A Dual System Model of Preferences Under Risk

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This article presents a dual system model (DSM) of decision making under risk and uncertainty according to which the value of a gamble is a combination of the values assigned to it independently by the affective and deliberative systems. On the basis of research on dual process theories and empirical research in Hsee and Rottenstreich (2004) and Rottenstreich and Hsee (2001) among others, the DSM incorporates (a) individual differences in disposition to rational versus emotional decision making, (b) the affective nature of outcomes, and (c) different task construals within its framework. The model has good descriptive validity and accounts for (a) violation of nontransparent stochastic dominance, (b) fourfold pattern of risk attitudes, (c) ambiguity aversion, (d) common consequence effect, (e) common ratio effect, (f) isolation effect, and (g) coalescing and event-splitting effects. The DSM is also used to make several novel predictions of conditions under which specific behavior patterns may or may not occur.

Keywords: dual process systems, preferences, risk, choice, uncertainty

Existing models of decision making under risk such as the expected utility (EU) theory (Von Neumann & Morgenstern, 1944), prospect theory (Kahneman & Tversky, 1979), rank-dependent utility theory (Quiggin, 1981; Schmeidler, 1989), cumulative prospect theory (CPT; Tversky & Kahneman, 1992), and their variants take a unitary perspective toward the human mind. They assume a single system of thought that assigns subjective values to the outcomes of a risky prospect, perceives probabilities or cumulative probabilities in specific systematic ways, and then combines the two in an algorithmic fashion in order to arrive at a valuation for the given gamble. This article departs from the unitary perspective and presents the dual system model (DSM), a descriptive model of preferences under risk from the dual process perspective. In the psychology literature, there is extensive research on dual process theories, but not much attention has been paid to dual process–based models of individual decision making under risk and uncertainty. The result of the proposed formulation is a behaviorally grounded model that provides a parsimonious account of a wide range of behavioral phenomena and has the advantage of incorporating within its framework (a) different thinking dispositions, a feature of the decision maker; (b) affective content of the outcomes, a feature of the risky gambles; and (c) different task construals, a feature of the nature of the task. Standard models such as EU and CPT are silent on these important dimensions of decision making.

The dual process paradigm of reasoning states that there are two fundamentally different ways of processing information, one variously labeled as intuitive, automatic, natural, narrative, and experiential and the other seen as analytical, verbal, deliberative, and rational. Stanovich and West (2000) referred to the former system collectively as *System 1* and the latter collectively as *System 2*. Psychological research on decision making has increasingly demonstrated that dual process models are more successful in explaining behavior than are unitary models in a wide variety of settings (e.g., Chaiken & Trope, 1999; Epstein, 2003; Hogarth, 2001; Kahneman, 2003; Sanfey, Loewenstein, McClure, & Cohen, 2006). Also, the neuroscience literature increasingly shows evidence of multiple separable neural systems in the brain that contribute to decision making and behavior (Damasio, 1994; LeDoux, 1996; Sanfey et al., 2006). Economics, too, is being increasingly influenced by a multiple systems approach to decision making (Bernheim & Rangel, 2004; Bracha & Brown, 2008; Loewenstein & O'Donoghue, 2004).

There are several dual process theories (Deutsch & Strack, 2006; Epstein, 1973, 1994; Evans & Over, 1996; Hammond, 1996; Hogarth, 2005; Klein, 1998; Levinson, 1995; Pollock, 1991; Reber, 1993; Sloman, 1996; E. R. Smith & DeCoster, 2000) with considerable overlap of content and structure (see Chaiken & Trope, 1999, for a comprehensive review of the field). Building on the commonalities in the different dual process theories, I assume an associative affect-based mode of decision making (System A) and a deliberative rule-based mode of decision making (System D). Processing in System A is intimately influenced by mood and emotional states of mind and involves how one feels about a particular prospect. On the other hand, processing in System D is analytical in nature and can involve computational operations. Hence, the affective system is driven by preconscious, less effortful, experiential considerations, and the deliberative system is driven by conscious, more effortful, numerical and logical considerations.

Decision making behavior is influenced by both systems to varying degrees, depending on the individual, the nature of the outcomes, and the task construal. Individuals can have different thinking dispositions (Epstein, Pacini, Denes-Raj, & Heier, 1996), the outcomes in a gamble can be more or less affective in nature, and the task itself can be construed in ways that may influence the

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two systems to different extents (Stanovich & West, 2000). In general, emotional arousal and relevant experience shift the balance of influence toward System A. The way in which the relative influence of the two systems is captured in the model is treated in detail in the Model Formulation section.

The model draws motivation from empirical regularities observed in Hsee and Rottenstreich (2004) and Rottenstreich and Hsee (2001) and demonstrates how it accounts for a wide variety of empirical phenomena reported in the literature and makes several empirically testable predictions along the way. The general structure of the model is given in Figure 1. The stimulus, a risky gamble G, is directed to the two systems of processing: the affective system and the deliberative system. In general, each system then assigns a value to the gamble, independent of each other, on the basis of its own method of evaluation. The final output that drives behavior is then a combination of the valuations of the two systems. It is important to note that the model does not imply that people necessarily engage both systems of processing in all situations. Any specific formulation should allow for the usage of only one system or the other.

The next section presents the assumptions and the model formulation. This is followed by analysis of various applications and a conclusion section.

Model Formulation

Value Functions

Hsee and Rottenstreich (2004) examined the effect of the magnitude or scope of a stimulus on its perceived subjective value. They assumed a dual process mode of valuation similar to the one envisaged in this article and proposed that people value stimuli by two psychological processes: *valuation by calculation* and *valuation by feeling*. They showed that under a wide range of decision-making circumstances, people were generally insensitive to the magnitude of the stimulus when they relied on their feelings and displayed relatively constant sensitivity to scope when they relied on calculation. A general representation of the two different value functions, one based on calculation and the other based on feelings, is shown in Figure 2. For our purposes, the valuation by calculation corresponds to a preferential use of System D, and the valuation by feelings corresponds to a preferential use of System A.

In one of Hsee & Rottenstreich's (2004) experiments, two groups of participants were primed in order to enhance their tendency to engage in either valuation by calculation or valuation by feeling. Then they were asked to value either five or 10 Madonna compact discs (CDs) in a between-subjects design. When primed to calculate, participants valued the five-CD set at an
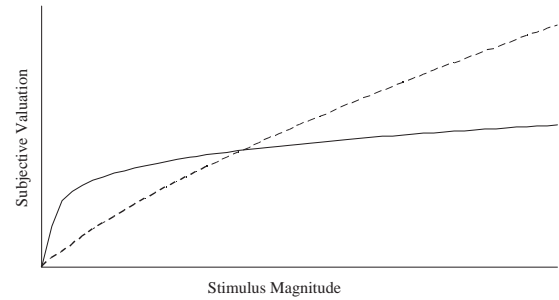

Figure 1. Model of decision making with dual systems.

Figure 2. Hypothetical value functions based on calculation (dotted line) and based on feeling (solid line). (Adapted from Hsee & Rottenstreich, 2004.)

average of $15.10 and the 10-CD set at an average of $28.81, showing significant sensitivity to the magnitude of the stimulus. On the other hand, when primed to feel, they valued the five-CD set at an average of $22.64 and the 10-CD set at an average of $19.77, showing insensitivity to the number of CDs in the stimulus. Similar results were found when relatively affect-rich (i.e., a music book) or affect-poor (i.e., cash) stimuli were valued or when the affective nature of the stimuli presentation was varied (e.g., using cute pictures vs. using dots to represent the number of pandas rescued in a task soliciting donations for the rescue effort). Participants consistently showed that in an affect-rich context, they were sensitive only to the presence or absence of stimuli, whereas in an affect-poor context, they were cognizant of the magnitude of the stimuli and accounted for it in their valuations. As evident in Figure 2, this dual valuation scheme may yield instances where feeling produces greater value at low magnitudes but calculation produces greater value at high magnitudes. Indeed, this was the case in the experiment with Madonna CDs mentioned above.

The neglect of magnitude in valuing nonmarket goods, such as the rescue of endangered species, has been researched extensively, and as Hsee and Rottenstreich (2004) observed, the results are largely consistent with those of their experiments (Baron & Greene, 1996; Desvousges et al., 1993; Frederick & Fischhoff, 1998; Kahneman, Ritov, & Schkade, 2000). The notion of using feelings as an input for valuation has been investigated by Slovic, Finucane, Peters, and MacGregor (2002) and Finucane, Alhakami, Slovic, and Johnson (2000), among others, and the current development follows from this research.

The research described motivates these first two key assumptions in the dual process model described here:

Assumption 1. System D values outcomes linearly, that is, $V_D(x) = kx$, where x is an outcome, such as dollars; $V_D(.)$ represents the value in System D; and k is a scaling constant.

Assumption 2. System A is nonlinear, monotonically increasing but with decreasing sensitivity to the magnitude of the stimuli. It is based on changes from a reference point, assumed to be the status quo. The affective value function, $V_A(x)$ for some outcome x with $V_A(0) = 0$, values outcomes according to a concave value function in the gain domain and a convex value function in the loss domain. Hence, the value function is similar in form to the prospect theory value function (Kahneman & Tversky, 1979), but it is not the same, because it represents value only in the affective system and captures behavior in conjunction with the
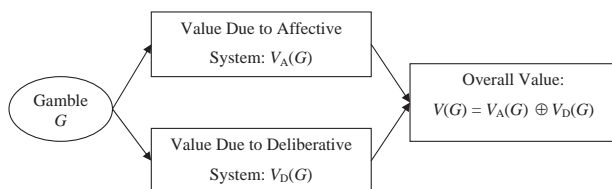
value function of the deliberative system, whereas the prospect theory value function is derived from the assumption of a unitary system of decision making. The loss aversion feature of prospect theory, for which the value function in the loss domain is steeper than that in the gain domain, is retained in the affective value function but not in the deliberative value function.

Practical valuations are a combination of the two value functions. Note that diminishing marginal utility is assumed to be a characteristic of only the affective system. The fact that the fifth cake one eats is less enjoyable than the first one or the 10th day of vacation is less enjoyable than the first day is a result of processing in the affective system. The cold and calculating deliberative system is not susceptible to the feeling of satiation that is intimately linked to the affective system.

Probability Perceptions

The psychophysics of probability perception is represented by an inverse S-shaped probability weighting function in which small probabilities are overweighted and large probabilities are underweighted (Abdellaoui, 2000; Bleichrodt & Pinto, 2000; Camerer & Ho, 1994; Gonzalez & Wu, 1999; Kahneman & Tversky, 1979; Kilka & Weber, 2001; Prelec, 1998; Preston & Baratta, 1948; Tversky & Fox, 1995; Tversky & Kahneman, 1992; Wu & Gonzalez, 1996). Rottenstreich and Hsee (2001) showed that the shape of the probability weighting function depended on the affective nature of the outcomes. Relatively affect-rich outcomes produced a curve that was significantly more S-shaped than that produced by relatively affect-poor outcomes (see Figure 3). Hence, for affect-rich outcomes people were more sensitive to their possibility of occurrence and less sensitive to intermediate levels of probability. On the other hand, affect-poor outcomes showed relative sensitivity to intermediate probability levels.

In one experiment, the majority of participants preferred $50 cash to a kiss from a favorite movie star, but when the same outcomes were presented as 1% chance lotteries, the majority preferred the kiss lottery. Similar results were obtained in a pricing task with low-probability $500 coupons redeemable for a European vacation (affect-rich) or tuition payments (affect-poor). The European vacation lottery was priced significantly higher than the tuition lottery. However, when high probability was used with

Figure 3. Hypothetical affect-poor (dotted line) and affect-rich (solid line) probability weighting functions. (Adapted from "Money, Kisses, and Electric Shocks: On the Affective Psychology of Risk," by Y. Rottenstreich and C. K. Hsee, 2001, *Psychological Science, 12,* p. 186. Copyright 2001 by Wiley–Blackwell. Adapted with permission.)

the same coupons, the affect-rich lottery was valued lower than the affect-poor one. The same paradigm was also demonstrated in the loss domain.

According to Rottenstreich & Hsee (2001), this affect-driven account of probability weighting occurs because affect-rich outcomes "elicit greater degrees of hope and fear and, therefore, larger jumps at the endpoints" (p. 186). Their explanation follows from the observation that mental imagery, which underlies emotion, is insensitive to intermediate probabilities (Elster & Loewenstein, 1992). Hence, the mental image of a car crash is equally frightening whether the probability is 1% or 10%, and when affective reactions depend on such mental images, it is conceivable that processing in the affective system is insensitive to intermediate probabilities. This is also consistent with the *risk as feelings* model of Loewenstein, Weber, Hsee, and Welch (2001) and findings in Slovic, Monahan, and MacGregor (2000) and Camerer (1992).

These considerations motivate these next two key assumptions in the model:

Assumption 3. System D perceives probability, a learned mathematical concept, without any distortion, that is, $w(p) = p$, which is identical to the formulation in the EU theory.

Assumption 4. System A recognizes only whether an outcome is possible and is entirely insensitive to probabilities. Hence, every outcome with nonzero probability gets equal weight in this system.

The probability distortion effects that one observes in practice can be viewed as a result of the combined output from the two systems. Assumptions 1 and 3 lead to the valuation of a risky prospect in System D, and Assumptions 2 and 4 lead to the valuation of a risky prospect in System A.

The DSM Model

Let $(p_1, x_1; p_2, x_2 \ldots; p_n, x_n)$ represent gamble G with n outcomes, such that p_i represents the probability of the ith outcome, x_i, of the gamble. Note that x_i could be a risky gamble itself. Assume also that $p_i > 0$ for all i values and $\Sigma p_i = 1$. According to Assumption 2, the value function in System D can be represented as $V_D(x_i) = kx_i$, and according to Assumption 4, System D perceives probability without any distortion. Following from these two assumptions, the valuation of gamble G in the deliberative system is given by

$$V_D(G) = \sum_i p_i V_D(x_i) = k\sum_i p_i x_i. \tag{1}$$

Hence, the value of a gamble in System D is simply the scaled expected value of the gamble.

On the basis of Assumptions 1 and 3, the valuation in System A is given by

$$V_A(G) = \sum_i \frac{1}{n} V_A(x_i). \tag{2}$$

The two values are combined to produce a single value of the gamble, which then drives decision-making behavior. It is proposed that, in the interest of parsimony and analytical tractability, the overall value of the gamble is simply the convex combination of the values in the two systems. Note that both the values are in
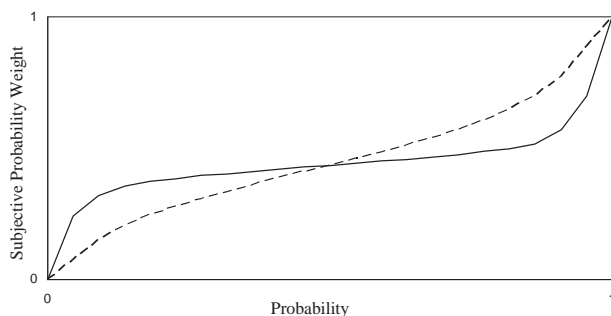
terms of utiles. When γ is the weight given to System A, $\gamma \in [0, 1]$, then

$$V(G) = \gamma V_A(G) + (1 - \gamma)V_D(G) = \gamma \frac{1}{n}\sum_i V_A(x_i)$$

$$+ (1 - \gamma)k\sum_i p_i x_i. \quad (3)$$

This is stated formally in the following definition:

Definition 1.   For two gambles, G_i and G_j, $G_i \succeq G_j$, where $\succeq$ represents weak preference, if and only if $V(G_i) \geq V(G_j)$, where $V(.)$ is given by Equation 3.

An example of this, assuming only positive outcomes and a power value function for the affective system, is the following:

$$V(G) = \gamma \frac{1}{n}\sum_i x_i^m + (1 - \gamma)k\sum_i p_i x_i, \quad (4)$$

where $m < 1$. The estimation of the parameters is a measurement exercise and can be taken up in future research. See Appendix for the formulation when outcomes are continuous.

The goal of the above formulation is to capture both routes of thinking and explain as much as possible in as simple a way as possible. Admittedly, it may not capture the actual process of decision making in its entirety, but I believe that the simplicity outweighs the costs of having a more complex formulation. Hence, in the deliberative system one may be aware that $100 million is only marginally better than $10 million, and in the affective system one may respond to a 1/10 chance of a plane crash differently compared with a 1/1,000,000 chance.[1] The point is to make the model streamlined so that it can be used to analyze and predict behavior in a parsimonious way.

In the Properties and Applications section, the discrete form of the model will be applied to a wide variety of behavioral phenomena reported in the literature, and the usefulness of the model in explaining them and deriving new predictions will be demonstrated. The way the model can be applied to cases of ambiguity, where information about the probability of gambles is either unavailable or incomplete, is illustrated in the Ambiguity Aversion section. However, before discussing the properties and applications of the model, I'd like to discuss the psychological interpretation of γ.

Psychological Interpretation of γ

The parameter γ can be interpreted as the relative extent of involvement of System A in decision making under risk. A comprehensive analysis of what factors influence the two systems and to what extent is beyond the scope of this article. It is, however, proposed that γ is influenced by at least the three following factors:

1. Individual thinking dispositions: Researchers have used different thinking dispositions to predict individual differences in a variety of thinking tasks over and above cognitive ability (Cacioppo & Petty, 1982; S. M. Smith & Levin, 1996; Stanovich & West, 1998). It has been demonstrated that within the dual process paradigm, there are reliable individual differences in the two thinking styles

(e.g., Epstein et al., 1996). Epstein and colleagues (1996) have also developed the Rational–Experiential Inventory to measure different thinking predispositions. It is proposed that in the framework of the current model, people who lay more stress on how they feel about the available options before making a decision will have higher γ values than will those who lay more stress on logical or quantitative analysis. Hence, on one extreme are the risk-neutral EU maximizers, and on the other extreme are the people basing their decisions on only the possible outcomes of each option with no regard for probabilities. Presumably, most people will fall somewhere between these two extremes.

2. Nature of the outcomes: The more the affective nature of the outcomes, the higher the value of γ. Hence, $100 for a fancy dinner at one's favorite restaurant will, on average, induce higher γ than will $100 for groceries. Because diminishing marginal utility need not depend systematically on the affective content of the outcomes, it is assumed that the valuation of outcomes in System A does not depend on the nature of the outcomes. In addition to the affective nature of the outcomes, the magnitude of the stakes can also make a difference. Camerer (1992) found that gambles with large stakes ($10,000 or $25,000) induced choice behavior consistent with a pronounced S-shaped probability weighting function, and when stakes were small ($5 or $10), behavior was consistent with a weighting function with small curvatures. This is also implied in venture theory (Hogarth & Einhorn, 1990). Rottenstreich and Hsee (2001) interpret this result as a consequence of large cash prizes engendering "more significant emotional reactions than small cash prizes" (p. 189). Though it is possible that the value of γ is higher when stakes are orders of magnitude higher, it is assumed that γ remains constant locally; that is, there is no change in the weight given to the affective system for small to medium changes in outcome values.

3. Task construal: The nature of the task and the way it is construed influences the relative involvement of one system or the other. There is likely to be more affective involvement for decisions made for oneself rather than those made for some other entity. Hence, decisions for the self are likely to have higher γ. This leads to the prediction that decision makers such as managers in companies will exhibit more risk-neutral behavior than will people making personal decisions (Faro & Rottenstreich, 2006; Hsee & Weber, 1997). Decision problems that are contextualized, such as risky options for medical procedures, are likely to have higher affective involvement and thereby higher γ than are abstract problems posed in the laboratory. Another important factor that could drive relative influence of the two systems is temporal proximity. Decision problems needing immediate attention and having immediate resolution are likely to

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[1] I thank Nick Chater for the stated examples.

have greater affective involvement and a higher γ. Indeed, people were found to exhibit less risk-averse behavior for lotteries further out in time (Noussair & Wu, 2006; Sagristano, Trope, & Liberman, 2002; Shelley, 1994). The deliberative system is logical and calculative. It uses tools and techniques such as different mathematical operations, and the more data it has, the more it can apply this acquired knowledge. In the case of ambiguity, for example, for which specific probability data are not available, System D does not have much to work with. This increases the relative influence of System A, increasing the γ for ambiguous gambles. Another example is that of joint or separate evaluations. Compared with separate evaluations, joint evaluations provide more data to System D and reduce γ. This is consistent with the speculation in Rottenstreich and Hsee (2001) that joint evaluations should produce greater sensitivity to stimuli magnitude than would separate evaluations.

In the applications that follow in this article, γ is assumed to remain unchanged in tasks with the same features, such as choices between risky gambles. However, it can change when the nature of the task changes, but the way it varies between tasks should be consistent with some existing theory or should appeal to intuition. In other words, while applying the model, care must be taken not to vary γ in an arbitrary fashion.

The next section presents several well-known decision problems to which the model is applied. The section demonstrates how the model can be used to explain different behavioral phenomena and includes predictions concerning new testable behavior patterns. The decision problems addressed are not reviewed in great detail, because they have large literatures of their own. The intention is to summarize the decision problems and analyze them from the perspective of the DSM.

Properties and Applications

Monotonicity and Transitivity

It is easy to see that the DSM preserves monotonicity and transitivity as long as γ remains unchanged in a given task. In decision tasks that are temporally and spatially confined—for example, choices between gambles in a laboratory session—this is likely to be true. However, γ for an individual may not remain constant for decisions involving different contextual situations over a period of time. Hence, people might still exhibit violations of monotonicity and transitivity if several decisions over a period of time are considered.

Consider, for example, Gambles A = (.5, 100) and B = (.5, 81), where (p, x) denotes an outcome of x with probability p, otherwise zero. Assuming $k = 1$ and a power value function for the affective system with $m = 0.5$ in Equation 4, $V(A) = 50 - 45\gamma_A$ and $V(B) = 40.5 - 36\gamma_B$. Monotonicity will be violated—that is, $V(B) > V(A)$—when $\gamma_A > 0.2 + 0.8\gamma_B$. Hence, monotonicity will not be violated unless there is an affective trigger that alters the balance of influence of the two systems. The analysis for transitivity follows a similar line of reasoning.

Nontransparent Stochastic Dominance

The principle of dominance states that if one option is better than another in one state and at least as good in all other states, then the dominant option should be chosen. Relatedly, first-order stochastic dominance states that, for Risky Prospects A and B, A is preferred to B if the cumulative distribution of A is to the right of the cumulative distribution of B, that is, the probability of earning any given amount or higher is greater in Option A than in Option B. Tversky and Kahneman (1986) reported that the dominance rule is obeyed when its application is transparent, but masking of dominance in a way such that the dominated option yields a better outcome in some identified state results in significant violation of dominance. The decision problems from Tversky and Kahneman (1986) in Table 1 illustrate their point.

In Problem 1, it is easy to see that Option B dominates Option A, and all participants chose B over A. Problem 2 is identical to Problem 1, but the dominance relation is masked. Option C is formed by coalescing the last two states in Option A, and Option D is formed by coalescing the second and third states in Option B. The formulation of Problem 2 "enhances the attractiveness of C, which has two positive outcomes and one negative, relative to D, which has two negative outcomes and one positive" (Tversky & Kahneman, 1986, pp. 264–265). The result was that 58% of respondents chose the dominated Option C. If the affective system weighs all outcomes equally, then Gamble C with two good outcomes and one bad outcome will be perceived as better than Gamble D, which has one good and two bad outcomes. Hence, Gamble C will be preferred by the affective system, and Gamble D will be preferred by the dominance-satisfying deliberative system. This may lead to violation of dominance under specific conditions, which is formally analyzed below.

Birnbaum (1997, 2005) used a modified paradigm involving three outcome gambles, all of which had positive outcomes, to generate substantial violations of stochastic dominance. Consider Gamble 1 = (.90, 96; .05, 14; .05, 12) and Gamble 2 = (.85, 96; .05, 90; .10, 12), where Gamble 1 dominates Gamble 2 but has one good outcome and two bad outcomes compared with two good

Table 1
Transparent and Nontransparent Dominance Examples From Tversky and Kahneman (1986)

Problem and choice	Gamble branches				
	1	2	3	4	5
Problem 1: Transparent dominance					
Option A					
Probability	.90	.06	.01	.01	.02
Outcome	$0	$45	$30	–$15	–$15
Option B					
Probability	.90	.06	.01	.01	.02
Outcome	$0	$45	$45	–$10	–$15
Problem 2: Nontransparent dominance					
Option C					
Probability	.90	.06	.01	.03	
Outcome	$0	$45	$30	–$15	
Option D					
Probability	.90	.07	.01	.02	
Outcome	$0	$45	−$10	–$15	

outcomes and one bad outcome in Gamble 2. About 74% of participants chose Gamble 2, thereby violating dominance (Birnbaum, 2005).

One can analyze the problem from the perspective of the DSM by generalizing the gambles to $G_1 = (p_1, x; p_2, y; p_3, z)$ and $G_2 = (q_1, x; q_2, y'; q_3, z)$ such that $x > y' > y > z$ and $q_1 + q_2 \leq p_1$. Under these conditions, G_1 dominates G_2 and this representation is the nontransparent version of the problem with G_1 having one good outcome, x, and two bad outcomes, y and z, and G_2 having two good outcomes, x and y', and one bad outcome, z. The dominance relationship becomes transparent when the gambles are represented as $G_1 = [q_1, x; q_2, x; p_1 - (q_1 + q_2), x; p_2, y; p_3, z]$ and $G_2 = [q_1, x; q_2, y'; p_1 - (q_1 + q_2), z; p_2, z; p_3, z]$. In the transparent representation of the problem, the states of the world are represented identically, making it easy to do a statewise comparison of the outcomes, which then makes the dominance relationship apparent. Transparency can assume many forms, but for the purposes of this discussion it is defined as follows:

Definition 2. If two gambles have an equal number of branches and the corresponding ranked branches have identical probabilities, then a dominance relationship between the risky choices is transparent.

Proposition 1. The DSM preserves transparent dominance as defined above.

Proof. Applying the DSM to G_1 and G_2 results in $V_A(G_1) - V_A(G_2) = (1/5)[V_A(x) - V_A(y') + V_A(x) - V_A(z) + V_A(y) - V_A(z)] > 0$ and $V_D(G_1) > V_D(G_2)$ as $V_D(.)$ preserves dominance. Hence, $V(G_1) > V(G_2)$. QED.

Proposition 2. For $G_1 = (p_1, x; p_2, y; p_3, z)$ and $G_2 = (q_1, x; q_2, y'; q_3, z)$ where $x > y' > y > z$ and $q_1 + q_2 \leq p_1$, the necessary and sufficient condition for violation of nontransparent dominance under the DSM is

$$\gamma > \theta, \text{ where } \theta = \frac{1}{1 + \dfrac{V_A(G_2) - V_A(G_1)}{V_D(G_1) - V_D(G_2)}}. \tag{5}$$

Proof. Violation of stochastic dominance occurs if $V(G_2) > V(G_1)$. Note that $y' > y$ implies $V_A(G_2) > V_A(G_1)$, and because $V_D(.)$ preserves dominance, $V_D(G_1) > V_D(G_2)$, implying that $\theta \in (0, 1)$. Applying the DSM formulation and going through simple algebraic manipulations yield the above result. QED.

Note that γ is a characteristic of the decision maker and θ is a function of the gambles under consideration. Hence, violation of dominance is possible under certain plausible conditions when the relative influence of System A exceeds a specific value driven by the decision situation. The following, hitherto untested, prediction can thus be made:

Prediction 1. People whose thinking disposition is more affectively oriented are more likely to violate dominance.

It is important to note that, given some frequency distribution of γ values over [0, 1] in the population for this problem, as the value of θ increases, the incidence of violation will decrease because there will be fewer people satisfying the condition for violation. This feature can be used to do a comparative statics analysis on θ, which yields the following predictions (assuming that γ remains constant for small to medium changes in the outcome values):

Prediction 2. Under the assumption of a power value function, $V_A(x) = x^m$, violations will increase as the value of m

increases (curvature of the affective value function decreases) because $\partial\theta/\partial m < 0$.

Partially differentiating θ with respect to m reveals that as the value of m increases the value of θ decreases. As explained above, a decrease of θ implies an increase in violations of dominance. Hence, increasing m will increase violations. This logic has been used to make a number of predictions in the rest of the article.

Prediction 3. Violations will decrease with increasing value of x because $\partial\theta/\partial x > 0$.

Prediction 4. Violations will increase with increasing value of y' because $\partial\theta/\partial y' < 0$.

Prediction 5. Violations will decrease with increasing value of y because $\partial\theta/\partial y > 0$.

Prediction 6. Violations will increase with increasing value of z because $\partial\theta/\partial z < 0$.

Prediction 7. With q_2 or q_3 fixed, violations will increase with increasing value of q_1 because $\partial\theta/\partial q_1 < 0$.

Prediction 3 has empirical support in Study 4 reported in Birnbaum (2005). When y' was increased from \$20 to \$90, violations increased from 53% to 66%. Prediction 6 has empirical support in Studies 1, 2, and 3 reported in Birnbaum (2005). In Studies 1 and 2, q_2 was fixed at 5%, and when q_1 was increased from 25% to 85%, violations rose from 35% to 74%. In Study 3, q_3 was fixed at 10%, and when q_1 was increased from 15% to 85%, violations rose from 60% to 74%.

Standard choice theories such as the EU theory, RDU, rank- and sign-dependent utility theory (Luce & Fishburn, 1991), the CPT (Tversky & Kahneman, 1992), lottery-dependent utility theory (J. L. Becker & Sarin, 1987), aspiration level theory (Lopes & Oden, 1999), and generalized utility theory (Machina, 1982) do not allow violation of stochastic dominance and cannot explain the above findings.

Risk Attitudes

The issue of risk attitudes is analyzed with the simple two-outcome gamble $G = (p, x)$ assuming a power value function of $V_A(x) = x^m$. The results extend to gambles with multiple outcomes. The value of gamble G is given by

$$V(G) = \gamma x^m/2 + (1 - \gamma)kpx. \tag{6}$$

It can be readily seen that risk neutrality results when $\gamma = 0$. What is not obvious, however, is that risk neutrality for $\gamma > 0$ results as well, as stated in the next proposition.

Proposition 3. For gamble $G = (p, x)$ and value function $V_A(x) = x^m$, there exists a value of $m = m^*$, where $m^* = -ln(2)/ln(p)$, at which risk neutrality is obtained for all values of $\gamma \in [0, 1]$.

Proof. If c is the certainty equivalent (CE) of gamble G, then

$$\gamma x^m/2 + (1 - \gamma)kpx = \gamma c^m + (1 - \gamma)kc. \tag{7}$$

Setting $c = px$ (the expected value [EV] of the gamble), the condition for risk neutrality, leads to $c^m = x^m/2$, which on solving for m yields $m^* = -ln(2)/ln(p)$. QED.

Proposition 3 states that individuals having $m = m^*$ will be risk-neutral for the gamble (p, x) irrespective of the balance of affective and deliberative processing. Risk-averse or risk-seeking behavior is characterized in the next proposition.

Proposition 4. For gamble $G = (p, x)$, value function $V_A(x) = x^m$, and m^* as defined in Proposition 3, $m < m^*$ implies risk aversion and $m > m^*$ implies risk seeking. Moreover, when $m < m^*$, risk aversion increases with γ, and when $m > m^*$, risk seeking also increases with γ.

Proof. See Appendix.

Propositions 3 and 4 imply that the curvature of the affective value function, which measures how rapidly an individual approaches satiation, is an important determinant of risk attitude. Also, the increased use of System A can lead to both increased risk aversion and risk seeking, depending on the curvature of the affective value function. Following is a brief discussion of the fourfold pattern of risk attitudes to illustrate these points.

Empirical evidence suggests that people are (a) risk seeking for gains and risk-averse for losses of low probability and (b) risk-averse for gains and risk seeking for losses of high probability (Fishburn & Kochenberger, 1979; Kahneman & Tversky, 1979; Tversky & Kahneman, 1992). Because this pattern of risk attitudes occurs over a wide range of payoffs, explanations based on the shape of the utility function (Friedman & Savage, 1948; Markowitz, 1952) have been rejected. Instead, an inverse S-shaped pattern of probability transformation, where low probabilities are overweighted and high probabilities are underweighted, has been suggested to explain this phenomenon. The DSM, which proposes a dual system–based dichotomy of valuation of risky prospects, also implies a similar pattern of behavior.

Table 2 gives the CEs of the gamble (.05, 100) at different values of γ and m, assuming, for simplicity, $k = 1$. In this gamble, $m^* = 0.23$, and at this value CE = EV for all values of γ. Note that, as per Proposition 4, risk seeking occurs only for $m > 0.23$, in which situations the degree of risk seeking increases with γ at all levels of m. Likewise, when $m < 0.23$, risk aversion, which then increases with γ, occurs. Hence the following prediction:

Prediction 8. Individuals with affective value functions with sufficient degree of curvature ($m < m^*$) will not be risk seeking for low-probability gains, as predicted by the fourfold pattern of risk attitudes.

Prediction 8 implies that people who reach satiation rapidly are unlikely to be consistent with the four-fold pattern. The high-probability end is shown in Table 3, which gives the CEs of the gamble (.95, 100) at different values of γ and m. Because $m < 1$ is an assumption in the model and $m^* = 13.5$ in this case, risk aversion results for all values of m and γ consistent with the predictions of the fourfold pattern. Interestingly, the analysis

Table 2
Predicted Certainty Equivalents of Gamble (.05, 100) at Different Values of γ and m

m	γ					
	0	.2	.4	.6	.8	1
0.2	5.00	4.97	4.92	4.83	4.60	3.31
0.23	5.00	5.00	5.00	5.00	5.00	5.00
0.4	5.00	5.30	5.76	6.55	8.30	17.68
0.6	5.00	6.23	7.97	10.68	15.77	31.50
0.8	5.00	8.58	13.06	19.03	27.66	42.04
1	5.00	14.00	23.00	32.00	41.00	50.00

Table 3
Predicted Certainty Equivalents of Gamble (.95, 100) at Different Values of γ and m

m	γ					
	0	.2	.4	.6	.8	1
0.2	95.0	94.7	94.2	93.2	90.2	3.3
0.4	95.0	94.3	93.0	90.6	84.1	17.7
0.6	95.0	93.2	90.3	85.3	73.9	31.5
0.8	95.0	90.8	85.0	76.6	63.7	42.0
1	95.0	86.0	77.0	68.0	59.0	50.0

shows that the DSM implies a higher incidence of risk aversion for high-probability gains than the incidence of risk seeking for low-probability gains. This is consistent with the subcertainty hypothesis of prospect theory, which states that underweighting of large probabilities is more prevalent than the overweighting of small probabilities (Tversky & Kahneman, 1986).

The analysis for negative payoffs follows an identical methodology, yielding the corresponding results, and is consequently omitted here.

Ambiguity Aversion

An ambiguous decision problem is one in which information is known to be missing (Frisch & Baron, 1988). The most common demonstration of an ambiguity attitude is found in experiments in which participants choose between a risky gamble, for which probabilities are known, and an ambiguous gamble, for which the probabilities are either completely unknown or there is incomplete information about them (Ellsberg, 1961). Consider a choice between Gamble A, in which an urn contains 50 black and 50 white balls and the participant will receive prize W if a ball drawn at random is black, and Gamble B, in which an urn contains 100 black and white balls in an unknown proportion and the decision maker will receive prize W if a ball drawn at random is black. Most people choose the unambiguous urn in Gamble A in this decision problem, thereby exhibiting ambiguity aversion, which leads to violations of subjective EU (Savage, 1954) under specific conditions (Ellsberg, 1961). This problem and various variants have shown that ambiguity aversion is a robust phenomenon and that people are immune to written arguments against their inconsistent choices (Slovic & Tversky, 1974) and are even willing to pay relatively large premiums (10%–20% of expected value) to avoid ambiguity (S. W. Becker & Brownson, 1964; Bernasconi & Loomes, 1992; Curley & Yates, 1989; MacCrimmon & Larsson, 1979).

This problem can be analyzed from the perspective of the DSM by considering an unambiguous Urn U containing 100 black and white balls with a known proportion p of black balls and an ambiguous Urn A containing the same total number of black and white balls but in uncertain proportions. The decision maker needs to choose either Urn U or Urn A, and if a ball drawn at random from the chosen urn is black, then the person will receive prize W. The specification of the ambiguous urn is such that p lies at the center of the range of black balls. For example, with $p = .4$, or 40 black balls, in the unambiguous urn, the ambiguous urn could have 0 to 80 black balls or 10 to 70 black balls and so forth. The degree

of ambiguity depends on the range of black balls and increases with increasing range. Let γ_U and γ_A be the relative use of the affective system while valuing the unambiguous and ambiguous urns, respectively. As discussed earlier, because important data about the decision problem, relevant for the deliberative system, are missing in the ambiguous case, the reliance on the affective system increases with ambiguity and results in $\gamma_A > \gamma_U$, with $\gamma_A - \gamma_U$ increasing with increasing degree of ambiguity. In valuing the ambiguous urn according to the deliberative system, one assumes that a discrete uniform second-order probability (SOP) distribution is formed over the specified range of black balls in the urn, that is, $P(i) = 1/r$, where $P(i)$ is the probability that there are i number of black balls in the ambiguous urn, where i belongs to range r of black balls. Using the deliberative system, one can then proceed to compute the expected value of the gamble from the compound lottery, thus formed, in the usual way. The concept of using SOP in choice models in general and ambiguity models in particular is not new and has been used before, albeit in a unitary system setup, by Marschak (1975); Chew, Karni, and Safra (1987); Segal (1987); Kahn and Sarin (1988); Klibanoff, Marinacci, and Mukerji (2005); and Nau (2006; see Camerer & Weber, 1992, for an early review of empirical evidence and models dealing with ambiguity). Note that the affective system value function is blind to probabilities and will not be directly affected by ambiguity.

Using a power value function for the affective system and applying the DSM, the value of the unambiguous Urn U is given by

$$V(U) = \gamma_U W^m/2 + (1 - \gamma_U)kpW. \tag{8}$$

The value of the ambiguous Urn A is given by

$$V(A) = \gamma_A W^m/2 + (1 - \gamma_A)\frac{kW}{r}\sum_{i \in r} i/n$$
$$= \gamma_A W^m/2 + (1 - \gamma_A)kpW. \tag{9}$$

For gains ($W > 0$), choosing the unambiguous urn, $V(U) > V(A)$, yields the following necessary and sufficient condition for ambiguity aversion:

$$2kpW^{1-m} > 1. \tag{10}$$

For losses ($W < 0$), assuming loss aversion parameter λ, where $\lambda > 1$, the following necessary and sufficient condition for ambiguity aversion results:

$$2kp|W|^{1-m}/\lambda < 1. \tag{11}$$

The ambiguity premium (AP), the amount people are willing to give up in order to avoid ambiguity, is captured by $V(U) - V(A)$, the incremental value of the unambiguous urn. For gains, it is given by

$$AP = (\gamma_A - \gamma_U)(kpW - W^m/2). \tag{12}$$

For losses, it is given by

$$AP = (\gamma_A - \gamma_U)(\lambda|W|^m/2 - kpW). \tag{13}$$

Inspecting Inequalities 10 and 11 leads to the following predictions:

Prediction 9. Ambiguity-averse behavior increases with increasing p for gains and decreases with increasing p for losses. Consequently, people are likely to be ambiguity seeking for low-probability gains and high-probability losses and ambiguity-averse for high-probability gains and low-probability losses.

Prediction 10. Ambiguity-averse behavior increases with increasing W for gains and decreases with increasing magnitude of W for losses.

Because $\lambda > 1$, all ambiguity seekers for gains ($2kpW^{1-m} < 1$) will be ambiguity-averse for losses and some will be ambiguity-averse for both gains and losses. This leads to the following prediction:

Prediction 11. Ambiguity averters for losses outnumber ambiguity seekers for gains. Equivalently, ambiguity averters for gains outnumber ambiguity seekers for losses.

Because $\gamma_A - \gamma_U$ increases with increasing degree of ambiguity, Equations 12 and 13 lead to the following prediction:

Prediction 12. In a problem with given p, ambiguity aversion increases with increasing degree of ambiguity for ambiguity-averse people, and ambiguity seeking increases with increasing degree of ambiguity for ambiguity seeking people.

The literature provides some evidence of Prediction 9 (Curley & Yates, 1985, 1989; Di Mauro & Maffioletti, 2004; Einhorn & Hogarth, 1986; Hogarth & Einhorn, 1990; Kahn & Sarin, 1988; Kramer & Budescu, 2005) and weaker evidence of Prediction 10 (Hogarth & Einhorn, 1990). Kramer and Budescu (2005) reported that in contrast to the case of comparing precise probabilities with ambiguity, participants showed higher ambiguity aversion at low p than at high p when they chose between different degrees of ambiguity. Under the present analysis, the DSM cannot account for this contrary finding. Experimental evidence in Cohen, Jaffray, and Said (1985) supports Prediction 11. They found 59% ambiguity aversion, 35% indifference, and 6% ambiguity preference for gains and 25% ambiguity aversion, 42% indifference, and 33% ambiguity preference for losses. However, the experiment was not designed to directly test the specific prediction, and further empirical research is needed to support it. On a related point, there is some evidence in the literature that ambiguity aversion is lower for losses than for gains (Cohen et al., 1985; Einhorn & Hogarth, 1986; Hogarth & Einhorn, 1990), but the DSM does not make any definite predictions on this point. Prediction 12 finds conflicting evidence in the works of S. W. Becker and Brownson (1964); Larson (1980), and Curley and Yates (1985). In summary, some evidence addressing the predictions exists, but it is not definitive, and more empirical research is required in order to test them.

Common Consequence Effect (Allais Paradox)

The most famous example of the violation of the common consequence axiom is the Allais paradox (Allais, 1953). The common consequence axiom states that if an outcome occurs with the same probability in two gambles, then that outcome should not play any role in choice decisions. Participants choose between gambles $G_1 = (.10, 5 \text{ million}; .89, 1 \text{ million})$ and $G_2 = (1, 1 \text{ million})$, and between $L_1 = (.10, 5 \text{ million})$ and $L_2 = (.11, 1 \text{ million})$. The common choice pattern of G_2 over G_1 and L_1 over L_2 can be shown to violate the common consequence axiom of the EU theory. Under the EU theory, $G_2 \succ G_1$ implies $U(1) > .10U(5) + .89U(1)$ or $.11U(1) > .10U(5)$, but $L_1 \succ L_2$ implies $.11U(1) <$

.10U(5), thereby leading to a contradiction. The next proposition analyzes a general version of the common consequence effect from the perspective of the DSM.

Proposition 5. Let $G_1 = (p_1, y; p_2, x)$, $G_2 = (1, x)$, $L_1 = (p_1, y)$, and $L_2 = (1 - p_2, x)$, where $y < x$. Assuming $p_1 y > (1 - p_2)x$, because p_1 and $1 - p_2$ are typically close together (e.g., .10 and .11 in the example above), the necessary and sufficient conditions for the common consequence effect ($G_2 \succ G_1$ and $L_1 \succ L_2$) are

$$2V_A(x) > V_A(y) \text{ and} \tag{14}$$

$$\gamma > \frac{1}{1 + \dfrac{2V_A(x) - V_A(y)}{3k(p_1 y + p_2 x - 1)}} = \psi. \tag{15}$$

Proof. See Appendix.

The intuition in this problem is that between L_2 and G_2 there is an increase in both the deliberative and the affective values. In contrast, between L_1 and G_1 there is an increase in the deliberative value, but the affective value reduces, such that under enough affective influence, the net result is an increase that is significantly smaller than that between L_2 and G_2. Hence, if the increment in value from L_2 to G_2 is high enough relative to the increment from L_1 to G_1, then the preference of L_1 over L_2 in the second choice can be converted to a preference of G_2 over G_1 in the first, thus resulting in the common consequence effect.

Proposition 5 requires that the affective valuation of G_2 be higher than that of G_1 in order to avoid G_1 dominating G_2 (necessary Condition 14), and the influence of the affective system needs to be high enough for the violation (Condition 15), failing which G_1 and L_1 will be chosen, thereby satisfying the common consequence axiom.

Condition 15 allows us to make the following prediction:

Prediction 13. People whose thinking disposition is more affectively oriented are more likely to display the common consequence effect.

Note that Ψ is between 0 and 1, and because violations are likely to reduce with increasing Ψ, we can make the following additional predictions:

Prediction 14. Violations will decrease with increasing y, because $\partial\Psi/\partial y > 0$.

Prediction 15. Violations will decrease with increasing p_1 or p_2, keeping the other fixed, because $\partial\Psi/\partial p_1 > 0$ and $\partial\Psi/\partial p_2 > 0$.

Testing of these predictions can be taken up in future research.

Common Ratio Effect

The common ratio effect, originally due to Allais (1953), is analyzed with an example from Kahneman and Tversky (1979). Participants chose between gambles $G_1 = (1, 3000)$ and $G_2 = (.8, 4000)$, and between $L_1 = (.25, 3000)$ and $L_2 = (.20, 4000)$. Note that the payoffs and ratios of probability in G_1 and G_2 are the same as in L_1 and L_2. It can easily be shown that the frequent choice of G_1 over G_2 and L_2 over L_1 constitutes a violation of the EU theory. Under the EU theory, $G_1 \succ G_2$ implies U(3000) > .8U(4000), but $L_2 \succ L_1$ implies .25U(3000) < .20U(4000) or U(3000) < .8U(4000), thereby leading to a contradiction.

Under the DSM, assuming $V_A(x) = x^m$, the necessary and sufficient conditions for the common ratio effect in the above example are

$$3000^m > 4000^m/2 \text{ and} \tag{16}$$

$$\gamma > \frac{1}{1 + \dfrac{3000^m - 4000^m/2}{200k}}. \tag{17}$$

Condition 16 is satisfied for all values of $m < 1$ and Condition 17 yields $\gamma > .47$ for $m = 0.8$. The intuition in applying the DSM to the common ratio effect is that when one goes from G_2 to L_2, the deliberative value decreases but there is no change in the affective value. However, when one goes from G_1 to L_1, both the deliberative and the affective values decrease due to G_1 being a sure thing. Hence, if G_1 is preferred over G_2 and if in moving from the first choice to the second one G_1 reduces more than G_2 does, then L_2 may be preferred over L_1 in the second choice, thus resulting in the common ratio effect.

As with dominance and the common consequence effect, we can predict that people whose thinking disposition is more affectively oriented are more likely to display the common ratio effect.

The Isolation Effect (Reduction of Compound Lotteries)

The reduction axiom states that whether a risky option is expressed as a single-stage gamble or a multistage compound gamble should not affect preferences in any way. This is an essential feature in most utility theories. However, several violations of the reduction property have been observed (Bar-Hillel, 1973; Bernasconi & Loomes, 1992; Conlisk, 1989; Kahneman & Tversky, 1979; Keller, 1985; Ronen, 1973). People tend to cancel the common elements of two gambles and "isolate" the uncommon elements, which in certain cases leads to this type of violation. For example, Kahneman and Tversky (1979) found that most people preferred $G_2 = (.80, 0; .2, 4000)$ to $G_1 = (.75, 0; .25, 3000)$, but when G_2 is expressed in a multistage form, such as $G_2' = (.75, 0; .25[.2, 0; .8, 4000])$, that is, a first stage where there is a 75% chance of nothing and a 25% chance of moving to a second stage where there is an 80% chance of winning 4000, then the preference is reversed. The general explanation is that when Gamble A is stated in a multistage form, people cancel out the 75% chance of nothing in the two gambles and choose Gamble B because they prefer gamble $(1, 3000)$ to gamble $(.8, 4000)$.

Under the DSM, assuming $V_A(x) = x^m$, the necessary and sufficient conditions for the isolation effect in the above example are

$$3000^m > 4000^m/2 \text{ and} \tag{18}$$

$$\gamma > \frac{1}{1 + \dfrac{3000^m - 4000^m/2}{100k}}. \tag{19}$$

Condition 18 is satisfied for all values of $m < 1$, and Condition 19 yields $\gamma > .31$ for $m = 0.8$. The intuition in the application of the DSM to the isolation effect is that the valuation of G_2', the compound lottery, is less than that of its simple form, G_2, in

System A but is the same in System D. Hence, in valuing the compound lottery, the affective system applies its valuation to the outcome twice, once while reducing it to its simple form and then once again on the simple form. Because probabilities play no part in the valuation, this reduces the affective value of the compound lottery in the problem by half. Because the value of G_2' is less than that of G_2, the isolation effect results if the value of G_1 lies between those of G_2' and G_2.

As in the earlier applications, one can predict that people whose thinking disposition is more affectively oriented are more likely to display the isolation effect.

Juxtaposition and Event-Splitting Effects

This is an example of the violation of description invariance, the assumption that different logically equivalent representations of the same choice problem should lead to the same preference behavior. *Event-splitting effects* refers to the empirical finding that "the subjective weight given to an outcome depends on the number of states of the world in which it occurs" (Starmer & Sugden, 1993, p. 235). Specifically, if an event on which an outcome is dependent is split into two subevents, then the subjective weight given to that outcome increases. The DSM predicts the event-splitting effect.

Consider the gamble $G = (p, x; 1 - p, 0)$ and the gamble where the first event is split into two: $G' = (p', x; p'', x; 1 - p, 0)$ such that $p' + p'' = p$. According to the DSM, the subjective weight assigned to both the gambles by System D is the same. However, System A values gamble G at $V_A(x)/2$ and gamble G' at $2V_A(x)/3$, thereby predicting the event-splitting effect. This assumes, reasonably, that γ remains the same while evaluating the two gambles.

One of the predictions of regret theory is the juxtaposition effect, in which choices over prospects are systematically influenced by the juxtaposition of outcomes in the payoff matrix. Starmer and Sugden (1993) found that the juxtaposition effects reported in empirical studies were primarily due to event-splitting effects. Hence, models allowing for event-splitting are required to explain this description invariance phenomenon.

Discussion and Conclusion

This article proposes a dual process–based model of decision making under risk and uncertainty in which the value of a gamble under consideration is a combination of the values assigned to it independently by the affective and deliberative systems of processing. Past research has shown that under many circumstances human behavior can be explained better by a dual processing mode than by a unitary one. Research has also shown that priming people on one system or the other or using outcomes having different affective content influences the shape of the value function as well as the probability weighting function. The DSM model draws motivation from this empirical research and research on dual processes and dichotomizes valuations of risky gambles into its dual process components. The result is a deterministic model of choice under risk based on the value maximization paradigm that is parsimonious and analytically tractable. This model has the additional feature of incorporating affective contexts in risky decision making.

Related Models

There are a few models in the economics literature that make use of the paradigm presented here, though they either address different kinds of problems or use different approaches from the model presented here. One class of models uses a dual-self approach to address issues related to self-control and addiction from the perspective of intertemporal choice (Bernheim & Rangel, 2004; Fudenberg & Levine, 2006; Thaler & Shefrin, 1981). In general, they assume one self that is concerned with long-term utility and another that is myopic and concerned with immediate self-satisfaction. The models then go on to specify interactions between the two systems in order to explain behavior related to addiction and self-control. The conceptualization of System 1 is more visceral in nature and differs significantly from the conceptualization in this article.

A more general model is that of Loewenstein and O'Donoghue (2004), in which they conceptualize deliberative and affective systems similar to the ones here. However, the utility in the deliberative system roughly corresponds to that in the EU model, the standard economic model, and the treatment of the affective system is geared toward motivational drives rather than subjective feeling states. Also, the deliberative system is influenced by both cognitive and affective states. An individual's behavior is determined by the interaction between the two systems, whereby the deliberative system influences the decisions of the affective system by exerting willpower, which is costly. Hence, a key driver of decision making is the availability and cost of willpower.

Bracha and Brown (2008) directly deals with individual decision making under risk and uncertainty. In their formulation, the rational process and the emotional process jointly determine choice behavior. The rational process is the same as the EU model, and the emotional process forms risk perceptions. The interaction of the two systems is modeled as an intrapersonal simultaneous-move game and is presented in the context of choices in the insurance markets. The pure-strategy Nash equilibria of the game, if they exist, represent the possible choices made by the individual.

The model that most closely resembles the development reported here is the prospective reference theory (PRT; Viscusi, 1989), which interestingly is not explicitly based on dual processes. This model assumes that people have a prior probabilistic belief concerning the outcomes of the gamble and that, upon being presented with the outcome probabilities, the probabilistic belief is updated in standard Bayesian fashion. In the symmetric case, the prior belief is assumed to be a uniform distribution over the outcomes; that is, all outcomes have equal probabilities. The value of the gamble G is given by (with somewhat different notations from the original)

$$V(G) = \frac{\gamma}{\gamma + \xi}\left(\frac{1}{n}\right)\sum_{i=1}^{n} U(x_i) + \frac{\xi}{\gamma + \xi}\sum_{i=1}^{n} p_i U(x_i). \quad (20)$$

Aside from the premises from which the PRT and the DSM are built up, there are two important differences between the model formulations. The first difference is that in the PRT, both additive terms have the same utility function, whereas they are different in the DSM. The second difference is in the interpretation of the multiplicative terms. The γ and ξ in the PRT represent the information content in the decision maker's prior beliefs and stated

probabilities, respectively. Hence, the multiplicative terms are the relative information content of prior and stated probabilities, unlike in the model here, where they represent the relative influence of the two processing systems.

The DSM is applied to a wide variety of empirical phenomena and not only has been shown to account for the classical violations of the EU model, such as the common ratio effect and the common consequence effect, but also explains nontransparent stochastic dominance, the fourfold pattern of risk attitudes, the isolation effect, and coalescing and event-splitting effects. It can also be applied to decision making under conditions of ambiguity, in which precise information about probabilities is not available. In addition, the DSM can be used to predict changes in behavior patterns with changes in specific features of the gambles under consideration. Several predictions have been made in this article, some of which have been addressed in past empirical research to varying degrees and others of which remain to be tested. Future research can focus on three areas: empirically determining the parameter values, testing the predictions, and applying the model to more domains of human behavior to generate and test more predictions.

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Appendix

Case of Continuous Outcomes and Proofs of Propositions

Model formulation for continuous outcomes. Let $X \in R$, the set of real numbers, be a random variable taking values between l and u, where $l < u$, and let gamble G be represented by $p(x)$, the probability density function over X, $x \in X$. Note that one limitation of the model is that l and u, the lower and upper bounds of the possible outcomes, need to be finite, that is, $l > -\infty$ and $g < \infty$, because uniform distributions needed for System A require finite support. This should not pose a serious problem, because most outcomes in the known world are finite. The continuous versions of the value functions are provided in the following equations:

$$V_D(G) = k \int_l^u xp(x)dx \tag{A1}$$

$$V_A(G) = \frac{1}{u-l} \int_l^u V_A(x)dx \tag{A2}$$

$$V(G) = \gamma \frac{1}{u-l} \int_l^u V_A(x)dx + (1-\gamma)k \int_l^u xp(x)dx \tag{A3}$$

Proof of Proposition 4. Beginning with $m = m^*$, introduce a small increment, Δm, on both sides of Equation 7. Note that when

$\Delta m > 0$, the increment of value on the left-hand side, $\gamma x^{m^*}(x^{\Delta m} - 1)/2$, is greater than the increment of value on the right-hand side, $\gamma(px)^m[(px)^{\Delta m} - 1]$, because $(px)^{m^*} = x^{m^*}/2$. Hence, to maintain equality in Equation 11, one must have $c > px$, which thereby leads to risk seeking behavior. Exactly the opposite is true when $\Delta m < 0$. Also note that the difference in increment, $\gamma x^{m^*}[x^{\Delta m} - (px)^{\Delta m}]/2$, increases with increasing γ. Hence $|c - px|$ must increase with γ to satisfy equality in Equation 11, which thereby leads to the stated result. QED.

Proof of Proposition 5. Applying the DSM to G_1, G_2, L_1, and L_2 and setting $V(G_2) > V(G_1)$ and $V(L_1) > V(L_2)$, one gets the following inequalities:

$$\gamma[2V_A(x) - V_A(y)] > (1-\gamma)3k[p_1 y - (1-p_2)x] \text{ and} \tag{A4}$$

$$\gamma[V_A(y) - V_A(x)] > (1-\gamma)2k[(1-p_2)x - p_1 y]. \tag{A5}$$

If $p_1 y > (1 - p_2)x$ is assumed, the Inequality A5 is satisfied as $y > x$ implies $V_A(y) > V_A(x)$. Because the right-hand side of Inequality A4 is positive, then $2V_A(x) > V_A(y)$ is a necessary condition (Condition 14). Solving Inequality A4 for γ yields Condition 15. QED.