# Quantum Fisher information as a non-classical measurement in the radical pair mechanism of the avian compass model

Victor Onofre[1], Jose Luis Meza[1], F. Rojas[2]
[1] *Posgrado de Ciencias Fisicas*
[2] *Departamento de Fisica Centro de Nanociencias y Nanotecnologia UNAM*
*Ensenada, Baja California , 22830 Mexico*
*victor.onofre@uabc.edu.mx*

## Abstract

The leading hypothesis to explain how migratory birds can detect the direction of the Earth's magnetic field (EMF) is the Radical-Pair(RP) mechanism, which is by now a well established theoretically and experimentally mechanism [4]. A radical pair is a bipartite system of atoms or molecules that has an odd number of electrons were the key terms involved are the Zeeman effect, the hyperfine interaction and the Singlet-Triplet interconversion; consequently, are magnetically sensitive. In this work, we focus on the interaction between nuclear and electron spin of the radical pair under the influence of the EMF. Obtaining expressions of the Quantum Fisher Information (QFI) as a non-classical measurement and the expected value of the singlet state as a function of the temperature and angle of the EMF.

## Introduction

The radical pair mechanism is a promising hypothesis to explain the extraordinary phenomenon of the avian compass model [3].This mechanism has been supported by a series of behavioral experiments, which indicate that the avian compass depends on both inclination and light intensity [4].

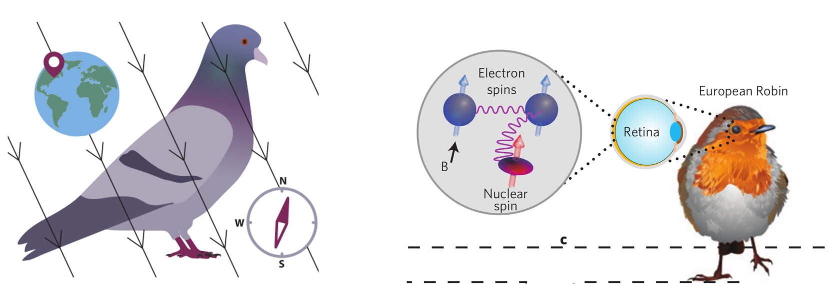

**Figure 1.** The avian compass based in the radical pairs explains how migratory birds can detect the direction of the EMF.

The Earth's magnetic field is described by:

$$\vec{B} = B_0(\sin\theta\cos\phi, \sin\theta\cos\phi\cos\theta)$$

$$\phi = 0 \quad \vec{B} = (B_x, 0, B_z)$$

$$B_x = B_0\sin\theta \quad B_y = 0$$

$$B_z = B_0\cos\theta$$

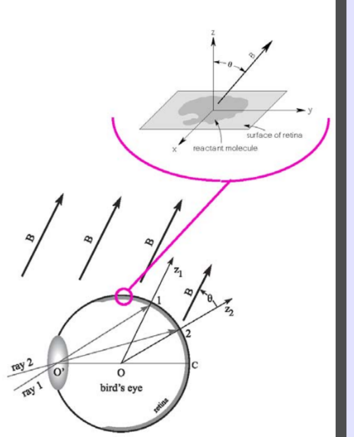

**Figure 2.** Interaction of the magnetic field in the retina of the bird.

## Quantum Fisher Information

The classical fisher information (CFI) is a measure of how quickly a probability distribution changes with respect to some parameter. While the quantum fisher information(QFI) is how quickly a quantum state (represented by a density matrix) changes with respect to some operator and is defined as [2]:$F(\rho, H) = Tr(\rho L^2)$ where $L$ is the symmetric logarithmic derivative determined as $i[\rho, H] = \frac{1}{2}(L\rho + \rho L)$ using the spectral decomposition $\rho = \sum_m \lambda_m |m\rangle\langle m|$ can be written as:

$$F(\rho, H) = \frac{1}{2}\sum_{m,n}\frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j}|\langle i|H|j\rangle|^2 \tag{4}$$

Eq.(4) is a qualitative non-classical indicator of the system.

**Non-classical measurement:Discord-like quantum correlation**

The quantum correlation based on Local Quantum Fisher Information (LQFI), through the minimum $Fisher_A = min_{H_A} F(\rho, H_A)$, this property is null or vanish if the bipartite state is classic-quantum (CQ) or classical-classical (CC). For the type $2 \times N$, it is sufficient to consider the general operator $H_a = \vec{n}\sigma$, $n$ is a unit vector and $\sigma$ the Pauli matrices. The minimum can be calculated as

$$Fisher_A = \lambda_{min}(W) \tag{5}$$

,the minimum eigenvalue of the $3 \times 3$ matrix $W$ defined as

$$(W)_{ij} = \frac{1}{2}\sum_{m,n}\frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j}\langle\Psi_i|\sigma_{iA}|\Psi_j\rangle\langle\Psi_j|\sigma_{jA}|\Psi_i\rangle \tag{6}$$

where Eq.(6) is a quantitative non-classical indicator of the system, i.e, a measurement of the "quantumness".In the case of the radical pair $\lambda_i = P_i = \frac{e^{-\beta E_i}}{Z}$, $Z = \sum_i e^{-\beta E_i}$ and $E_i$ are the eigenvalues of the Hamiltonian (Eq.(2)), $\rho$ in the spectral decomposition using the Eigenbase of Eq.(2) is of the form $\rho = P_1|\varphi_\pm\rangle_1|\phi\rangle_1 + P_2|\varphi_\pm\rangle_1|\phi\rangle_2 + P3|\varphi_\pm\rangle_2|\phi\rangle_1 + P_4|\varphi_\pm\rangle_2|\phi\rangle_2$

## Hamiltonian

The radical pair Hamiltonian is of the form [1]:

$$H = \gamma\vec{B}\cdot(\vec{S_1} + \vec{S_2}) + \vec{S_1}\cdot\boldsymbol{A}\cdot I \rightarrow H = H_1 + H_2 \quad H_1 = \gamma\vec{B}\cdot\vec{S_1} + \vec{S_1}\cdot\boldsymbol{A}\cdot I \quad H_2 = \gamma\vec{B}\cdot\vec{S_2} \tag{1}$$

where $\gamma = \frac{1}{2}\mu_B g_s$ is the gyromagnetic ratio with $\mu_B$ being the Bohr Magneton and $g_s$ the $g$ factor of the electron. The spin operators are $S_1 = (S_x^1, S_y^1, S_z^1)$, $S_2 = (S_x^1, S_y^1, S_z^1)$.The $I$ is the spin operator for the nuclei,$\boldsymbol{A}$ is the anisotropic HF tensor with a diagonal form $\boldsymbol{A} = diag(A_x, A_y, A_z)$. The eigensystem of this Hamiltonian is:

$$\begin{aligned}
&H_{1\,Eigenvectors} \rightarrow |\varphi_\pm\rangle_1 = \cos\theta_\pm/2\,|1\rangle + \sin\theta_\pm/2\,|0\rangle \quad |\varphi_\pm\rangle_2 = \sin\theta_\pm/2\,|1\rangle - \cos\theta_\pm/2\,|0\rangle \\
&H_{1\,Eigenvalues} \rightarrow \quad E_1 = w_\pm, E_3 = -w_\pm \quad where \quad w_\pm = \alpha B_\pm \\
&sin\theta_\pm = \frac{B_x}{B_\pm} \quad \cos\theta_\pm = \frac{(B_z \pm A_z/\gamma)}{B_\pm} \quad B_\pm = \sqrt{B_x^2 + (B_z \pm A_z/\gamma)^2} \\
&H_{2\,Eigenvectors} \rightarrow |\phi\rangle_1 = \cos\theta/2\,|1\rangle + \sin\theta/2\,|0\rangle \quad |\phi\rangle_2 = \sin\theta/2\,|1\rangle - \cos\theta/2\,|0\rangle \\
&H_{2\,Eigenvalues} \rightarrow \quad E_2 = w_0, E_4 = -w_0 \quad where \quad w_0 = \gamma B_0
\end{aligned} \tag{2}$$

## Results

The expected value of the Singlet state($|S\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$) and the QFI from Eq.(5) was obtained, evaluating the functions in terms of $T$ and $\theta$:

$$< S >= Tr[\rho(|S\rangle\langle S|)] = \frac{1}{4} + [\frac{1}{4} - \frac{1}{2}(P_1 + P_4)\cos(\theta_\pm - \theta)] \tag{3}$$

We have used the Quantum Fisher Information as a non-classical measurement in a thermal ensemble, from Fig. 3 it can be seen that at low temperatures there is no correlation with the $< S >$, so Fisher information comes from another mechanism. In contrast, at high temperatures the Fisher Information is affected in the direction of $\pi$, emerging a correlation with the $< S >$.

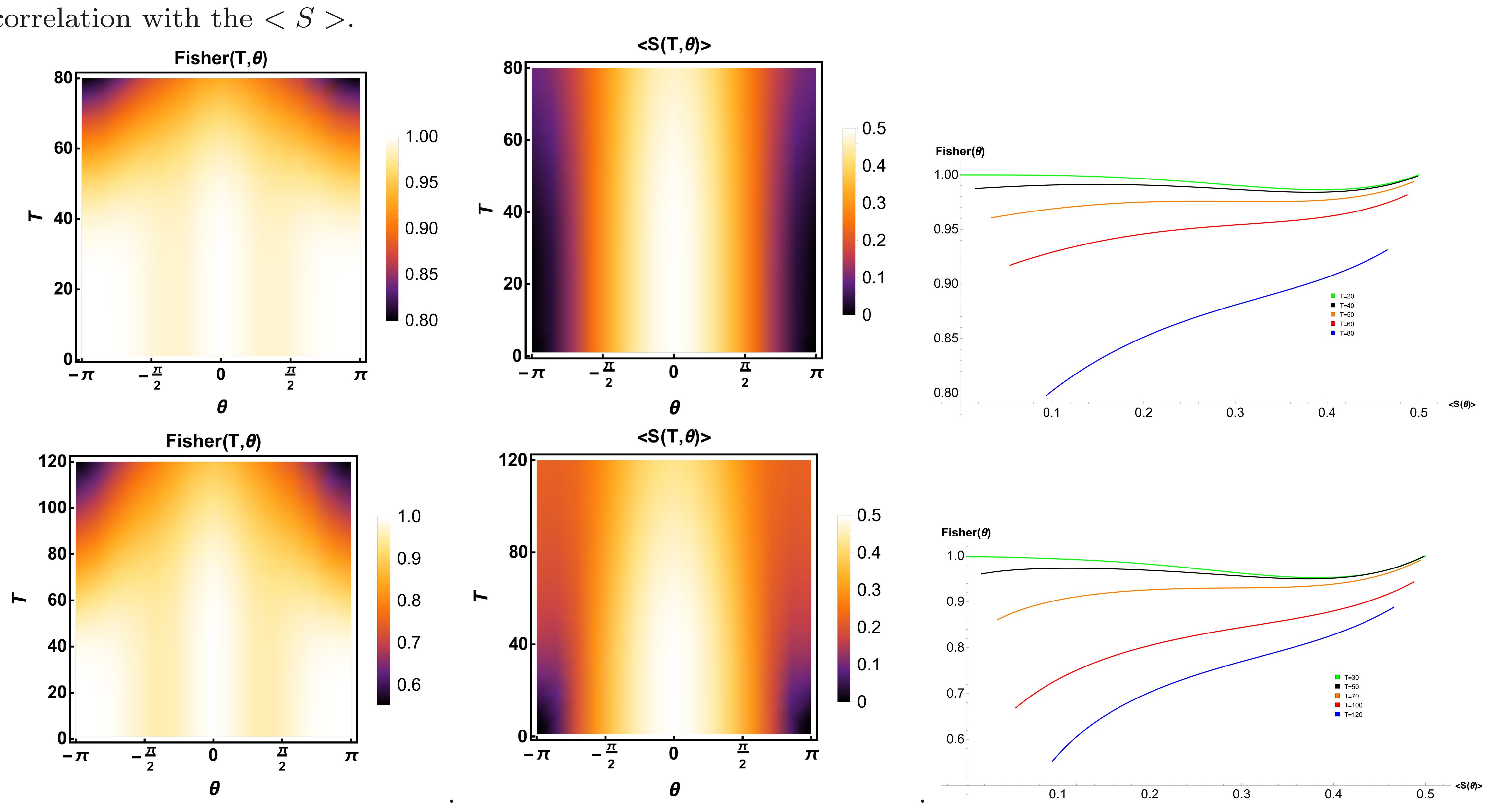


**Figure 3.** QFI from Eq.(5) (Left), Expected value of the Singlet state (Middle) and a parameterization of the form $(x, y) = (< S(T,\theta) >, Fisher_A(T,\theta))$ where $0 \leq \theta \leq \pi$ (Right), with a variation of the temperature ($T$) and angle($\theta$). In the three cases of the top part $B_0 = 46\mu T$ and in the part of below $B_0 = 96\mu T$

## Conclusions

The expressions of the QFI as a non-classical measurement and the expected value of the singlet state as a function of the temperature and angle of the EMF show a similitude at high temperatures, differing at low ones. The differences in low temperatures are an indicator of the possible use of "quantumness" and not only entanglement in the System. This property can be very useful in the non-stationary system, which will be developed in future works.

## References

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