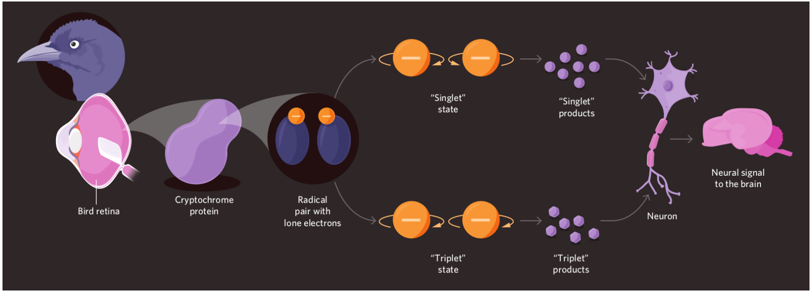
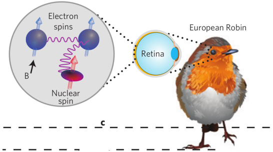
Master's Thesis

Victor Onofre

September 21, 2023

Thesis research: Discord-like quantum correlations in the mechanism of radical pairs for magnetoreception in birds





$$|s\rangle = \text{singlet state}$$
$$|S\rangle = \text{Singlet shelf}$$

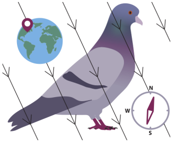


Image ref: Left[1], Center[2], Right[3]

[1]Lambert et al.,Nature Phys 9 (2013) - [2]Offord, The scientist (2019) - [3]Christiansen et al.,Annu. Rev. Neurosci. 42 (2019)

Mechanism of radical pairs

The radical pair Hamiltonian is of the form[2]:

$$H = \gamma \vec{B} \cdot (\vec{S_1} + \vec{S_2}) + \vec{S_1} \cdot \boldsymbol{A} \cdot \vec{J_0} \tag{1}$$

The master equation allows us to study these dynamics,

$$\dot{\rho}(t) = \frac{d\rho}{dt} = \frac{-i}{\hbar}[H, \rho] + k \sum_i P_i \rho P_i^\dagger - \frac{1}{2}(\sum_i \rho P_i^\dagger P_i + P_i^\dagger P_i \rho) \tag{2}$$

Where the coefficients k represents the dissipation rates of the system and the projection operators are defined as[3]:

$$\begin{aligned} P_1 &= \left|S\right\rangle \left\langle s, \uparrow\right|, \quad & P_2 &= \left|S\right\rangle \left\langle s, \downarrow\right| \quad & P_3 &= \left|T\right\rangle \left\langle t_+, \uparrow\right|, \quad & P_4 &= \left|T\right\rangle \left\langle t_+, \downarrow\right| \\ P_5 &= \left|T\right\rangle \left\langle t_-, \uparrow\right|, \quad & P_6 &= \left|T\right\rangle \left\langle t_-, \downarrow\right| \quad & P_7 &= \left|T\right\rangle \left\langle t_0, \uparrow\right|, \quad & P_8 &= \left|T\right\rangle \left\langle t_0, \downarrow\right| \end{aligned} \tag{3}$$

the final populations of $|S\rangle$ and $|T\rangle$ give the singlet and triplet yield

[2,3] Gauger et al.,Phys. Rev. Lett. (2011)

Singlet yield

$$\text{Singlet Yield} = \Phi_{t,k}(\theta) = Tr[\rho_{t,k}(\theta))S] \quad S = |S\rangle\langle S| \tag{4}$$

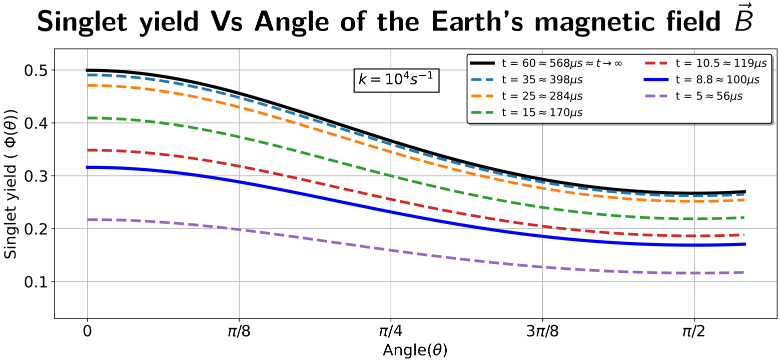


Figure: Singlet yield as a function of the angle (θ) of the Earth's magnetic field for k (Dissipation constant) equal to $k = 10^4 s^{-1}$. This is the constant equivalent to the experimental lifetime of the radical pairs, which is equivalent to time $t = 8.8 \approx 100\mu s$

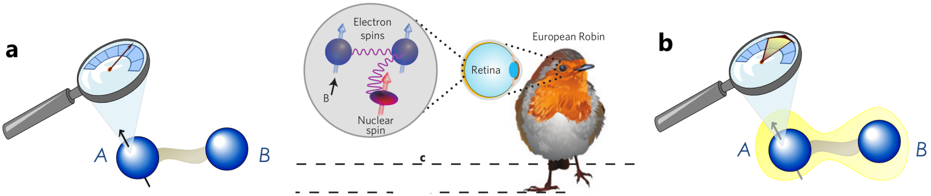
Quantifiers of non-classical correlations

$$\text{Local Quantum Uncertainty}^3 = \mathbf{LQU}(\rho_{AB}) = U_A(\rho_{AB}) = \mathbf{1} - \boldsymbol{\lambda_{max}(W_{AB})} \tag{5}$$

$$(W_{AB})_{ij} = Tr[\sqrt{\rho_{AB}}(\sigma_{iA} \otimes \mathbb{I}_B)\sqrt{\rho_{AB}}(\sigma_{jA} \otimes \mathbb{I}_B)]$$

$$\text{Local quantum Fisher information}^4 = \mathcal{Q}(\rho_{AB}) = \mathbf{1} - \boldsymbol{\lambda_{max}(W'_{AB})}, \tag{6}$$

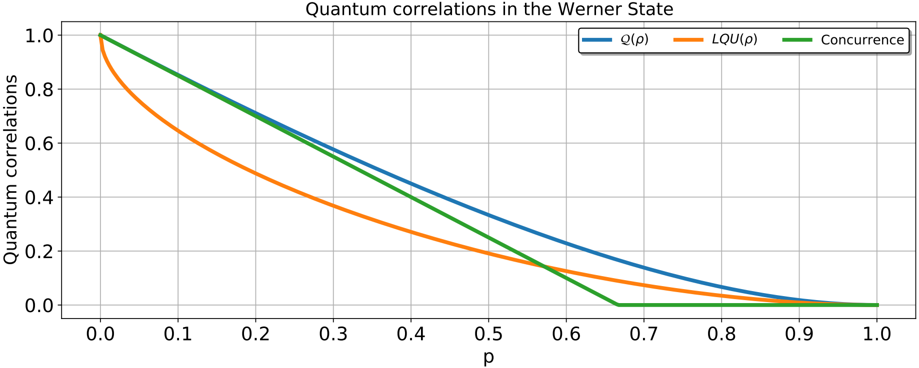
$$(W'_{AB})_{mn} = \sum_{i,j} \frac{2\lambda_i\lambda_j}{\lambda_i+\lambda_j} \langle\psi_i|\, \sigma_{m,A} \otimes \mathbb{I}_B \,|\psi_j\rangle \langle\psi_j|\, \sigma_{n,A} \otimes \mathbb{I}_B \,|\psi_i\rangle$$



[3] Girolami et al.,Phys. Rev. Lett. 110 (2013) - [4] Slaoui, Phys. Lett. A 383 (2019)

$\mathcal{Q}$ and LQU in the Werner State

$$\rho_{werner} = (1-p)\left|\phi_{singlet}\right\rangle\left\langle\phi_{singlet}\right| + \frac{p\mathbb{I}}{4} \quad p \in [0,1], \quad \left|\phi_{singlet}\right\rangle = \frac{1}{\sqrt{2}}(\left|01\right\rangle - \left|10\right\rangle) \tag{7}$$



Efficiency of $\mathcal{Q}$ and LQU in the mechanism of radical pairs

$$\textbf{Efficiency of } \boldsymbol{\mathcal{Q}_\theta(\rho(t))} = E_{\mathcal{Q}}(t) = 1 - \mathcal{Q}_\theta(\rho(t)), \tag{8}$$

$$\textbf{Efficiency of } \boldsymbol{LQU_\theta(\rho(t))} = E_{LQU}(t) = 1 - LQU_\theta(\rho(t)), \tag{9}$$

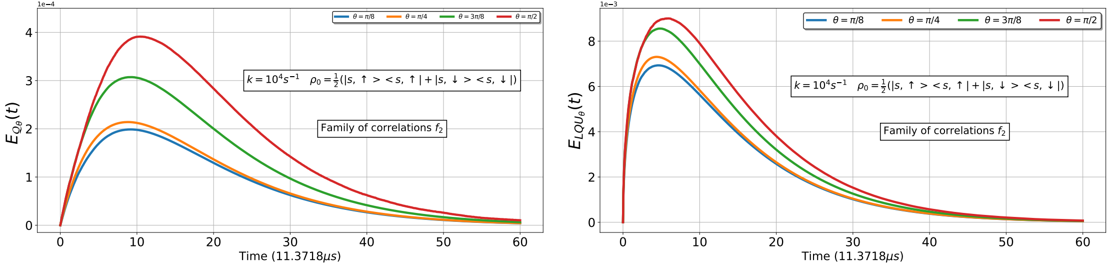


Figure: $k = 10^4 s^{-1}$, $\theta = \pi/8, \pi/4, 3\pi/8, \pi/2$

Conclusions

- The dependence of the angle of the earth's magnetic field with the singlet yield was shown,being consistent with the different studies on the mechanism of radical pairs.
- We found a relationship of $\mathcal{Q}$ and LQU with the Earth's magnetic field similar to that of the singlet yield. Showing a decrease as a function of the angle, especially with the family of correlations f_2:

$$\{S_1 - I^{\downarrow}\} = \mathbf{SubsystemA_2} \qquad \{S_2 - I^{\uparrow}, S_2 - I^{\downarrow}\} = \mathbf{SubsystemB_2}$$

.

- These results are an indication that quantum correlations may function as a resource in the radical pair mechanism.