E1 222 Stochastic Models and Applications
Problem Sheet 4–1

1. Let $\{X_n\}$ be *iid* with density $f(x) = e^{-x+\theta}, x > \theta > 0$. Let $N_n = \min(X_1, \cdots X_n)$. Does N_n converge in probability?

2. Given $P[X_n = 0] = 1-n^{-2}$, $P[X_n = e^n] = n^{-2}$. Show that X_n converge almost surely but not in r^{th} mean.

3. Given $P[X_n = 0] = 1-1/n$, $P[X_n = n^{1/2r}] = 1/n$, X_n are independent. Show that $E|X_n|^r \to 0$ but the sequence does not converge to zero almost surely.

4. Let $\Omega = [0,\ 1]$ and let P be the usual length measure. Let $X_n = n^{0.25} I_{[0,\ 1/n]}$, $n = 1, 2, \cdots$, where I_A denotes indicator of event A. Does the sequence converge in (i) probability, (ii) r^{th} mean for some r?

5. Let $\{X_n\}$ be random variables with $EX_n = m_n$ and $\text{Var}(X_n) = \sigma_n^2$. Find some sufficient condition such that $X_n - m_n$ converges to zero in (i). probability, (ii) almost surely.

6. Let $X_1, X_2, \cdots,$ be random variables with distributions

$$\begin{aligned} F_{X_n}(x) &= 0 && \text{if} \quad x < -n \\ &= \tfrac{x+n}{2n} && \text{if} \quad -n \le x \le n \\ &= 1 && \text{if} \quad x \ge n \end{aligned}$$

   Does $\{X_n\}$ converge in distribution?

7. Let $\Omega = [0, 1]$. Consider a sequence of binary random variables: $X_{nk},\ k = 1, \cdots, n, n = 1, 2, \cdots$. That is, the sequence is $X_{11}, X_{21}, X_{22}, X_{31}, X_{32}, X_{33}, \cdots$. These random variables are defined by

$$X_{nk}(\omega) = 1 \ \text{ iff } \ \frac{k-1}{n} \le \omega < \frac{k}{n},\ 1 \le k \le n, n = 1, 2, \cdots$$

   Show that the sequence converges to zero in probability but it does not converge with probability one