Affective modulation of weighting function

Victor Møller Poulsen, Studie Nr.: 201707639

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1 Description

Both Expected-utility theory and prospect theory posit that humans maximize some version of utility. They do so by combination of the values of two functions (Rottenstreich & Hsee, 2001). A value function v transforms objective value to subjective utility, and a weighting function w distorts probabilities (Gonzalez & Wu, 1999; Rottenstreich & Hsee, 2001). Expected-utility and prospect theory combine these two paramters in the simplest way possible (Rottenstreich & Hsee, 2001)

$$\sum w(p_i)v(i),$$

where p stands for probability and i stands for the i^{th} gamble.

Prospect Theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992) (PT) is arguably the main model of human decision making (Newell et al., 2015). It advances theorizing from expected-utility by postulating that losses and gains are evaluated as changes in wealth rather than in regard to end states (Newell et al., 2015).

In Kahneman and Tversky (1979) we find the familiar (non-linear) S-shaped value function v which is concave for the gains domain and convex for losses (where it is steeper as well). The weight function w is the identity, $w(p) = p$ in expected-utility theory (Rottenstreich & Hsee, 2001) whereas a non-linear probability distortion is proposed in prospect theory (Kahneman & Tversky, 1979). Here w is stylized as being reverse S-shaped, meaning that it is concave for low probabilities and convex for high probabilities Gonzalez and Wu (1999). This means that people underweight changes in probability in the middle of the spectrum (e.g. $[0.2-0.8]$) while overweighting changes in probability close to the end-points (e.g. $[0.0 - 0.2], [0.8 - 1.0]$). These general characteristics of the weighting function are empirically well documented (Tversky & Kahneman, 1992; Wu & Gonzalez, 1996).

1.1 Prior work

There is evidence to support the notion that the affect of outcomes modulates the parameters of both v (Hsee & Rottenstreich, 2004) and w (Rottenstreich & Hsee, 2001). A main finding is that the S-shape of the weighting function w appears to be more pronounced for high-affect than low-affect outcomes under uncertainty (Rottenstreich & Hsee, 2001). This was shown as a preference reversal in which a high-affect outcome was preferred for low probability (1%) whereas a low-affect outcome was preferred for high probability (100%) (Rottenstreich & Hsee, 2001). The finding that affect appears to modulate both v and w has subsequently been modelled as an interaction between an affective system and a deliberative system Mukherjee (2010, 2011).

1.2 Focus and parameterization

In this article we focus exclusively on the weighting function w while ignoring both the value function v and the combination of the two functions. We also restrict ourselves to the gains domain. The article will follow much of the methodological strategy for estimating the parameters of w presented in Gonzalez and Wu (1999) in order to validate the affective distortion hinted at in Rottenstreich and Hsee (2001). The weighting function w can be parameterized in different ways, notably with either one or two parameters. This article follows Gonzalez and Wu (1999) by parameterizing w with two parameters; δ and γ.

The δ parameter will vary based on *elevation* (intercept) (Gonzalez & Wu, 1999), which here simply refers to the overall perceived attractiveness of outcomes under uncertainty.

The γ parameter will vary based on *curvature* (slope) (Gonzalez & Wu, 1999) and is what we are primarily interested in for our purposes. It follows as a direct prediction from Rottenstreich and Hsee (2001) that the curvature (γ) should be modulated by changes in the affective level of outcomes.

The model is:

$$\log \frac{w(p)}{1 - w(p)} = \gamma \log \frac{p}{1 - p} + \tau.$$

where solving for $w(p)$ and setting $\delta = \exp(\tau)$ gives us

$$w(p) = \frac{\delta \cdot p^\gamma}{\delta \cdot p^\gamma + (1 - p)^\gamma}.$$

The above equations are taken from Rottenstreich and Hsee (2001).

1.3 Methodology

but a thorough investigation of this effect is lacking. This study consits of two sub-studies. In the first study subjects will evaluate 10 items on a scale of affect. In the second study subjects will indicate their certainty equivalence (CE) as to gambles involving these questions. Based on this, the parameters of the weighting function

$$w(p) = \frac{\delta \cdot p^\gamma}{\delta \cdot p^\gamma + (1 - p)^\gamma}$$

are estimated for each of the 10 items, and it is calculated whether level of affect (obtained in study 1) modulates the parameters of the weighting function. Note that the above is the two-parameter weighting function suggested in (Gonzalez &

Wu, 1999).

2 Hypotheses

H_1: It is expected that the 10 questions in study 1 will - on average - obtain significantly different ratings as to affective quality. This is necessary for the follow-up study to make sense.

H_2 It is hypothesized that the γ parameter will be higher for items that are rated as being higher in affect. (estimate of size of effect).

H_3: It is expected that the δ parameter will not be systematically modulated by the level of affect of items.

3 Design Plan

Study type: Observational Study.

Blinding: No blinding is involved in this study.

3.1 Study Design

Study 1: All subjects will rate all items (see Appendix 1) as to the level of affect they feel with regards to them.

Study 2: All participants indicate their certainty equivalence (CE) for all combinations of items (10) and certainty levels (1%, 5%, 15%, 30%, 50%, 70%, 85%, 95%, 99%). This results in 90 observations per participant.

4 Sampling Plan

Existing Data: Registration prior to creation of data.

Data collection procedures: Participants will be recruited through online channels (e.g. facebook, student groups, etc.). Participants must be at least 18 years old to participate. In the first experiments subjects will be payed 30 DKK for agreeing to participate in an approx. 10 minute online survey. In the second experiment subjects will be payed 150 DKK for agreeing to participate in an approx. 60 minute online survey.

Sample size:

Study 1: 30 participants.

Study 2: 50 participants.

Sample size rationale:

Power analysis? Credibility/Density interval 95% assuming data generating process?

5 Variables

5.1 Manipulated variables

Study 1: No manipulated variables.

Study 2: Levels of uncertainty are manipulated, and are given as $0.01, 0.05, 0.15, 0.3, 0.5, 0.7, 0.85, 0.95, 0.9$
Levels of affect differ for each item (obtained in Study 1).

5.2 Measured variables

Study 1: The single outcome variable will be the rating of affect level. This will be measured on a scale of $0 - 100$ using a slider.
Study 2: The single outcome variable is the price that subjects indicate that they are willing to pay for a ticket in a lottery (combination of probability of outcome). This will indicate their certainty equivalence (CE). This will be measured on a scale of $0 - 500$ dollars using a slider. The max is 500 dollars since the lottery tickets by definition cannot be worth more than this (see Appendix 2).

5.3 Indices

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6 Analysis Plan

All analysis is performed in the programming language R (R Core Team, 2020) using $Rstudio$ (RStudio Team, 2020).

Study 1: The affect ratings will be ordered based on group-level means?

Study 2: A bayesian generalized nonlinear mixed effects model is fit to the data using the R package $brms$ (Bürkner, 2018). This is done to estimate the unobserved parameters δ and γ from the independent variable probability/uncertainty and the dependent variable $w(p)$ which is the observed certainty equivalence (CE). Weakly informative priors are specified for both γ and δ (see Github).

7 Discussion

References

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