ES 156 Assignment 7

Welcome to Problem Set 7! You have three option in this problem set:

1. Solve the problems below. The purpose of these problems is for you to familiarize yourself with the DTFT and its many properties.
2. Do the "Shazam" Python programming exercise, where you get to implement a simplified version of Shazam. You can find the notebook on Canvas.
3. Do both the programming exercise and the problems below! In this case, you get to skip a future problem set (or, if you do all PSets, the lowest grade will be replaced). Recall that we only consider the 9 highest scores out of the 10 problem sets. If you do both components, we would consider the 9 highest scores out of 11.

We only kindly ask that you pick one of the options above and stick to it. For example, please don't do half of the problems below, and then half of the lab.

Problems

In the following problems please give enough detail/explanation so that we can follow the thinking that leads to your answers.

1. **DTFT Calculation.** Let u[n] be the unit step sequence, defined as $u[n] = 1$ for $n \geq 0$ and $u[n] = 0$ otherwise. Calculate the DTFT of the following discrete-time signals:
   (a) $x[n] = u[n-2] - u[n-4]$
   (b) $h[n] = (\frac{1}{2})^{-n} u[-n-1]$
   (c) $y[n] = x[n] * h[n]$
   (d) $z[n] = y[n - n_0]$
2. **DTFT Properties Proof.** Prove the following properties of DTFT:
   (a) $x[-n] \overset{\text{DTFT}}{\longleftrightarrow} X(e^{-j\omega})$.
   (b) If $x[n]$ is real, then $X(e^{j\omega}) = X^*(e^{-j\omega})$.
   (c) If $x[n]$ is real and symmetric ($x[n] = x[-n]$), then $X(e^{j\omega})$ is real.
   (d) If $x[n]$ is real and antisymmetric ($x[n] = -x[-n]$), then $X(e^{j\omega})$ is purely imaginary.
3. **IDTFT of An Ideal Bandpass Filter.** Let X be a periodic function with period 2π, such that

$$X(e^{j\omega}) = \begin{cases} 1, \; 1 < |\omega| < 2 \\ 0, \; |\omega| \leq 1 \text{ or } 2 \leq |\omega| \leq \pi \end{cases}$$

   Find the inverse discrete-time Fourier transform of X. Hint: use Example 5.12 from the textbook and the frequency shifting property.

4. **DTFT Property Application.**

   (a) Compute the DTFT of $x[n] = n2^{-n}u[n]$, using the method for geometric series of the form $\sum_n nr^n$.

   (b) Compute again the DTFT of $x[n]$, but now use the DTFT property

   $$nx[n] \overset{DTFT}{\longleftrightarrow} j\frac{d}{d\omega}X(e^{j\omega}).$$

   Verify that the two methods give the same result.

5. **LTI System Characterized by Difference Equation.** A discrete-time causal, linear, time-invariant system is described by the following input-output equation:

   $$6y[n] - 5y[n-1] + y[n-2] = 6x[n].$$

   Here, x is the input to the system and y is the corresponding output.

   (a) Find the frequency response $H(e^{j\omega})$ of this system.

   (b) Find the impulse response $h[n]$ of this system.

   (c) Find the response of this system to the input signal $x_1[n] = \delta[n] - (1/3)\delta[n-1]$.

   (d) Find the response of this system to the input signal $x_2[n] = e^{j\pi n}$.

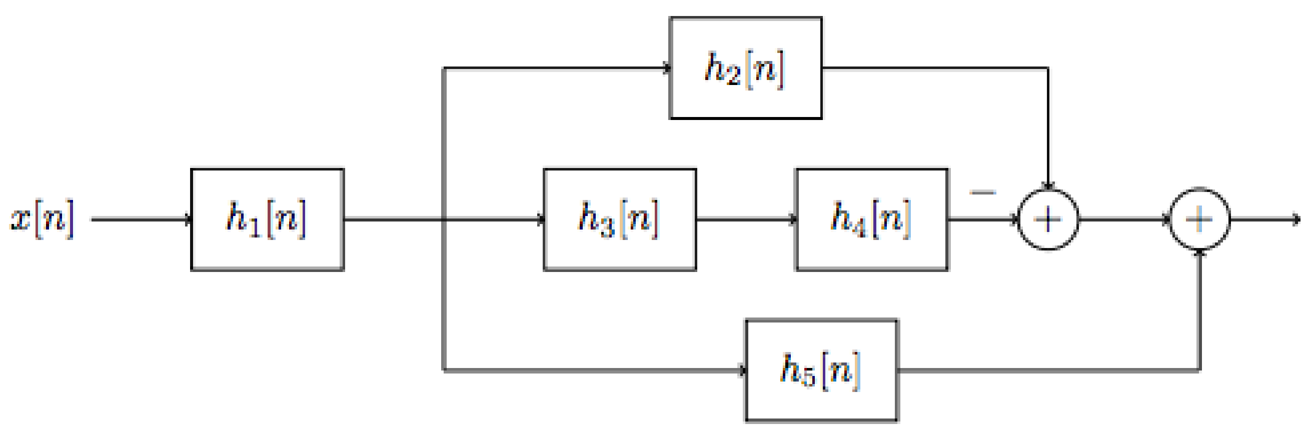

Figure 1: System for Problem 6

6. **Bonus 20 points** Solve problem 4.47 in the textbook. (If you do not have access to a book, please email the TFs for a copy of the problem.)