E1 222 Stochastic Models and Applications

Home Work Assignment 1

Submission Deadline: 9 PM on 13th September 2021

This is your first assignment. I will set up the assignment on the Teams. The assignment has to be submitted as a single PDF file. You do not have to submit solutions to all the problems. At 6 PM on 13th September I will choose two problems from this assignment and communicate it to you through a chat on the Teams. You need to submit solutions only to those two problems.

1. A rod of length l is tossed at random on a plane that is ruled with a series of parallel lines a distance of $2l$ apart. What is the probability that the rod will intersect one of the lines.

2. At a telephone exchange, the probability of receiving k calls in any time interval of length two minutes is given by the function $h(2, k)$. Assume that the event of receiving k_1 calls in a time interval I_1 is independent of the event of receiving k_2 calls in a time interval I_2, for all k_1 and k_2 whenever the intervals I_1 and I_2 do not overlap. Find an expression for the probability of receiving s calls in 4 minutes in terms of $h(2, k)$. Now suppose $h(2, k)$ is given by

$$h(2, k) = \frac{(2a)^k e^{-2a}}{k!}.$$

   where $a > 0$ is some real number. Now show that the probability of s calls in 4 minutes is given by $\frac{(4a)^s e^{-4a}}{s!}$

3. Consider the random experiment of rolling a pair of dice where we assume that all the 36 outcomes are equally likely. On this probability space let X be a random variable whose value is the sum of the numbers that showed up on the two dice. Find the probability mass function of X.

4. Consider the following special case of Polya's urn scheme. We start with one white and one black ball in the urn. We choose a ball at random and then put back that ball along with one more ball of the same colour. We keep repeating this process till the number of balls in

the urn are n. Let X_n denote the number of white balls in the urn when the total number of balls in the urn is n. Show that X_n is uniform over $\{1, 2, \cdots, (n-1)\}$.

5. Let X be a continuous random variable that has uniform distribution over $[-1,\ 2]$. Let $Y = |X|$. Find the probability density function of Y.

6. Suppose X is a continuous random variable with $E|X| < \infty$. Suppose the density function of X is symmetric about c. (That is, $f_X(c-x) = f_X(c+x), \forall x$). Show that $EX = c$.

7. Let X be a binomial random variable with parameters n and p. Let $Y = \max(0,\ X-1)$. Show that $EY = np - 1 + (1-p)^n$.

8. If X has beta density, find EX and $\mathrm{Var}(X)$.
   The beta density is given by
$$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1},\ 0 \leq x \leq 1.$$
   where $\Gamma(\cdot)$ is the gamma function and $a, b \geq 1$ are parameters.