape}] = 0.35$$

Should Ram get the car tested by the mechanic before making the decision?

7. The price of some commodity is Rs. 2 per gram this week. Next week the price would be either Rs.1 per gm or Rs. 4 per gram, each with probability 0.5. You have a capital of Rs.1000. What would be your strategy if (i) you want to maximize expected amount of money with you (next week), (ii) you want to maximize the expected quantity of the commodity with you.
8. Children from a school went to a picnic in four buses. Different buses carried different number of students. Define two random variables, X, Y , as follows. We select one of the four drivers at random and X is the number of students in the bus driven by that driver. We select a student at random and Y is the number of students in the bus in which the selected student travelled. Can you say whether $EX > EY$ or $EY > EX$ (or the information given is not sufficient to decide which of EX, EY is greater)?