Affective modulation of the weighting function

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1 Description

Both Expected-utility theory and prospect theory posit that humans maximize some version of utility. The theories get there by a combination of two functions (Rottenstreich & Hsee, 2001). A value function v transforms objective value to subjective utility, and a weighting function w distorts probabilities (Gonzalez & Wu, 1999; Rottenstreich & Hsee, 2001). Expected-utility and prospect theory combine these two paramters in the simplest way possible (Rottenstreich & Hsee, 2001)

$$\sum w(p_i)v(i),$$

where p stands for probability and i stands for the i^{th} gamble.

Prospect Theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992) (PT) is arguably the main model of human decision making (Newell et al., 2015). It advances theorizing from expected-utility by postulating that losses and gains are evaluated as changes in wealth rather than in regard to end states (Newell et al., 2015).

In Kahneman and Tversky (1979) we find the familiar (non-linear) S-shaped value function v which is concave for the gains domain and convex for losses (where it is steeper as well). The weight function w is the identity, $w(p) = p$ in expected-utility theory (Rottenstreich & Hsee, 2001) whereas a non-linear probability distortion is proposed in prospect theory (Kahneman & Tversky, 1979). Here w is stylized as being reverse S-shaped, meaning that it is concave for low probabilities and convex for high probabilities Gonzalez and Wu (1999). This means that people underweight changes in probability in the middle of the spectrum (e.g. $[0.2-0.8]$) while overweighting changes in probability close to the end-points (e.g. $[0.0 - 0.2], [0.8 - 1.0]$). These general characteristics of the weighting function are empirically well documented (Tversky & Kahneman, 1992; Wu & Gonzalez, 1996).

1.1 Prior work

There is evidence to support the notion that the affect of outcomes modulates the parameters of both v (Hsee & Rottenstreich, 2004) and w (Rottenstreich & Hsee, 2001). A main finding is that the S-shape of the weighting function w appears to be more pronounced for high-affect than low-affect outcomes under uncertainty (Rottenstreich & Hsee, 2001). This was shown as a preference reversal in which a high-affect outcome was preferred for low probability (1%) whereas a low-affect outcome was preferred for high probability (100%) (Rottenstreich & Hsee, 2001). The finding that affect appears to modulate both v and w has subsequently been modelled as an interaction between an affective system and a deliberative system Mukherjee (2010, 2011).

1.2 Focus and parameterization

In this article we focus exclusively on the weighting function w while ignoring both the value function v and the combination of the two functions. We also restrict ourselves to the gains domain. In Rottenstreich and Hsee (2001) they propose that the affective modulation can be estimated as an affect paramter a in the form:

$$w(p) = \frac{p^{1-a}}{p^{1-a} + (1-p)^{1-a}}.$$

where $a \in [0, 1]$ and larger values indicate greater affect and more curvature (Rottenstreich & Hsee, 2001). The issue with this one-parameter formulation is that it does not account for the fact that people generally show low *elevation*. What I mean by that is that the empirically observed weighting function w typically crosses the diagonal line at around 0.3 rather than 0.5 (Gonzalez & Wu, 1999). The one-parameter formulation fixes this point at 0.5 which can be seen from figure 1.

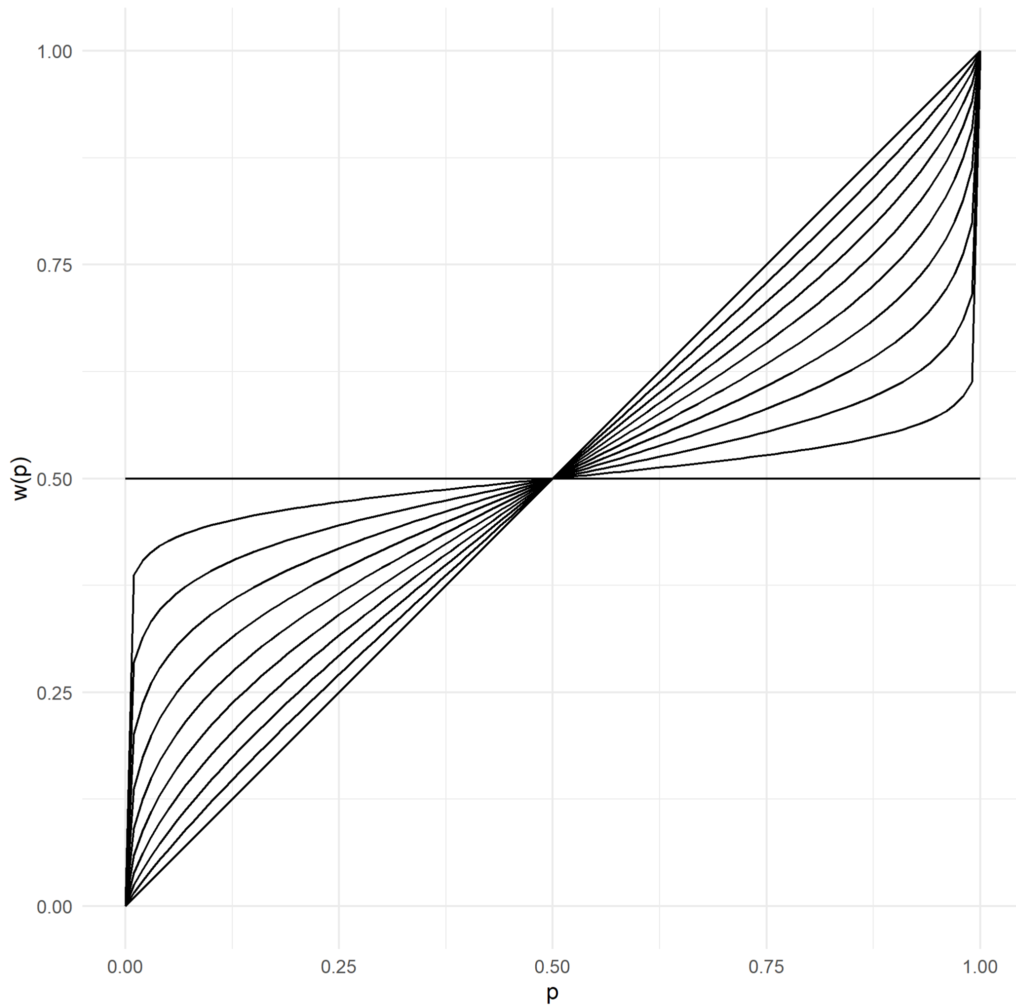


Figure 1: Data simulated from the model $w(p) = \frac{p^{1-a}}{p^{1-a}+(1-p)^{1-a}}$ with $a \in [0, 1]$. Diagonal line has $a = 0$, and the horizontal line has $a = 1$. Intermediate curves are generated for 0.2 increments of a. All values beside $a = 0$ show a probability distortion as compared to the objective probability. Note that all curves meet at $w(p) = 0.5, p = 0.5$. This is not empirically supported.

Instead of using the parameterization proposed in Rottenstreich and Hsee (2001) this paper will use the parameterization of w proposed in Gonzalez and Wu (1999).

They parameterize w with two parametrs; δ and γ.

The δ parameter will vary based on *elevation* (intercept) (Gonzalez & Wu, 1999), which here simply refers to the overall perceived attractiveness of outcomes under uncertainty.

The γ parameter will vary based on *curvature* (slope) (Gonzalez & Wu, 1999) and is what we are primarily interested in for our purposes. It follows as a direct prediction from Rottenstreich and Hsee (2001) that the curvature (γ) should be modulated by changes in the affective level of outcomes.

$p(w) = p$ for $\gamma = 1, \delta = 1$ with this parameterization. Higher δ corresponds to higher elevation, and higher α corresponds to *lower* curvature (unintuitively). See figure 2 for an illustration of how the δ and γ parameters independently modulate different aspects of the weighting function w.

The model is:

$$\log \frac{w(p)}{1 - w(p)} = \gamma \log \frac{p}{1 - p} + \tau.$$

where solving for $w(p)$ and setting $\delta = \exp(\tau)$ gives us

$$w(p) = \frac{\delta \cdot p^{\gamma}}{\delta \cdot p^{\gamma} + (1 - p)^{\gamma}}.$$

The above equations are taken from Rottenstreich and Hsee (2001).

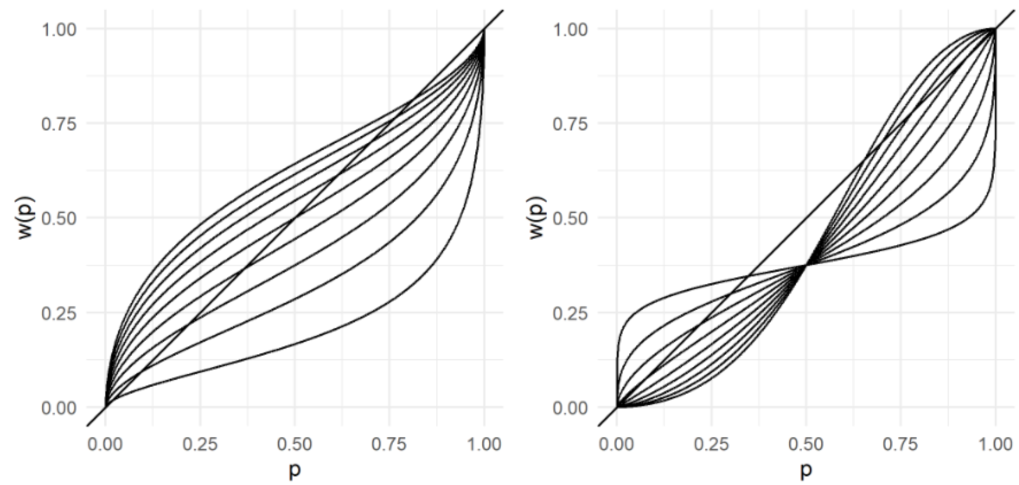


Figure 2: Data simulated from the model $w(p) = \frac{\delta \cdot p^{\gamma}}{\delta \cdot p^{\gamma} + (1-p)^{\gamma}}$ similarly to figure 4 of Gonzalez and Wu (1999). On the left: γ fixed at 0.6 and δ varied between 0.2 and 1.8. On the right: δ fixed at 0.6 and γ varied between 0.2 and 1.8. Shows that γ controls curvature and δ controls elevation. The identity function $w(p) = p$ is achieved for $\delta = 1, \gamma = 1$. Note.. gamma low has the opposite interpretation as compared to rottenstreich?

1.3 Methodology

Two studies are proposed to properly test the robustness of affect-level on the curvature (γ) of the weight function w.

In the first study, subjects will be asked to rate the affect-richness of 10 different items. All outcomes consist of coupons redeemable for various items, all worth $\$500$. The 10 items are designed to cover the full spectrum from affect-rich to affect-poor.

Example of expected high-affect item:

”If you won a $\$500$ coupon redeemable for a vacation abroad with a friend/partner how emotionally affected would you be?”

Example of expected low-affect item:
”If you won a $\$500$ coupon redeemable for insurance covering how emotionally affected would you be?”

For the full list of items see *Appendix A*. Participants will indicate how affect-rich each outcome is with a slider. Participants will see ”not affected at all” (left), ”somewhat affected” (middle) and ”very affected” (right). We will receive continuous ratings from 0 (affect poor) to 1 (affect rich). A mean affect rating across participants for each item will rank them from least affective to most affective. Three items (gambles) are then selected: The least affective item, the most affective item and the item in between these two extremes which separate them best (follow up).

In the second study, subjects will be presented with the three items which have been validated for affect-richness in the prior study. In this study however, the formulation around the items is that of a gamble. The formulation is the same for all items:

”You can buy a lottery ticket with an $[x]$ percent chance of winning a $\$500$ coupon redeemable for $[y]$ with a $[1 - x]$ percent chance of winning nothing. How much are you willing to pay for the lottery ticket?”

The three selected items are inserted as $[y]$ and 100 different probability levels: $x = 0.01, 0.02, \ldots, 0.99$ will be inserted as $[x]$ and the negation $[1 - x]$. With all possible combinations, this means that all participants will rate 3 items at 100 different levels of certainty each. As in experiment 1 participants will rate with a slider. This time ranging from $\$0$ to $\$500$ as it is neither logical to assign a value below $\$0$ or above $\$500$ to any of the gambles. The approach is somewhat different from Gonzalez and Wu (1999) but ultimately we estimate the same thing that they do; participants' certainty equivalence (CE). This simply is the amount of money they think that the gamble is worth.

Note that we are not directly measuring either δ or γ. What we do measure is the dependent variable $w(p)$ and the independent variable p for items ranked based on their affect-richness.

In order to infer the unmeasured paramters a bayesian (non)linear mixed effects model is proposed. The model is fitted in R with the $brms$ package. Here we can specify the previously mentioned formula:

$$w(p) \sim \frac{\exp(\tau) \cdot p^{\gamma}}{\exp(\tau) \cdot p^{\gamma} + (1-p)^{\gamma}}.$$

We have to specify that the model should be nonlinear. We can further specify that we would like to estimate specifically the value for τ and γ with random intercepts (partial pooling) for participants (ID) and with item as a main effect.

$$\tau \sim 0 + item + (1|ID),$$

$$\gamma \sim 0 + item + (1|ID).$$

We can then convert τ to δ by exponentiating the estimated value for τ for each item. For the analysis pipeline on simulated data, see Github.

After fitting the model, posterior samples for the three items are generated. Based on these, $.66$ and $.95$ credibility intervals are calculated and the distributions for each paramter (for each item) as a group-level effect is visualized.

2 Hypotheses

2.1 Study 1

There is no formal hypothesis connected with study 1. It is expected that participants will rate the items as having different affect-richness. If this is not achieved then study 2 does not make sense to conduct, and another attempt must be made to validate outcomes on a scale of affect. However, it does seem reasonable that the 10 different questions would cover the spectrum pretty well (see *Appendix A*) and as such mean ratings should differ significantly (follow up on significantly).

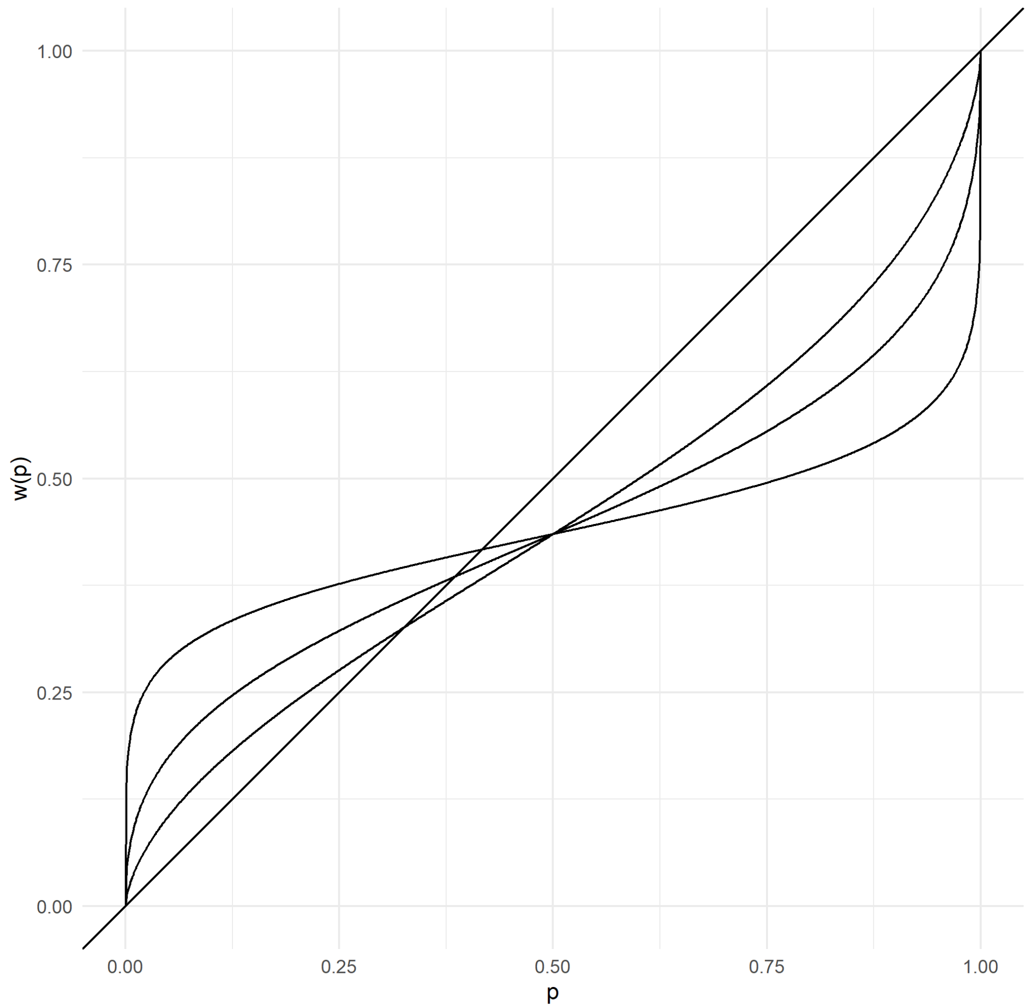
2.2 Study 2

Hypothesis 1: A directional effect is predicted for the γ parameter of the function:

$$w(p) = \frac{\delta \cdot p^{\gamma}}{\delta \cdot p^{\gamma} + (1-p)^{\gamma}}.$$

Recall that study 2 uses three three items (conditions) based on outcomes of varying affect-levels from study 1. We will refer to these as conditions low-affect A, medium-affect B, and high-affect C. Since Gonzalez and Wu (1999) report population $\gamma = 0.44$ (median) and use monetary gambles (low-affect) it is expected that our population γ will be close to this value. The minimally interesting effect for this study is shown in figure XXX, where γ values are $A = 0.44, B = 0.34, C = 0.24$. To evaluate whether γ has a directional effect we will sample the posterior of our model (for each parameter, for each condition) and compare the $.66$ and $.95$ credibility intervals. Some indication of an effect of affect on γ would be provided if the $.66$ credibility intervals do not overlap between conditions, and fall such that $A > B > C$ (recall, high-affect should mean low γ). Stronger evidence would be provided if this is the case for 0.95 credibility intervals. (is density intervals even a thing?).

Hypothesis 2: No direction of effect for the δ parameter (by item) is hypothesized. The δ parameter is not of interest to the main hypothesis (H1) and is mainly included in the analysis in order to control for elevation and properly estimate γ. If the three items differ in perceived overall value the δ parameter should capture this. This means that our γ distributions should still be interpretable even if the δ



Three curves shown, all with $\delta = 0.77$ as reported in Gonzalez and Wu (1999). γ levels $0.24, 0.34, 0.44$. The least curved line corresponds to $\gamma = 0.44$, as reported in Gonzalez and Wu (1999). For high-affect items γ should be lower, and as such we suggest 0.24 (for high affect) and 0.34 (for medium affect) as minimally interesting effects to detect.

parameter differs by item. The same analysis pipeline will be applied to δ as for γ but as suggested, it is not clear whether an effect would be interesting. The δ parameter is expected to have a value close to .77 which is the population median found for this parameter is Gonzalez and Wu (1999).

2.3 Simulation

In order to test the pipeline for the bayesian analysis, data simulation was conducted. Unfortunately, Gonzalez and Wu (1999) does not exactly report the values (i.e. distributional properties) that we need to generate data consistent with what they gathered. As such, it does not make sense to calculate power based on our simulations, and the simulation serves only the purpose of making clear how analysis on eventual data will be conducted.

Data is generated for 50 probability levels, $p = 0.01, 0.03, \ldots, 0.99$ crossed with 3 conditions (i.e. corresponding to the three items). Data is generated for 20 simulated subjects (ID).

Note that standard deviations vary between γ and δ, and between population level and individual variation. This qualitatively follows the results of Gonzalez and Wu (1999). Data is generated as a distribution of γ and δ for each condition. We generate 20 values (i) for each, corresponding to the number of participants. As we do not hypothesize that δ is modulated by condition (item) this can simply be generated as once.

$$
\begin{aligned}
\gamma_{A_i} &\sim norm(n = 30, m = 0.24, sd = 0.1) \\
\gamma_{B_i} &\sim norm(n = 30, m = 0.34, sd = 0.1) \\
\gamma_{C_i} &\sim norm(n = 30, m = 0.44, sd = 0.1) \\
\delta_i &\sim norm(n = 90, m = 0.77, sd = 0.2)
\end{aligned}
\tag{1}
$$

Based on these γ and δ values for participants per condition, we generate the final γ and δ values by adding individual noise for each probability level (j)

$$
\begin{aligned}
\gamma_{A_{ij}} &\sim norm(n = 50, m = \gamma_{A_i}, sd = 0.1) \\
\gamma_{B_{ij}} &\sim norm(n = 50, m = \gamma_{B_i}, sd = 0.1) \\
\gamma_{C_{ij}} &\sim norm(n = 50, m = \gamma_{C_i}, sd = 0.1) \\
\delta_{ij} &\sim norm(n = 150, m = \delta_i, sd = 0.3)
\end{aligned}
\tag{2}
$$

Each condition is assumed to have a "true" γ and δ parameter. In Gonzalez and Wu (1999) population $\gamma = 0.44$ (median) and population $\delta = 0.77$. As their gambles are with regards to money (low-affect) we expect to find similar parameter values for our lowest-affect condition (item 1). We

3 Design Plan

Study type: As the study does not have a manipulation (e.g. control and experimental) it should be classified as an observational study. This is true of both study 1 and study 2.

Blinding: No blinding is involved in this study.

3.1 Study Design

Study 1: All subjects will rate all items (see Appendix 1) as to the level of affect they feel with regards to them.

Study 2: All participants indicate their certainty equivalence (CE) for all combinations of items (10) and certainty levels (1%, 5%, 15%, 30%, 50%, 70%, 85%, 95%, 99%). This results in 90 observations per participant.

4 Sampling Plan

Existing Data: Registration prior to creation of data.

Data collection procedures: Participants will be recruited through online channels (e.g. facebook, student groups, etc.). Participants must be at least 18 years old to participate. In the first experiments subjects will be payed 30 DKK for agreeing to participate in an approx. 10 minute online survey. In the second experiment subjects will be payed 150 DKK for agreeing to participate in an approx. 60 minute online survey.

Sample size:

Study 1: 30 participants.

Study 2: 50 participants.

Sample size rationale:

Power analysis? Credibility/Density interval 95% assuming data generating process?

5 Variables

5.1 Manipulated variables

Study 1: No manipulated variables.

Study 2: Levels of uncertainty are manipulated, and are given as $0.01, 0.05, 0.15, 0.3, 0.5, 0.7, 0.85, 0.95, 0.9$

Levels of affect differ for each item (obtained in Study 1).

5.2 Measured variables

Study 1: The single outcome variable will be the rating of affect level. This will be measured on a scale of $0 - 100$ using a slider.

Study 2: The single outcome variable is the price that subjects indicate that they are willing to pay for a ticket in a lottery (combination of probability of outcome).

This will indicate their certainty equivalence (CE). This will be measured on a scale of $0 - 500$ dollars using a slider. The max is 500 dollars since the lottery tickets by definition cannot be worth more than this (see Appendix 2).

5.3 Indices

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6 Analysis Plan

All analysis is performed in the programming language R (R Core Team, 2020) using $Rstudio$ (RStudio Team, 2020).

Study 1: The affect ratings will be ordered based on group-level means?

Study 2: A bayesian generalized nonlinear mixed effects model is fit to the data using the R package $brms$ (Bürkner, 2018). This is done to estimate the unobserved parameters δ and γ from the independent variable probability/uncertainty and the dependent variable $w(p)$ which is the observed certainty equivalence (CE). Weakly informative priors are specified for both γ and δ (see Github).

7 Discussion

References

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8 Appendix

Appendix A

As all items in study 1 follow the same template:

”If you won a $\$500$ coupon redeemable for/at $[x]$ how emotionally affected would you be?”

The 10 proposed $[x]$ outcomes are:

- for a vacation abroad with a friend/partner.

- at a local shopping mall.
- for donation to a charity of your choice.
- for a cultural experience in your city.
- for insurance covering.
- for investing in the stock market.
- for job training.
- for $500.
- for spending on a present to someone you love.
- at your favorite restaurant.