Large Scale Machine Learning

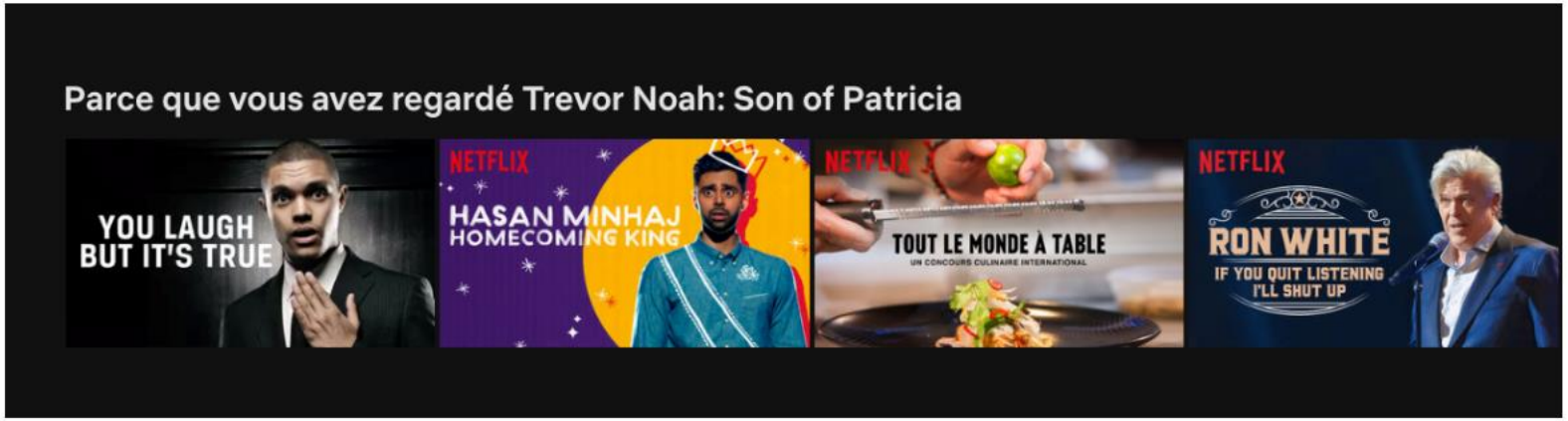
Matrix factorization

Recommender Systems

- Information overload
  - Many choices available
  - « The paradox of choice » (jam experiment, choice overload)

- Recommender system
  - Provide aid
  - Given a user and his « context » and a set of items => selection of items predicted to be « good » for the user

Recommender Systems



Les clients ayant acheté cet article ont également acheté

Recommender Systems: The Textbook
› Charu C. Aggarwal
Relié
EUR 50,31 ✓prime

Statistical Methods for Recommender Systems
Deepak K. Agarwal
★★★★★ 1
Relié
EUR 53,39 ✓prime

Recommender Systems Handbook
Francesco Ricci
Relié
EUR 224,03

Programming Collective Intelligence
› Toby Segaran
★★★★½ 3
Broché
EUR 26,55 ✓prime

Doing Bayesian Data Analysis: A Tutorial with R, JAGS, and Stan
John Kruschke
Relié
EUR 69,52 ✓prime

Deep Learning with Python
› Francois Chollet
★★★★★ 2
Broché
EUR 43,87

Collaborative filtering

- *« tell me what's popular among my peers »*
- One of the most often and successfully used techniques
- Widely applicable, does not need a lot of domain knowledge
- **Hypothesis**: Users who shared similar tastes in the past will continue to do so in the future

Collaborative filtering

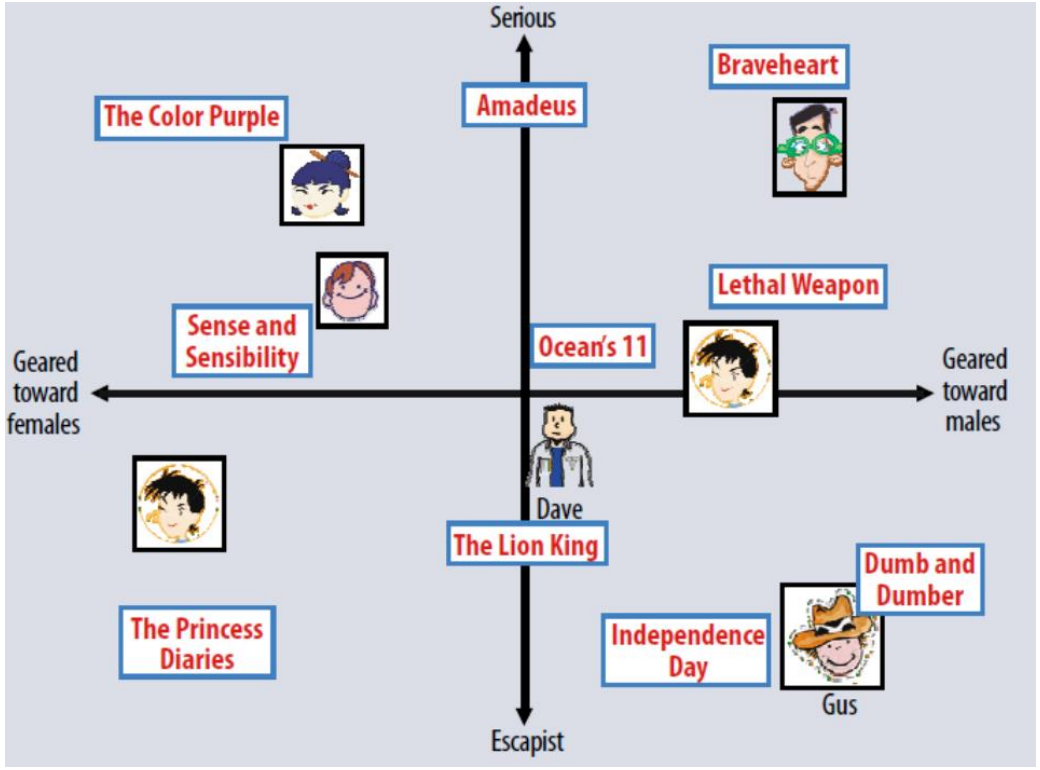
- **Most common setting:**
  - **Input**: matrix of user-item feedback or ratings (with missing values of course, this matrix very sparse)
  - **Output**: Predictions for missing values
- No need of item features (eg. movie genre, length or actors)

Collaborative filtering

- Explicit
  - The user rated explicitely the item (like/dislike, star rating …)
  - It requires effort from the user (friction)
  - Somewhat clear signal of what the user feels about the item
- Implicit
  - Click/Non Click, buying an item, visiting a page, viewing a video
  - Easier to collect, minimal friction
  - More « honest » (Netflix example: highly rated vs watched)

Recommended reading on implicit / explicit feedback: https://www.wired.com/2013/08/qq-netflix-algorithm/

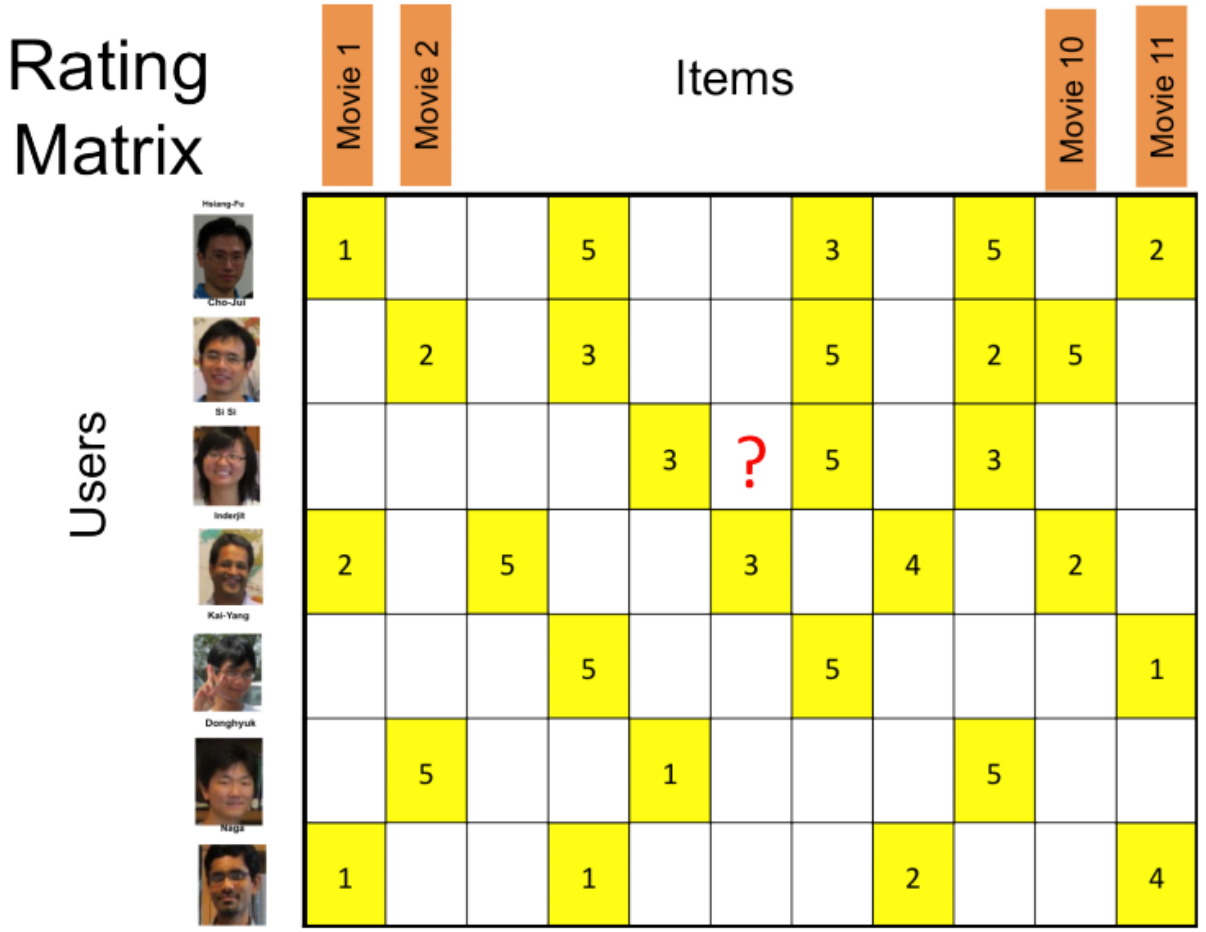
Matrix factorization



Matrix factorization

- Main idea: latent factors of users/items
  - Some users like action movies, romance, … at different proportions
  - Same can be said about movies
- Use these latent factors to
  - Predict interactions/ratings
  - Compute similarities between users and items

User-Item Matrix Factorization



User-Item Matrix Factorization

H^T

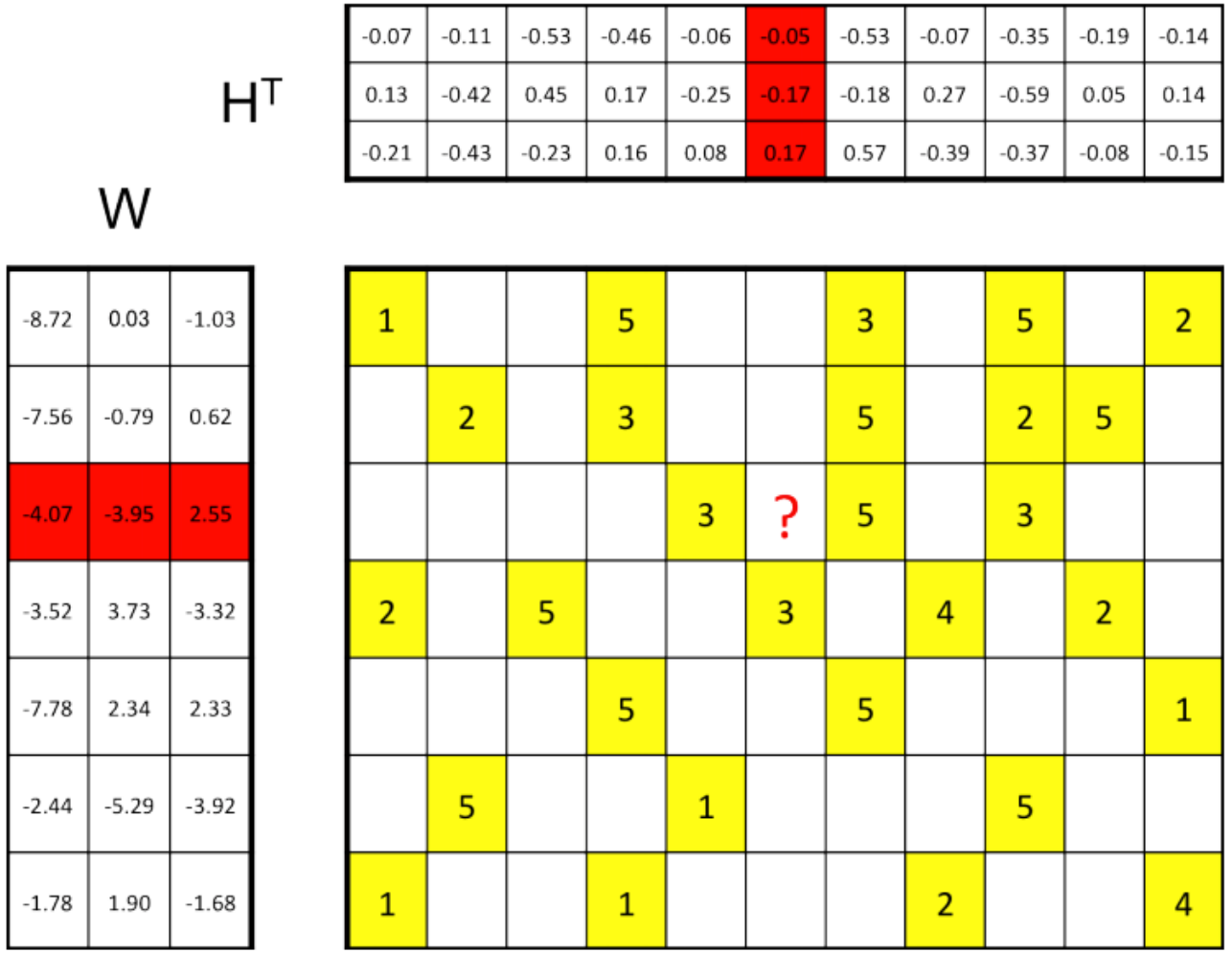
-0.07	-0.11	-0.53	-0.46	-0.06	-0.05	-0.53	-0.07	-0.35	-0.19	-0.14
0.13	-0.42	0.45	0.17	-0.25	-0.17	-0.18	0.27	-0.59	0.05	0.14
-0.21	-0.43	-0.23	0.16	0.08	0.17	0.57	-0.39	-0.37	-0.08	-0.15

W

-8.72	0.03	-1.03
-7.56	-0.79	0.62
-4.07	-3.95	2.55
-3.52	3.73	-3.32
-7.78	2.34	2.33
-2.44	-5.29	-3.92
-1.78	1.90	-1.68

1			5			3		5		2
	2		3			5		2	5	
				3		5		3		
2		5			3		4		2	
			5			5				1
	5			1				5		
1			1				2			4

User-Item Matrix Factorization



Unconstrained MF

$$\min_{\substack{U \in \mathbb{R}^{m \times k} \\ V \in \mathbb{R}^{n \times k}}} \sum_{(i,j) \in \Omega} \left(A_{ij} - u_i^T v_j\right)^2 + \lambda \left(\|U\|_F^2 + \|V\|_F^2\right)$$

- A m-by-n rating matrix
  - m number of users
  - n number of items
- u_i embedding (latent factors vector) for user $i \in \mathbb{R}^k$
- v_j embedding (latent factors vector) for item $j \in \mathbb{R}^k$
- $\Omega = \{(i,j) \mid A_{ij} \; is \; observed\}$
- Regularization terms to avoid overfitting

Unconstrained MF: Iterative optimization

- Use SGD (or second order methods to find the parameters)

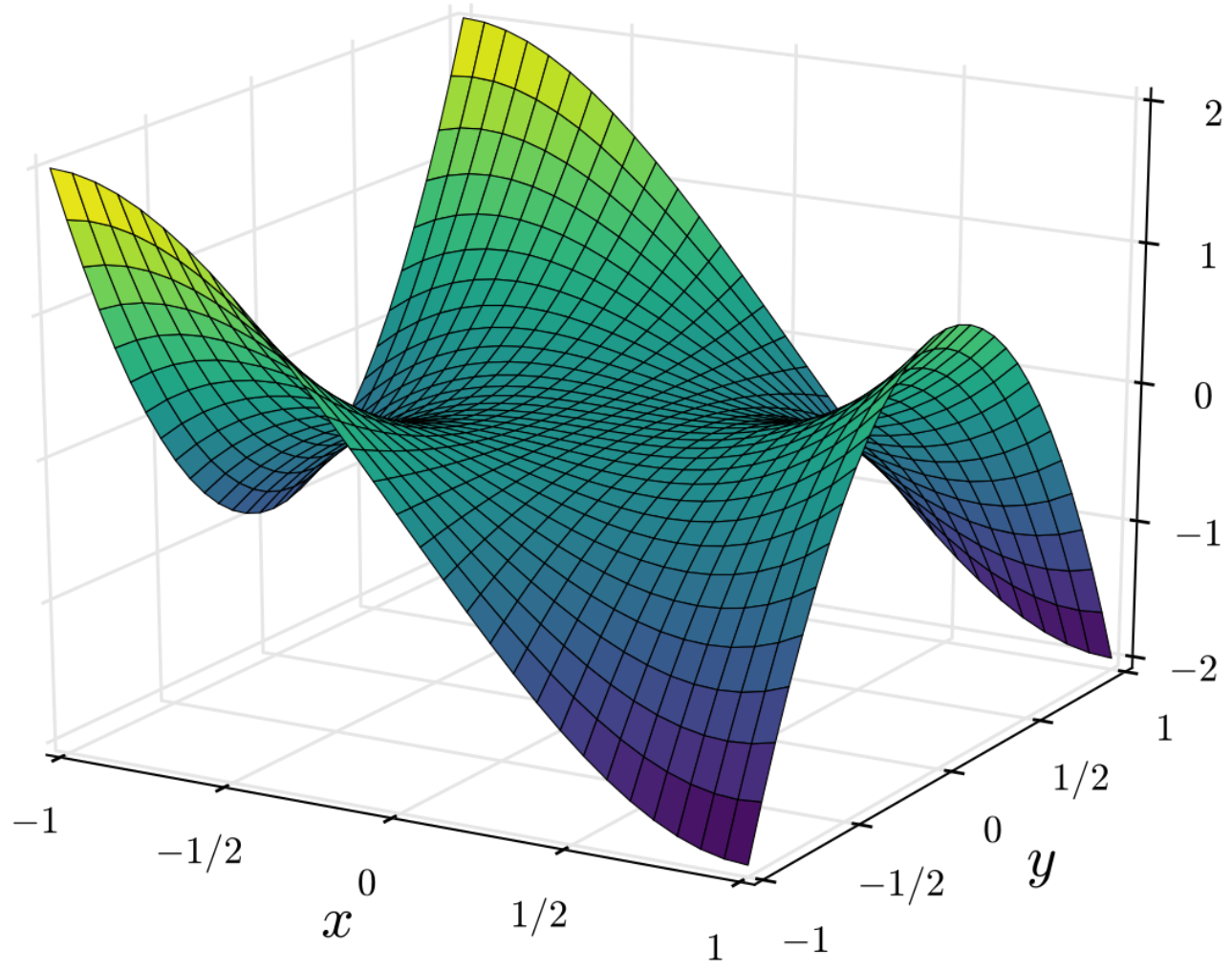
$$L(A_{ij}, u_i, v_j) = \left(A_{ij} - u_i^T v_j\right)^2 + \lambda \left(\|u_i\|^2 + \|v_j\|^2\right)$$

- For each non-missing entry A_{ij}
  - Read ith row of U and jth row of V
  - Update u_i and v_j

$$u_i \leftarrow u_i - \alpha \nabla_{u_i} L(A_{ij}, u_i, v_j)$$

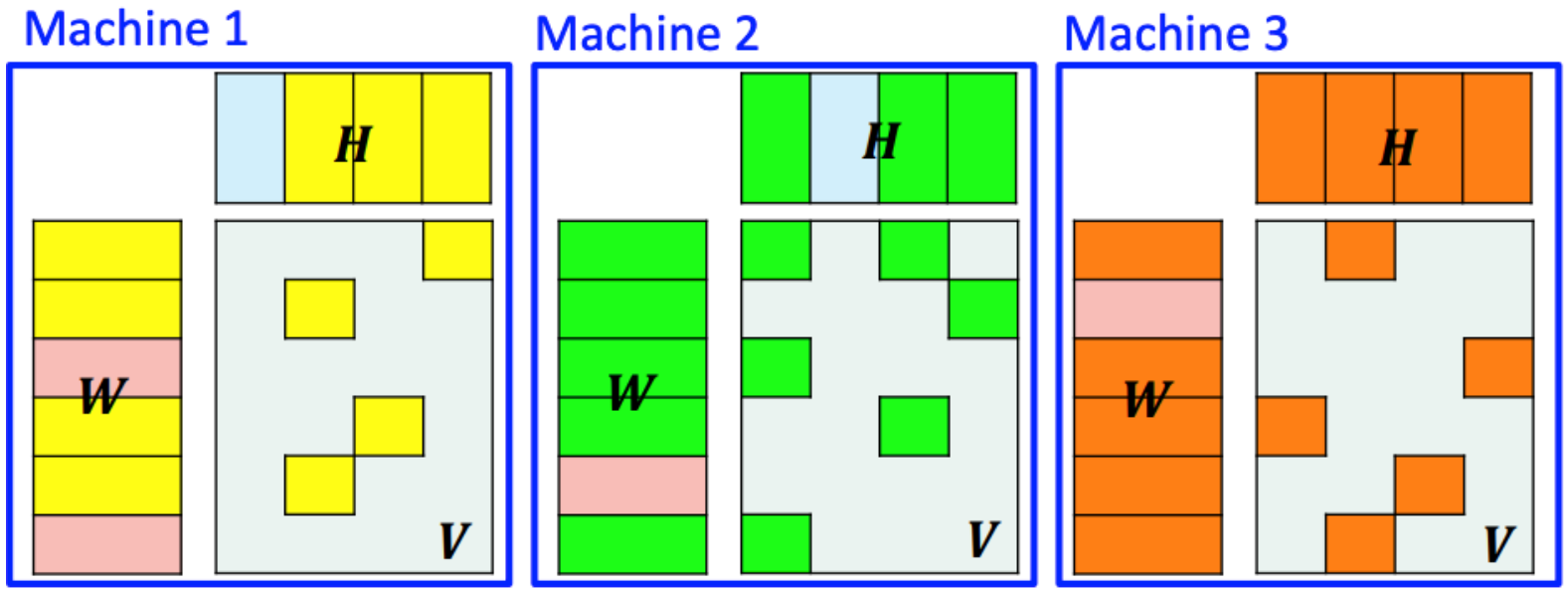
$$v_j \leftarrow v_j - \alpha \nabla_{v_j} L(A_{ij}, u_i, v_j)$$

Unconstrained MF: Iterative optimization



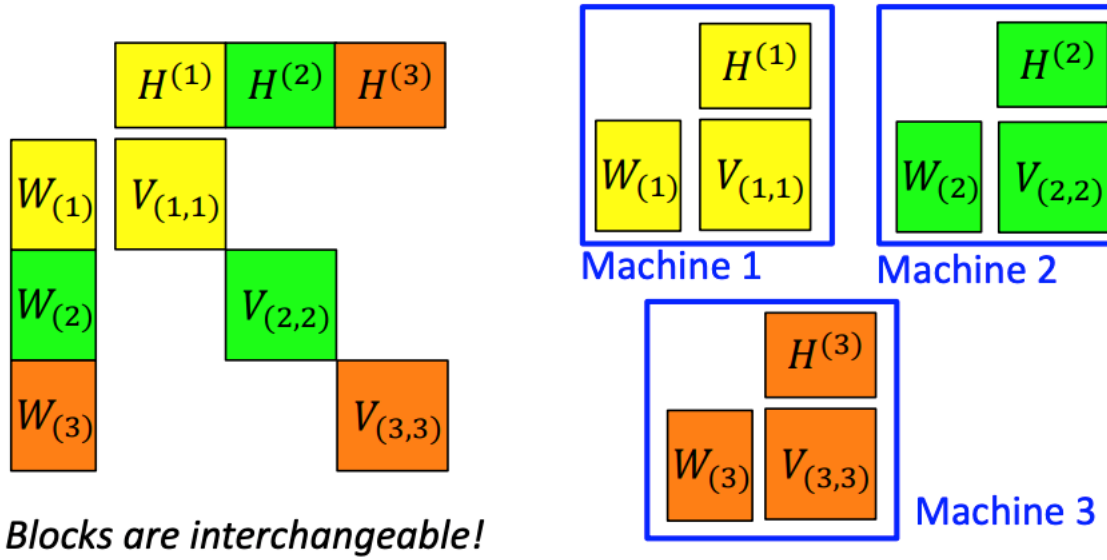
Unconstrained MF: Simple Parallel SGD

- Like in the previous lecture, we can distribute the work across multiple machines and average the gradient updates



Unconstrained MF: Simple Parallel SGD

- Even better, find set of independant or almost independant blocks



ALS: Alternative Least Squares

$$\min_{\substack{U \in \mathbb{R}^{m \times k} \\ V \in \mathbb{R}^{n \times k}}} L(A, U, V), \qquad L(A, U, V) = \sum_{(i,j) \in \Omega} \left(A_{ij} - u_i^T v_j\right)^2 + \lambda \left(\|U\|_F^2 + \|V\|_F^2\right)$$

- $L(A, U, V)$ is a non-convex function of U and V
  - Many local optima, saddle points …

- BUT $L(A, U, V)$ is a convex function of U (if we fix V), same thing for V (if we fix U)
  - One local optimum, that is also global
  - There is a closed form solution

ALS: Alternative Least Squares

- Repeat until convergence
  - Assume item factors V are fixed, solve for u_i

$$u_i = (V^T V + \lambda I)^{-1} V^T r_{i*}$$

  - Then assume user factors U are fixed, solve for V

$$v_j = (U^T U + \lambda I)^{-1} U^T r_{*j}$$

  - Alternate both steps for a few iterations

Distributed ALS: Block ALS

- This is a high level distribution strategy for ALS
  - Relies on the fact that U and V must fit in memory (not the ratings R)
  - Partition Ratings by user to create R_1 and by item to create R_2 (so we have two different copies)
  - Broadcast the two matrices U and V
  - Using R_1 and V we can update U
  - Using R_2 and U we can update V

Another tool: Singular Value Decomposition

- The Singular Value Decomposition of matrix A is the triplet (U, V, Σ) that verify

$$A = U \, \Sigma \, V^T$$

- Regular SVD: if $\boldsymbol{A}$ is $\boldsymbol{m} \times \boldsymbol{m}$, $\boldsymbol{U}$ and $\boldsymbol{V}$ and $\boldsymbol{\Sigma}$ are $\boldsymbol{m} \times \boldsymbol{m}$
- Truncated SVD: $\boldsymbol{A}$ is $\boldsymbol{m} \times \boldsymbol{m}$, $\boldsymbol{U}$ and $\boldsymbol{V}$ are $\boldsymbol{m} \times \boldsymbol{k}$, $\boldsymbol{\Sigma}$ is $\boldsymbol{k} \times \boldsymbol{k}$ $(k \ll m)$

(Note: https://fr.wikipedia.org/wiki/D%C3%A9composition_en_valeurs_singuli%C3%A8res)

User-Item Matrix Factorization with SVD

If A is a user-product matrix, then $A^T A$ is the product co-occurrence matrix. We can then observe that V contains the eigenvectors of that matrix:

$$A^T A = V \Sigma U^T \, \mathrm{U} \, \Sigma V^T = \mathrm{V} \Sigma V^T$$

From this, we can deduct $\mathrm{U} = \mathrm{X} \, \mathrm{V} \, \Sigma^T$!

Still, there are a few issues

- SVD doesn't handle missing values
- The cooccurrence signal is biased by product popularity, we need another measure
- We take full cooccurrence of items in timelines, the user intent might change through time

Item-Item matrix

- Let’s shift our attention away from the user-item matrix

- Can we build a pairwise item interaction matrix that makes sense ?
  - Co-counts: does not seem like a great idea, popular products will have a high interaction number with everything

  - Is there a quantity that can characterize the discrepancy between the observed cooccurence and the one they would have had if they were unrelated

Pointwise Mutual Information PMI

- The PMI of a pair of outcomes x and y belonging to discrete random variables X and Y

$$pmi(x, y) = \log \frac{p(x,y)}{p(x)\,p(y)}$$

  - 0 means the two variables are independent
  - Positive values means it's more "likely" to observe y if x if present (and vice versa) : $\frac{p(y|x)}{p(y)} > 1$
  - Negative values means it's less "likely" to observe y if x if present (and vice versa) : $\frac{p(y|x)}{p(y)} < 1$

(Note: Cool slides on PMI and word embeddings: https://web.stanford.edu/~jurafsky/li15/lec3.vector.pdf)

Pointwise Mutual Information PMI

- For two products x and y, we can estimate empirically the PMI in a given dataset

$$pmi(x,y) = \log \frac{\#(x,y)\ \#E}{\#x\ \#y}$$

  - $\#(x,y)$ number of times x and y were interacted with by the same user (cooccurrence)
  - $\#x$ $(\#y)$ number of times x appears in the dataset
  - $\#E$ total number of all interactions in the dataset

Singular Value Decomposition

- With Truncated SVD, in our case, each row of Matrix U will correspond to an embedding of size k of a product

- The embeddings will capture relevant characteristics of the products, and similar products will have similar embeddings

Singular Value Decomposition

- Singular Value Decomposition is very **expensive** for large matrices, both **memory** and **time** wise

- A growing area of research, ***randomized numerical linear algebra,*** combines probability theory with numerical linear algebra to develop fast, randomized algorithms with theoretical guarantees

- The main insight is that ***randomness*** is an algorithmic resource creating **efficient**, unbiased approximations of nonrandom operations.

Randomized SVD

- Consider the general problem of low-rank matrix approximation.
  - Given an $m \times n$ matrix $\boldsymbol{A}$, we want $m \times k$ and $k \times n$ matrices $\boldsymbol{B}$ and $\boldsymbol{C}$ such that $k \ll n$ and $\boldsymbol{A} \approx \boldsymbol{BC}$.
  - To approximate this computation using randomized algorithms, Halko et al propose a two-stage computation
    - **Step 1:** Compute an approximate basis for the range of $\boldsymbol{A}$. We want a matrix $\boldsymbol{Q}$ with ℓ orthonormal columns ($k \leq \ell \leq n$) that captures the action of $\boldsymbol{A}$. Formally, $\boldsymbol{A} \approx \boldsymbol{QQ}^{*}\boldsymbol{A}$
    - **Step 2:** Given such a matrix $\boldsymbol{Q}$—which is much smaller than $\boldsymbol{A}$—use it to compute our desired matrix factorization.

Randomized SVD – Step 2

- In the case of SVD, imagine we had access to $\boldsymbol{Q}$. Then randomized SVD is the following:
  - Given an orthonormal matrix $\boldsymbol{Q}$ such that $\boldsymbol{A} \approx \boldsymbol{Q}\boldsymbol{Q}^{*}\boldsymbol{A}$
    - Form $\boldsymbol{B} = \boldsymbol{Q}^{*}\boldsymbol{A}$ ($n \times \ell$ matrix)
    - Compute the SVD of $\boldsymbol{B}$, i.e. $\boldsymbol{B} = \widetilde{\boldsymbol{U}}\,\boldsymbol{\Sigma}\,\boldsymbol{V}^{*}$ **(complexity $n\ell^2$ instead of n^3 for $\boldsymbol{A}$)**
    - Set $\boldsymbol{U} = \boldsymbol{Q}\,\widetilde{\boldsymbol{U}}$
    - Return $\boldsymbol{U}, \boldsymbol{\Sigma}, \boldsymbol{V}^{*}$

Randomized SVD – Step 2

- The efficiency of this algorithm comes from $\boldsymbol{B}$ being small relative to $\boldsymbol{A}$.
  - Since $\boldsymbol{A} \approx \boldsymbol{Q}\,\underbrace{\boldsymbol{Q}^{*}\boldsymbol{A}}_{\boldsymbol{B}} = \boldsymbol{Q}\left(\widetilde{\boldsymbol{U}}\,\boldsymbol{\Sigma}\,\boldsymbol{V}^{*}\right)$

    setting $\boldsymbol{U} = \boldsymbol{Q}\widetilde{\boldsymbol{U}}$ produces a low-rank approximation, $\boldsymbol{A} \approx \boldsymbol{U}\,\boldsymbol{\Sigma}\,\boldsymbol{V}^{*}$
- Note that ***randomness*** only occurs in **Step 1** and that **Step 2 is *deterministic*** given $\boldsymbol{Q}$
- Thus, the algorithmic challenge is to efficiently compute $\boldsymbol{Q}$ through randomized methods

Randomized SVD – Step 1

- The goal of a randomized range finder is to produce an orthonormal matrix $\boldsymbol{Q}$ with as few columns as possible such that

$$\|(A - \boldsymbol{Q}\boldsymbol{Q} * \boldsymbol{A})\| \leq \varepsilon$$

- for some desired tolerance ε

Randomized SVD – Step 1

- Let ℓ be a sampling parameter indicating the number of Gaussian random vectors to draw for $\boldsymbol{\Omega}$.
- Draw an $n \times \ell$ Gaussian random matrix $\boldsymbol{\Omega}$
- Generate an $m \times \ell$ matrix $\boldsymbol{Y} = \boldsymbol{A\Omega}$
- Generate an orthonormal matrix $\boldsymbol{Q}$, e.g. using QR factorization $\boldsymbol{Y} = \boldsymbol{QR}$
- Note that the Algorithm takes as input a sampling parameter ℓ where $\ell \geq k$ ideally. Then $p = \ell - k$ is the *oversampling parameter*.

Randomized SVD – Step 1

- Intuition
  - A is a $m\ x\ n$ matrix of rank exactly k
  - Let's draw k random vectors $\{\ \omega^{(i)} : i = 1,2, \dots, k\ \}$ and form a set of $y^{(i)} = A\omega^{(i)}$
  - Because the random vectors $\{\ \omega^{(i)}\ \}$ form a linearly independant set, it is very likely that $\{\ y^{(i)}\ \}$ are also linearly independant
  - So now we have k independant vectors in the range of a matrix of size k, they then form a basis of that range
  - To produce an orthonormal basis, we just need to orthonomalize the $\{\ y^{(i)}\ \}$
  - Which is just running QR decomposision of $\boldsymbol{Y = A\Omega}$

  (Y is formed from $\{\ y^{(i)}\ \}$. and Ω is formed from $\{\ \omega^{(i)}\ \}$)

Randomized SVD – Step 1

- When the singular spectrum of **A** decays slowly, randomized SVD can have high reconstruction error, how can we increase the rate of spectrum decay ?

- Power iterations

  - Decompose $\left(\boldsymbol{AA^T}\right)^q A$ instead for which the singular value matrix is $\boldsymbol{\Sigma^{2q+1}}$

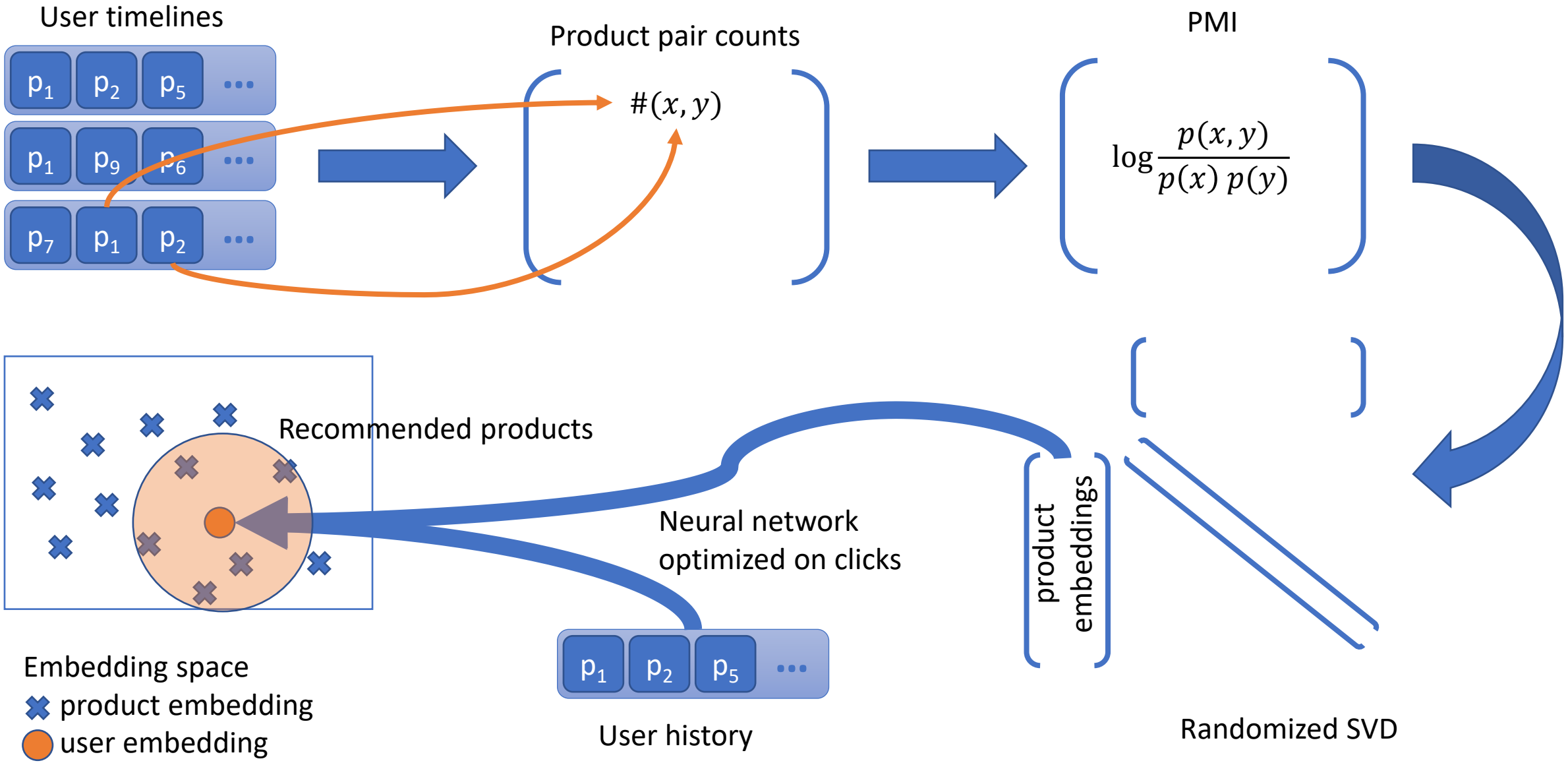
  - The main idea is that power iterations do not modify the singular vectors, but make the spectrum decay more rapidly

Randomized SVD

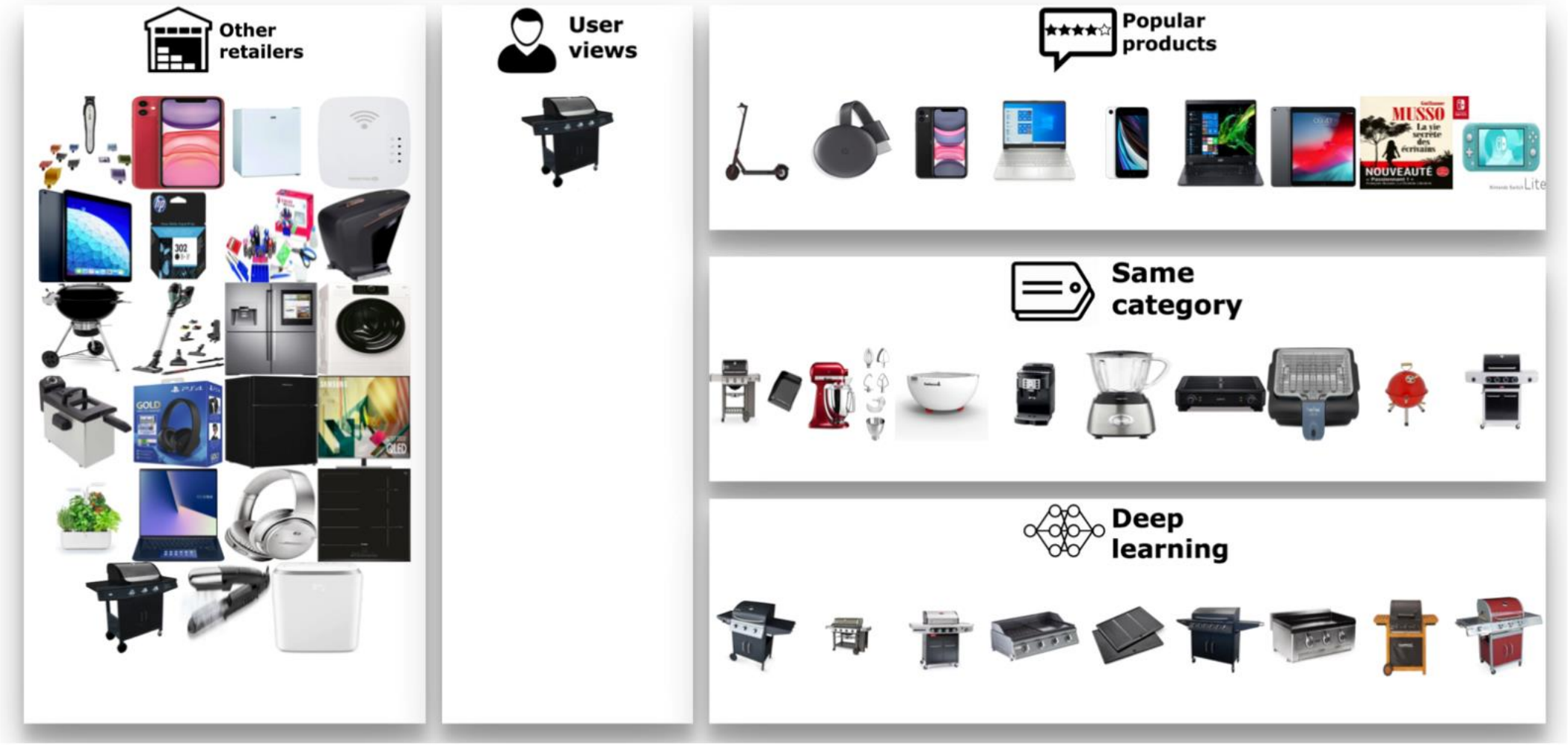
- This is what we use in production at Criteo !

- We open sourced Spark-RSVD
  - **Spark-RSVD** is a lib to compute approximate SVD decomposition of large sparse matrices (up to 100 million rows and columns) using an iterative algorithm for speed and efficiency.

Usage at Criteo



Criteo real example



Criteo real example