Affective modulation of weighting function

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1 Description

Both Expected-utility theory and prospect theory posit that humans maximize some version of utility. The theories get there by a combination of two functions (Rottenstreich & Hsee, 2001). A value function v transforms objective value to subjective utility, and a weighting function w distorts probabilities (Gonzalez & Wu, 1999; Rottenstreich & Hsee, 2001). Expected-utility and prospect theory combine these two paramters in the simplest way possible (Rottenstreich & Hsee, 2001)

$$\sum w(p_i)v(i),$$

where p stands for probability and i stands for the i^{th} gamble.

Prospect Theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992) (PT) is arguably the main model of human decision making (Newell et al., 2015). It advances theorizing from expected-utility by postulating that losses and gains are evaluated as changes in wealth rather than in regard to end states (Newell et al., 2015).

In Kahneman and Tversky (1979) we find the familiar (non-linear) S-shaped value function v which is concave for the gains domain and convex for losses (where it is steeper as well). The weight function w is the identity, $w(p) = p$ in expected-utility theory (Rottenstreich & Hsee, 2001) whereas a non-linear probability distortion is proposed in prospect theory (Kahneman & Tversky, 1979). Here w is stylized as being reverse S-shaped, meaning that it is concave for low probabilities and convex for high probabilities Gonzalez and Wu (1999). This means that people underweight changes in probability in the middle of the spectrum (e.g. $[0.2-0.8]$) while overweighting changes in probability close to the end-points (e.g. $[0.0 - 0.2], [0.8 - 1.0]$). These general characteristics of the weighting function are empirically well documented (Tversky & Kahneman, 1992; Wu & Gonzalez, 1996).

1.1 Prior work

There is evidence to support the notion that the affect of outcomes modulates the parameters of both v (Hsee & Rottenstreich, 2004) and w (Rottenstreich & Hsee, 2001). A main finding is that the S-shape of the weighting function w appears to be more pronounced for high-affect than low-affect outcomes under uncertainty (Rottenstreich & Hsee, 2001). This was shown as a preference reversal in which a high-affect outcome was preferred for low probability (1%) whereas a low-affect outcome was preferred for high probability (100%) (Rottenstreich & Hsee, 2001). The finding that affect appears to modulate both v and w has subsequently been modelled as an interaction between an affective system and a deliberative system Mukherjee (2010, 2011).

1.2 Focus and parameterization

In this article we focus exclusively on the weighting function w while ignoring both the value function v and the combination of the two functions. We also restrict ourselves to the gains domain. In Rottenstreich and Hsee (2001) they propose that the affective modulation can be estimated as an affect paramter a in the form:

$$w(p) = \frac{p^{1-a}}{p^{1-a} + (1-p)^{1-a}}.$$

where $a \in [0, 1]$ and larger values indicate greater affect and more curvature (Rottenstreich & Hsee, 2001). The issue with this one-parameter formulation is that it does not account for the fact that people generally show low *elevation*. What I mean by that is that the empirically observed weighting function w typically crosses the diagonal line at around 0.3 rather than 0.5 (Gonzalez & Wu, 1999). The one-parameter formulation fixes this point at 0.5 which can be seen from figure 1.

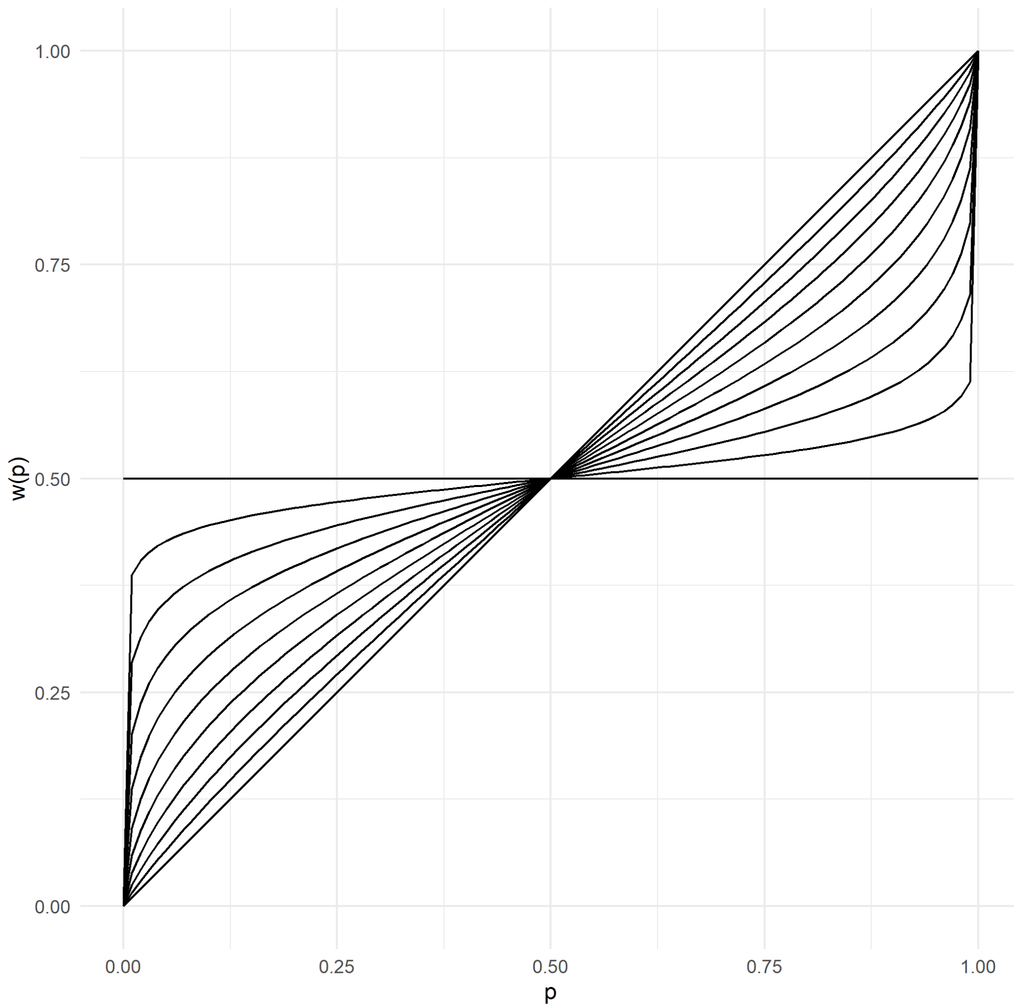

Figure 1: Data simulated from the model $w(p) = \frac{p^{1-a}}{p^{1-a}+(1-p)^{1-a}}$ with $a \in [0, 1]$. Diagonal line has $a = 0$, and the horizontal line has $a = 1$. Intermediate curves are generated for 0.2 increments of a. All values beside $a = 0$ show a probability distortion as compared to the objective probability. Note that all curves meet at $w(p) = 0.5, p = 0.5$. This is not empirically supported.

Instead of using the parameterization proposed in Rottenstreich and Hsee (2001) this paper will use the parameterization of w proposed in Gonzalez and Wu (1999).

They parameterize w with two parametrs; δ and γ.

The δ parameter will vary based on *elevation* (intercept) (Gonzalez & Wu, 1999), which here simply refers to the overall perceived attractiveness of outcomes under uncertainty.

The γ parameter will vary based on *curvature* (slope) (Gonzalez & Wu, 1999) and is what we are primarily interested in for our purposes. It follows as a direct prediction from Rottenstreich and Hsee (2001) that the curvature (γ) should be modulated by changes in the affective level of outcomes. See figure 2 for an illustration of how the δ and γ parameters independently modulate different aspects of the weighting function w.

The model is:

$$\log \frac{w(p)}{1 - w(p)} = \gamma \log \frac{p}{1 - p} + \tau.$$

where solving for $w(p)$ and setting $\delta = \exp(\tau)$ gives us

$$w(p) = \frac{\delta \cdot p^{\gamma}}{\delta \cdot p^{\gamma} + (1 - p)^{\gamma}}.$$

The above equations are taken from Rottenstreich and Hsee (2001).

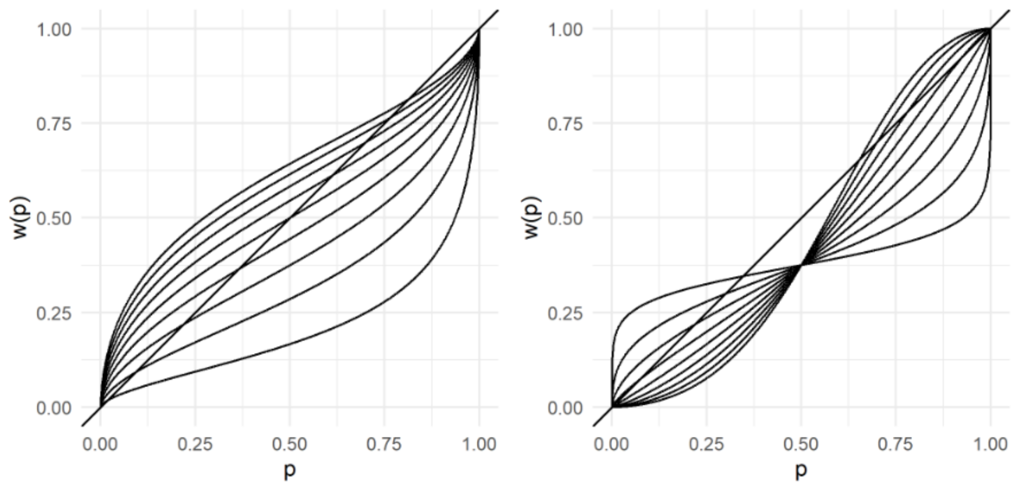

Figure 2: Data simulated from the model $w(p) = \frac{\delta \cdot p^{\gamma}}{\delta \cdot p^{\gamma} + (1-p)^{\gamma}}$ similarly to figure 4 of Gonzalez and Wu (1999). On the left: γ fixed at 0.6 and δ varied between 0.2 and 1.8. On the right: δ fixed at 0.6 and γ varied between 0.2 and 1.8. Shows that γ controls curvature and δ controls elevation. The identity function $w(p) = p$ is achieved for $\delta = 1, \gamma = 1$. Note.. gamma low has the opposite interpretation as compared to rottenstreich?

1.3 Methodology

Two studies are proposed to properly test the robustness of affect-level on the curvature (γ) of the weight function w.

In the first study, subjects will be asked to rate the affect-richness of 10 different items. All outcomes consist of coupons redeemable for various items, all worth $\$500$. The 10 items are designed to cover the full spectrum from affect-rich to affect-poor.

Example of expected high-affect item:

"If you won a $\$500$ coupon redeemable for a vacation abroad how emotionally affected would you be?”

Example of expected low-affect item:

”If you won a $\$500$ coupon redeemable for insurance covering how emotionally affected would you be?”

For the full list of items see Appendix A. Participants will indicate how affect-rich each outcome is with a slider. Participants will see ”not affected at all” (left), ”somewhat affected” (middle) and ”very affected” (right). We will receive continuous ratings from 0 (affect poor) to 1 (affect rich). A mean affect rating across participants for each item will rank them from least affective to most affective. Three items (gambles) are then selected: The least affective item, the most affective item and the item in between these two extremes which separate them best (follow up).

In the second study, subjects will be presented with the three items which have been validated for affect-richness in the prior study. In this study however, the formulation around the items is that of a gamble. The formulation is the same for all items:

”You can buy a lottery ticket with an $[x]$ percent chance of winning a $\$500$ coupon redeemable for $[y]$ with a $[1-x]$ percent chance of winning nothing. How much are you willing to pay for the lottery ticket?”

The three selected items are inserted as $[y]$ and 100 different probability levels: $x = 0.01, 0.02, \ldots, 0.99$ will be inserted as $[x]$ and the negation $[1-x]$. With all possible combinations, this means that all participants will rate 3 items at 100 different levels of certainty each. As in experiment 1 participants will rate with a slider. This time ranging from $\$0$ to $\$500$ as it is neither logical to assign a value below $\$0$ or above $\$500$ to any of the gambles. The approach is somewhat different from Gonzalez and Wu (1999) but ultimately we estimate the same thing that they do; participants' certainty equivalence (CE). This simply is the amount of money they think that the gamble is worth.

Note that we are not directly measuring either δ or γ. What we do measure is the dependent variable $w(p)$ and the independent variable p for items ranked based on their affect-richness.

In order to infer the unmeasured paramters a bayesian (non)linear mixed effects model is proposed. The model is fitted in R with the $brms$ package. Here we can specify the previously mentioned formula:

$$w(p) \sim \frac{\exp(\tau) \cdot p^{\gamma}}{\exp(\tau) \cdot p^{\gamma} + (1-p)^{\gamma}}.$$

We have to specify that the model should be nonlinear. We can further specify that we would like to estimate specifically the value for τ and γ with random intercepts (partial pooling) for participants (ID) and with item as a main effect.

$$\tau \sim 0 + item + (1|ID),$$

$$\gamma \sim 0 + item + (1|ID).$$

We can then convert τ to δ by exponentiating the estimated value for τ for each item. For the analysis pipeline on simulated data, see Github.

After fitting the model, posterior samples for the three items are generated. Based on these, $.66$ and $.95$ credibility intervals are calculated and the distributions for each paramter (for each item) as a group-level effect is visualized.

Hypothesis: A directional effect is expected for the population effect of the γ parameter. It is expected that item 1 (lowest affect) will have the highest γ value, and that item 3 (highest affect) will have the lowest γ value, with item 2 falling in the middle. The paper does not propose to calculate significance or bayes factor, but simply hypothesizes that the $.95$ and $.66$ credibility intervals will only overlap in the expected direction (see plot).
but a thorough investigation of this effect is lacking. This study consits of two substudies. In the first study subjects will evaluate 10 items on a scale of affect. In the second study subjects will indicate their certainty equivalence (CE) as to gambles involving these questions. Based on this, the parameters of the weighting function

$$w(p) = \frac{\delta \cdot p^{\gamma}}{\delta \cdot p^{\gamma} + (1-p)^{\gamma}}$$

are estimated for each of the 10 items, and it is calculated whether level of affect (obtained in study 1) modulates the parameters of the weighting function. Note that the above is the two-parameter weighting function suggested in (Gonzalez & Wu, 1999).

2 Hypotheses

H_1: It is expected that the 10 questions in study 1 will - on average - obtain significantly different ratings as to affective quality. This is necessary for the follow-up study to make sense.
H_2 It is hypothesized that the γ parameter will be higher for items that are rated as being higher in affect. (estimate of size of effect).
H_3: It is expected that the δ parameter will not be systematically modulated by the level of affect of items.

3 Design Plan

Study type: Observational Study.

Blinding: No blinding is involved in this study.

3.1 Study Design

Study 1: All subjects will rate all items (see Appendix 1) as to the level of affect they feel with regards to them.

Study 2: All participants indicate their certainty equivalence (CE) for all combinations of items (10) and certainty levels (1%, 5%, 15%, 30%, 50%, 70%, 85%, 95%, 99%). This results in 90 observations per participant.

4 Sampling Plan

Existing Data: Registration prior to creation of data.

Data collection procedures: Participants will be recruited through online channels (e.g. facebook, student groups, etc.). Participants must be at least 18 years old to participate. In the first experiments subjects will be payed 30 DKK for agreeing to participate in an approx. 10 minute online survey. In the second experiment subjects will be payed 150 DKK for agreeing to participate in an approx. 60 minute online survey.

Sample size:

Study 1: 30 participants.

Study 2: 50 participants.

Sample size rationale:

Power analysis? Credibility/Density interval 95% assuming data generating process?

5 Variables

5.1 Manipulated variables

Study 1: No manipulated variables.

Study 2: Levels of uncertainty are manipulated, and are given as $0.01, 0.05, 0.15, 0.3, 0.5, 0.7, 0.85, 0.95, 0.9$
Levels of affect differ for each item (obtained in Study 1).

5.2 Measured variables

Study 1: The single outcome variable will be the rating of affect level. This will be measured on a scale of $0 - 100$ using a slider.
Study 2: The single outcome variable is the price that subjects indicate that they are willing to pay for a ticket in a lottery (combination of probability of outcome). This will indicate their certainty equivalence (CE). This will be measured on a scale of $0 - 500$ dollars using a slider. The max is 500 dollars since the lottery tickets by definition cannot be worth more than this (see Appendix 2).

5.3 Indices

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6 Analysis Plan

All analysis is performed in the programming language R (R Core Team, 2020) using $Rstudio$ (RStudio Team, 2020).

Study 1: The affect ratings will be ordered based on group-level means?

Study 2: A bayesian generalized nonlinear mixed effects model is fit to the data using the R package $brms$ (Bürkner, 2018). This is done to estimate the unobserved parameters δ and γ from the independent variable probability/uncertainty and the dependent variable $w(p)$ which is the observed certainty equivalence (CE). Weakly informative priors are specified for both γ and δ (see Github).

7 Discussion

References

Bürkner, P.-C. (2018). Advanced Bayesian multilevel modeling with the R package brms. *The R Journal*, *10*(1), 395–411. https://doi.org/10.32614/RJ-2018-017

Gonzalez, R., & Wu, G. (1999). On the shape of the probability weighting function. *Cognitive psychology*, *38*(1), 129–166.

Hsee, C. K., & Rottenstreich, Y. (2004). Music, pandas, and muggers: On the affective psychology of value. *Journal of Experimental Psychology: General*, *133*(1), 23.

Kahneman, D., & Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, *47*(2), 263–291.

Mukherjee, K. (2010). A dual system model of preferences under risk. *Psychological review*, *117*(1), 243.

Mukherjee, K. (2011). Thinking styles and risky decision-making: Further exploration of the affect–probability weighting link. *Journal of Behavioral Decision Making*, *24*(5), 443–455.

Newell, B. R., Lagnado, D. A., & Shanks, D. R. (2015). *Straight choices: The psychology of decision making*. Psychology Press.

R Core Team. (2020). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing. Vienna, Austria. https://www.R-project.org/

Rottenstreich, Y., & Hsee, C. K. (2001). Money, kisses, and electric shocks: On the affective psychology of risk. *Psychological science*, *12*(3), 185–190.

RStudio Team. (2020). *Rstudio: Integrated development environment for r*. RStudio, PBC. Boston, MA. http://www.rstudio.com/

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, *5*(4), 297–323.

Wu, G., & Gonzalez, R. (1996). Curvature of the probability weighting function. *Management science*, *42*(12), 1676–1690.