

# Part 2

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## KRR for Dynamic Environments

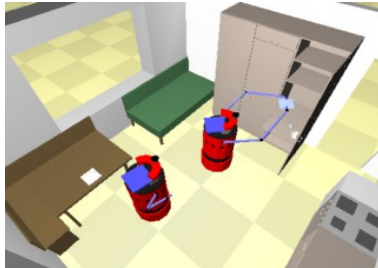
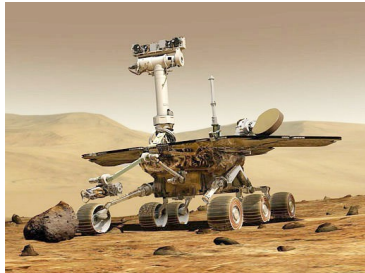
(Week 6 – 9)

# Dynamic Environments

Part 2 is concerned with KRR techniques for

- intelligent (virtual) agents
- intelligent robos
- ...

that interact with an environment



# Overview: Representation Techniques

## Week 6

- Representations for classical **planning problems**
  - deterministic environment; complete information

## Week 7

- **Logic programs** for problem representations
  - including planning problems, games

## Week 8

- **First-order logic** to describe dynamic environments
  - deterministic environment; (in-)complete information

## Week 9

- **State transition systems** to describe dynamic environments
  - nondeterministic environment; (in-)complete information

# Overview: Reasoning Techniques

## Week 6

- Planning algorithms

## Week 7

- Answer Set Programming

## Week 8

- Reasoning in first-order logic, Prolog

## Week 9

- Markov decision algorithms
- Decision making

# Planning

- Representations for classical planning
- Modern heuristics for state-space planning
- Planning graphs: a modern planning technique

## Background reading

*Automated Planning* by Malik Ghallab, Dana Nau, Paolo Traverso, Morgan Kaufmann 2004. Chapters 1, 2, 4 & 6

# Some Dictionary Definitions of “Plan”

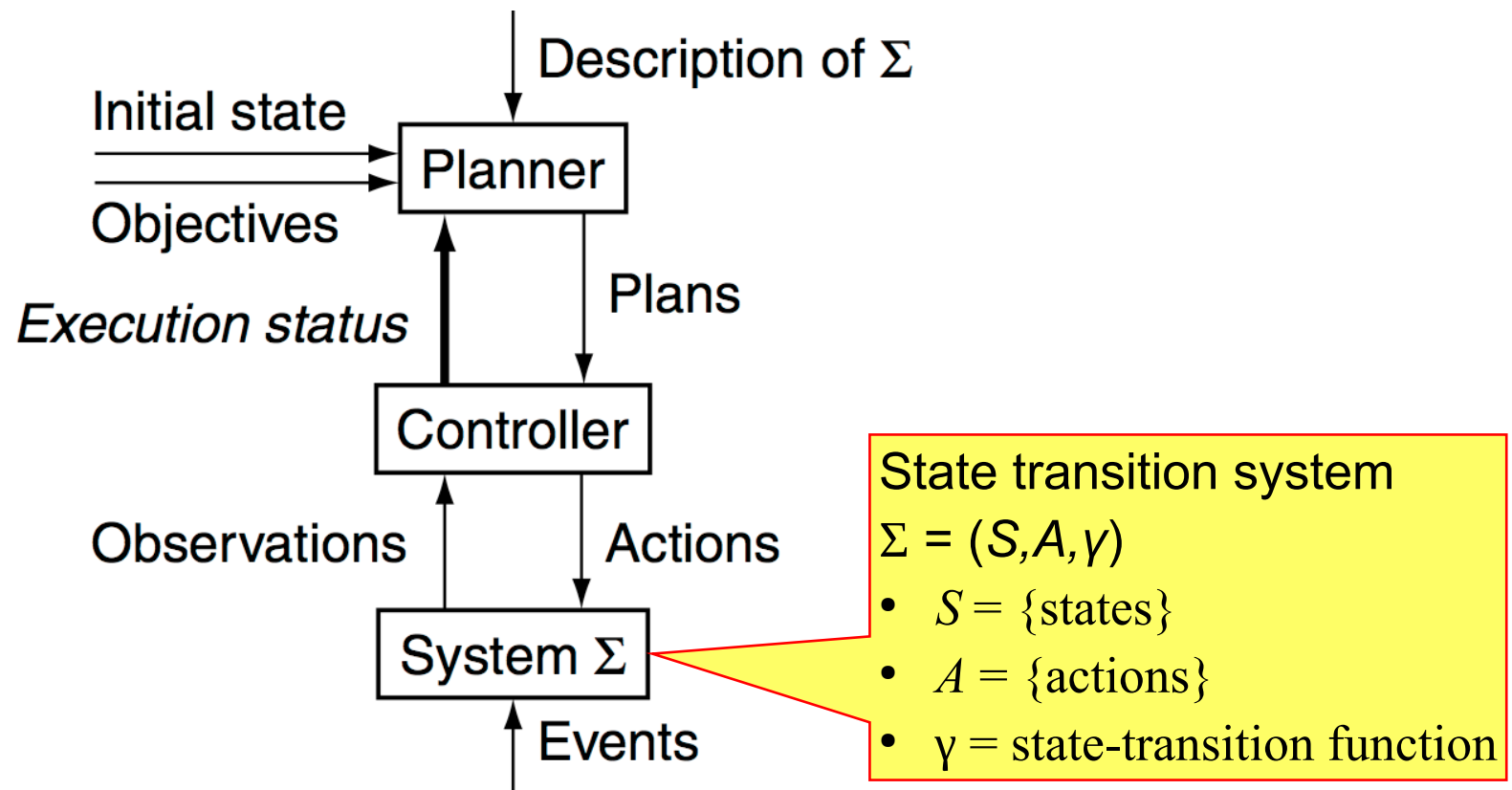
## **plan** *n.*

1. A scheme, program, or method worked out beforehand for the accomplishment of an objective: *a plan of attack.*
2. A proposed or tentative project or course of action: *had no plans for the evening.*

[a representation] of future behaviour ... usually a set of actions, with temporal and other constraints on them, for execution by some agent or agents.

– Austin Tate, *MIT Encyclopaedia of the Cognitive Sciences*, 1999

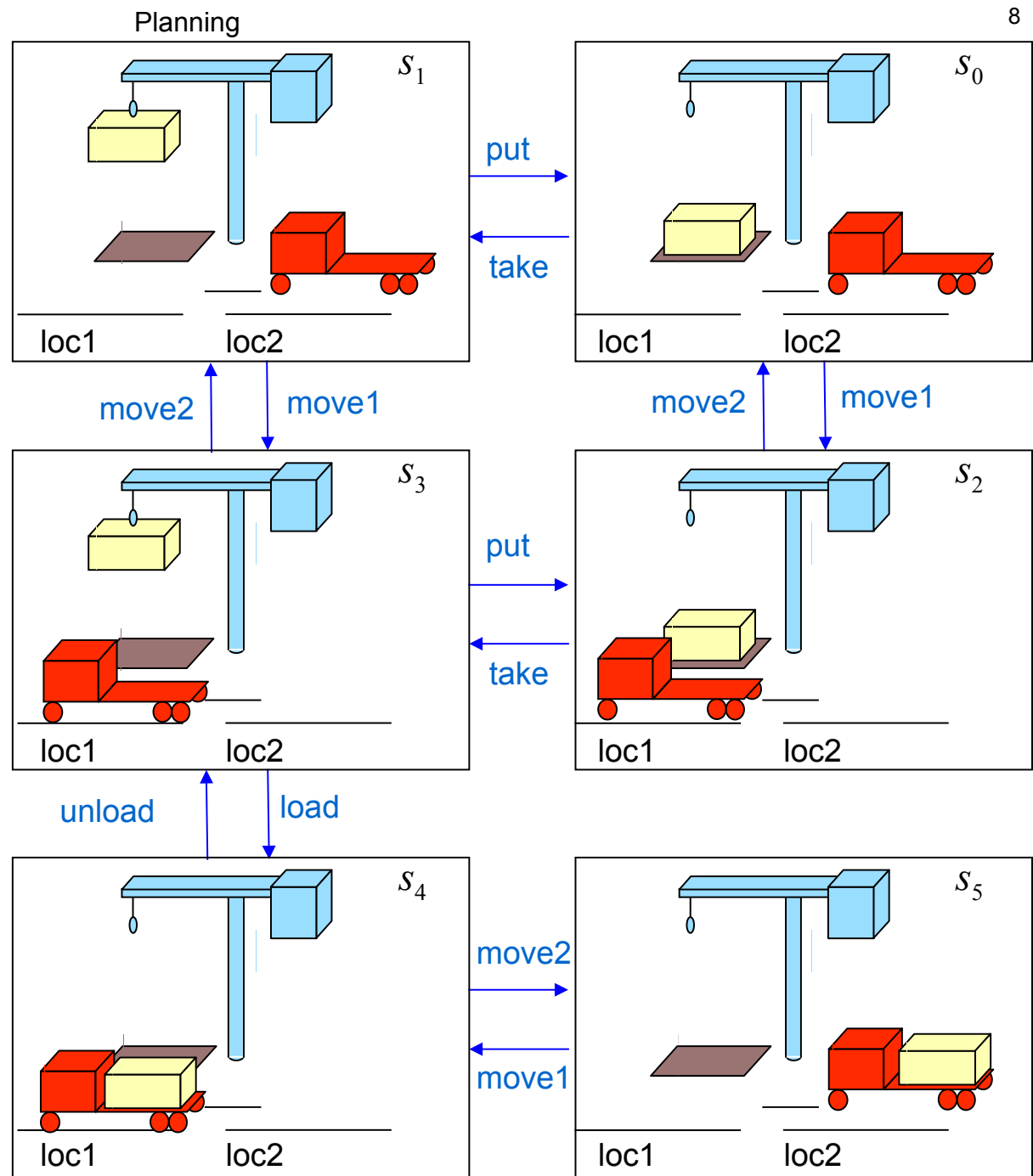
# Planning for an Agent/Robot in a Dynamic World



- $\Sigma$  is an abstraction that deals only with the aspects that the planner needs to reason about

# Example

- Example  $\Sigma = (S, A, \gamma)$ :
  - $S = \{s_0, \dots, s_5\}$
  - $A = \{\text{move1}, \text{move2}, \text{put}, \text{take}, \text{load}, \text{unload}\}$
  - $\gamma$ : see the arrows



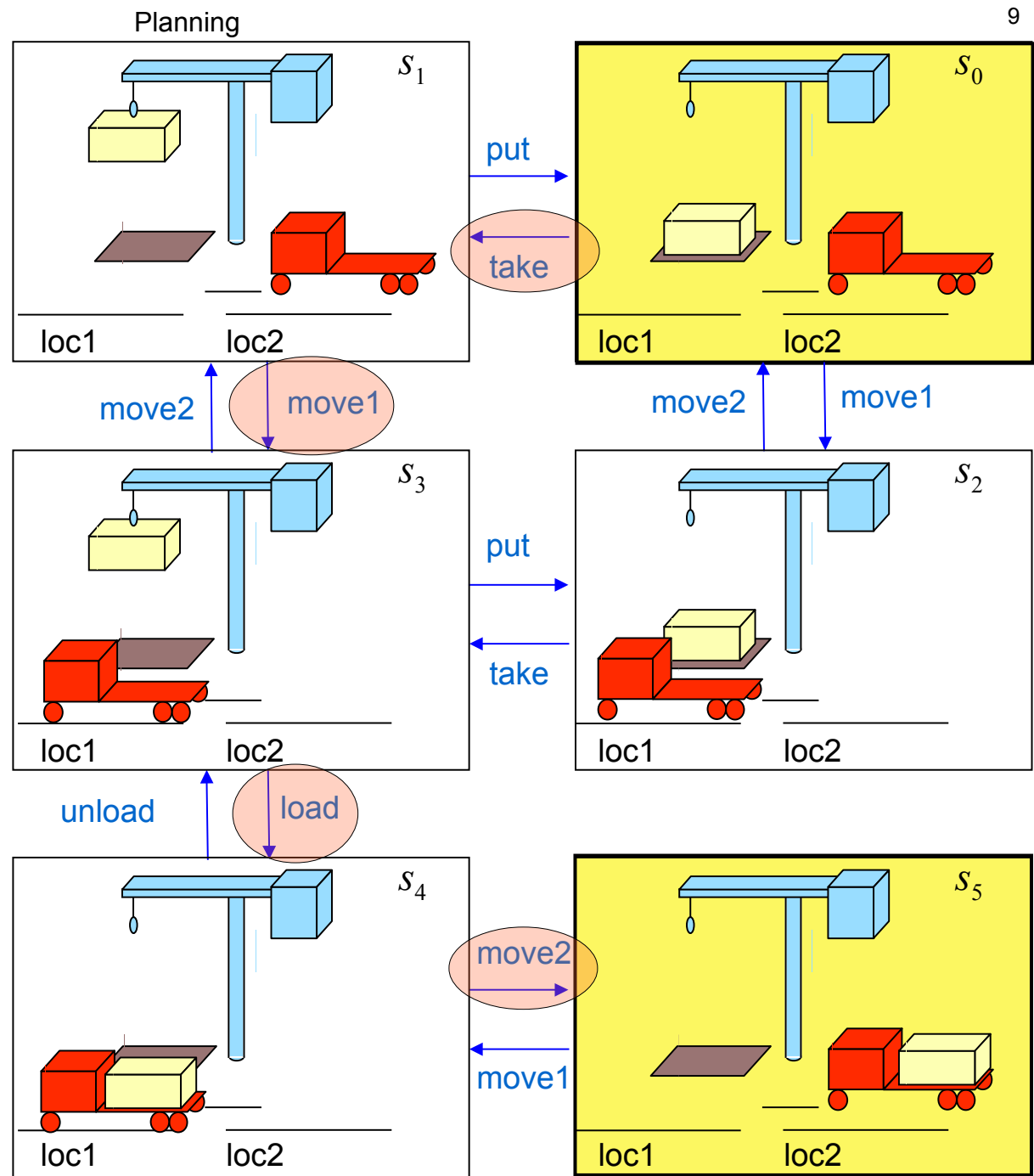
Dock Worker Robots (DWR) example



# Example

- Classical plan: a sequence of actions

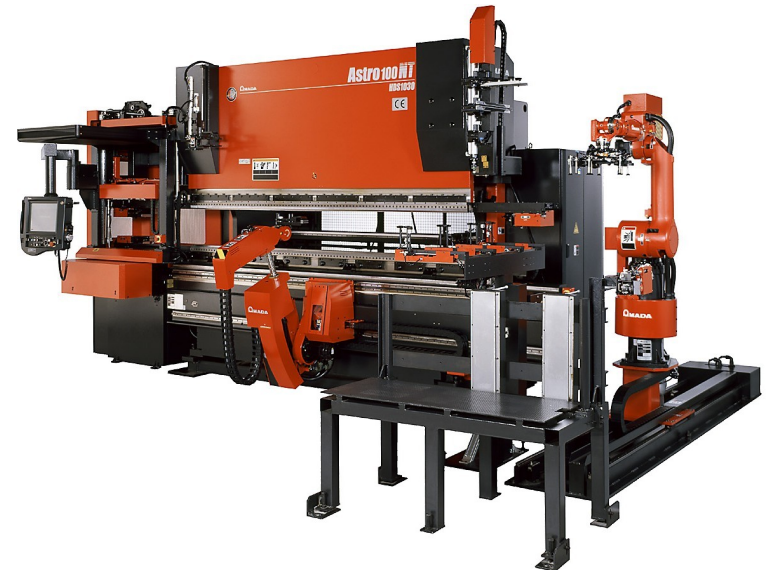
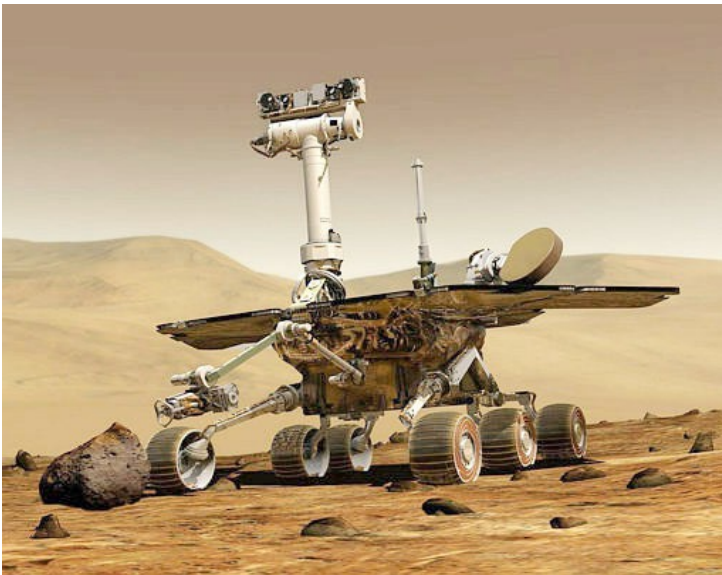
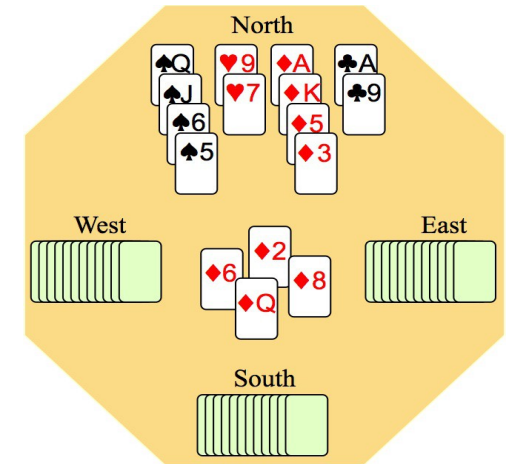
$\langle \text{take}, \text{move1}, \text{load}, \text{move2} \rangle$



Dock Worker Robots (DWR) example

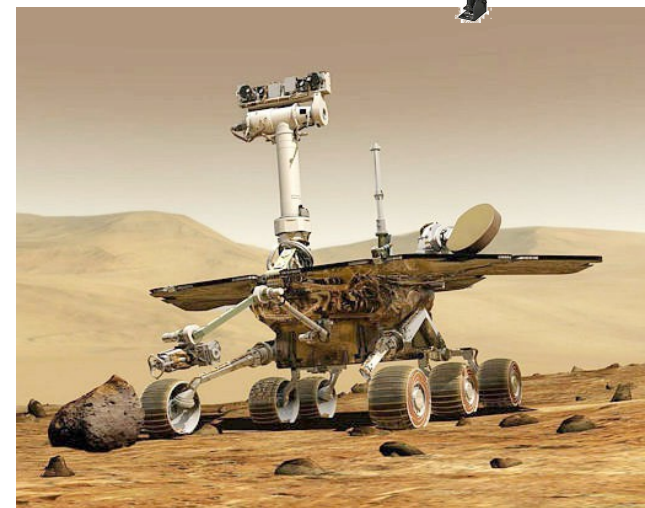
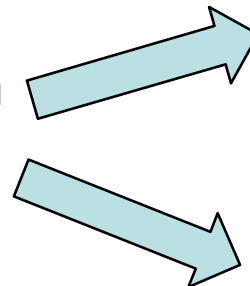
# Domain-Specific Planners

- Most successful real-world planning systems work this way
  - Mars exploration, sheet-metal bending, playing bridge, etc.
- Often use problem-specific techniques that are difficult to generalise to other planning domains



# Domain-Independent Planners

- No domain-specific knowledge except the description of the system  $\Sigma$
- In practice,
  - Not feasible to make domain-independent planners work well in all possible planning domains
- Make simplifying assumptions to restrict the set of domains
  - **Classical planning**
    - ➡ Historical focus of most research on automated planning

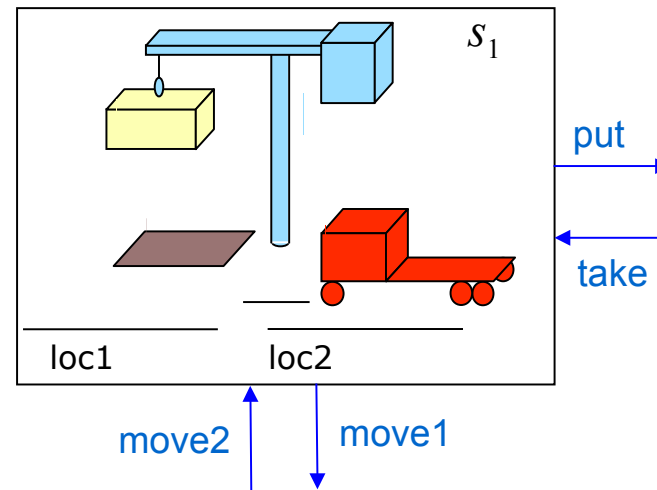


# Classical Planning

- Reduces to the following problem:  
Given  $\Sigma$ , initial state  $s_0$ , and goal states  $S_g$ ,  
find a sequence of actions  $(a_1, a_2, \dots, a_n)$  that produces  
a sequence of state transitions  $(s_0, s_1, s_2, \dots, s_n)$  such that  $s_n \in S_g$

## Is this trivial?

- Generalise the earlier example:
  - Five locations, three robot carts, 100 containers, three piles  
⇒  $10^{277}$  states



- Automated-planning research has been heavily dominated by classical planning. There are dozens of different algorithms.

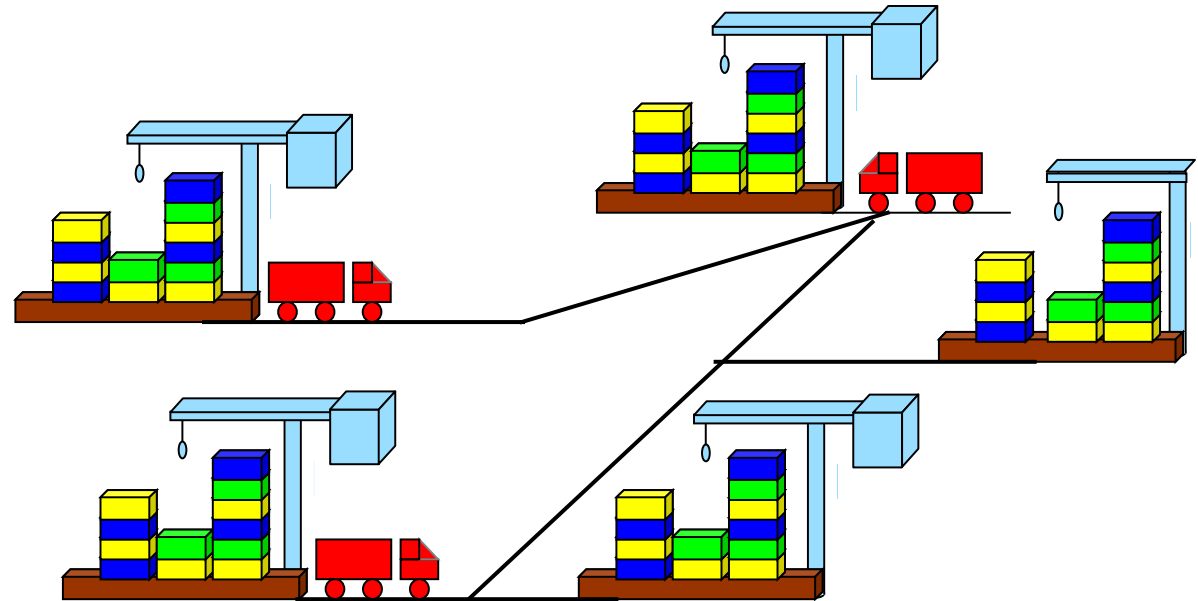
# Representations for Classical Planning

# Classical Representations: Motivation

- In most problems, far too many states to try to represent all of them explicitly as  $s_0, s_1, s_2, \dots$ 
  - ⇒ represent each state as a set of **atomic features**
- Define a set of **operators** that can be used to compute state-transitions
- Don't give all of the states explicitly
  - Just give the initial state
  - Use the operators to generate the other states as needed

# Classical Representation

- Language of first-order logic but without function symbols
  - ➡ finitely many predicate symbols and constant symbols
- Example: the DWR domain
  - Locations: l1, l2, ...
  - Containers: c1, c2, ...
  - Piles: p1, p2, ...
  - Robot carts: r1, r2, ...
  - Cranes: k1, k2, ...

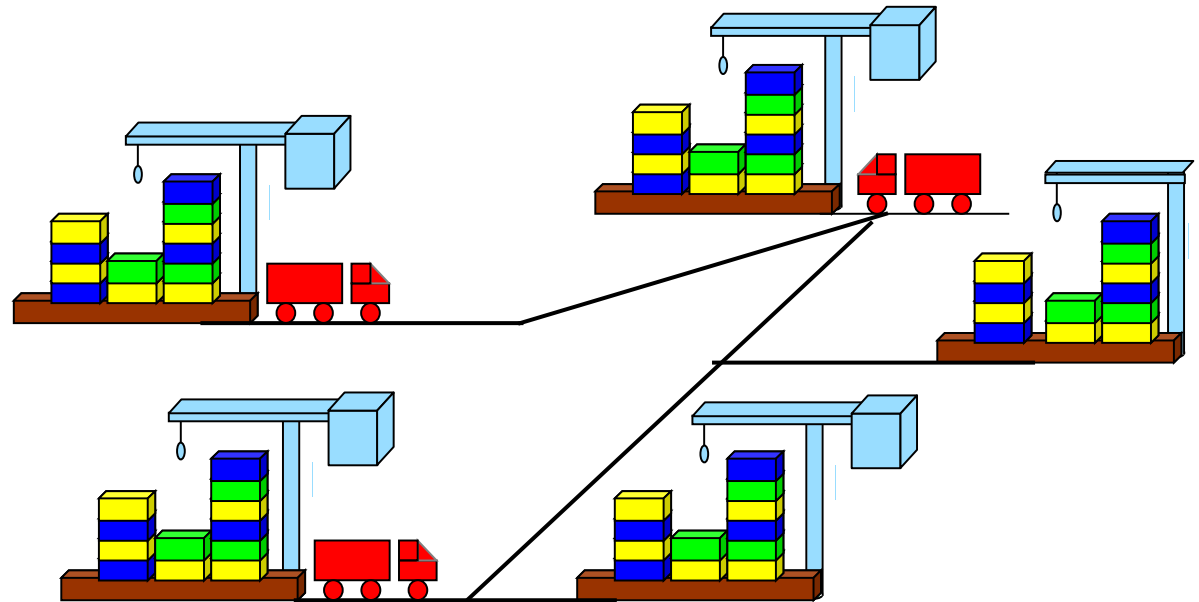


# Example (cont'd)

- **Fixed relations:** same in all states  
 $\text{adjacent}(l, l')$     $\text{attached}(p, l)$     $\text{belong}(k, l)$
- **Dynamic relations:** differ from one state to another

$\text{occupied}(l)$     $\text{at}(r, l)$   
 $\text{loaded}(r, c)$     $\text{unloaded}(r)$   
 $\text{holding}(k, c)$     $\text{empty}(k)$   
 $\text{in}(c, p)$     $\text{on}(c, c')$   
 $\text{top}(c, p)$     $\text{top}(\text{pallet}, p)$

- **Actions:**  
 $\text{take}(c, k, p)$     $\text{put}(c, k, p)$   
 $\text{load}(r, c, k)$     $\text{unload}(r)$     $\text{move}(r, l, l')$

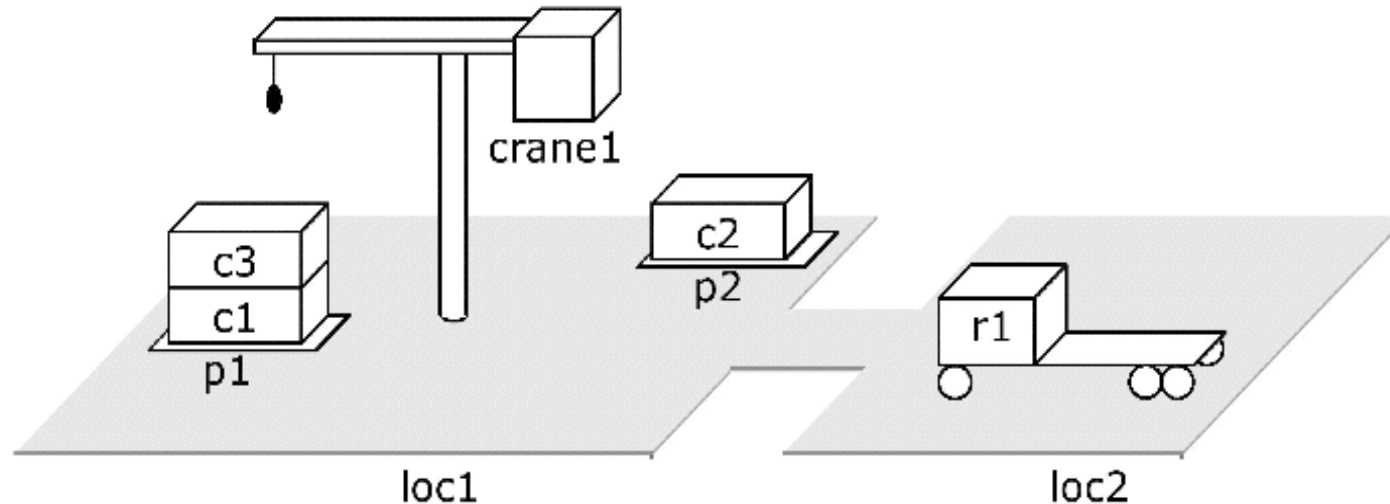




# States

A **state** is a set  $s$  of ground atoms

- The atoms represent the things that can be true in some states
- Only finitely many ground atoms, so only finitely many possible states



$$s_1 = \{ \text{attached}(p1, loc1), \text{in}(c1, p1), \text{in}(c3, p1), \text{top}(c3, p1), \\ \text{on}(c3, c1), \text{on}(c1, \text{pallet}), \text{attached}(p2, loc1), \text{in}(c2, p2), \\ \text{top}(c2, p2), \text{on}(c2, \text{pallet}), \text{belong}(\text{crane1}, loc1), \\ \text{empty}(\text{crane1}), \text{adjacent}(loc1, loc2), \text{adjacent}(loc2, loc1), \\ \text{at}(r1, loc2), \text{occupied}(loc2), \text{unloaded}(r1) \}$$

# Operators

An **operator** is a triple  $o = (\text{name}(o), \text{precond}(o), \text{effects}(o))$

- $\text{name}(o)$ : a syntactic expression of the form  $n(x_1, \dots, x_k)$ 
  - $(x_1, \dots, x_k)$  is a list of every variable symbol (parameter) that appears in  $o$
- $\text{precond}(o)$ : **preconditions**
  - literals that must be true in order to use the operator
- $\text{effects}(o)$ : **effects**
  - literals the operator will make true

## Example

```
take(k,l,c,d,p)
;; crane k at location l takes c off of d in pile p
precond: belong(k,l), attached(p,l), empty(k), top(c,p),
         on(c,d)
effects: holding(k,c), ¬empty(k), ¬in(c,p), ¬top(c,p),
        ¬on(c,d), top(d,p)
```

# Actions

An **action** is a **ground instance** (via a substitution) **of an operator**

```
take(k,l,c,d,p)
;; crane k at location l takes c off of d in pile p
precond: belong(k,l), attached(p,l), empty(k), top(c,p),
         on(c,d)
effects: holding(k,c), ¬empty(k), ¬in(c,p), ¬top(c,p),
        ¬on(c,d), top(d,p)
```

- Let  $\sigma = \{k/\text{crane1}, l/\text{loc1}, c/c3, d/c1, p/p1\}$
- Then  $\text{take}(k,l,c,d,p)\sigma$  is the following action:

**take(crane1,loc1,c3,c1,p1)**

precond: belong(crane1,loc1), attached(p1,loc1), empty(crane1),  
top(c3,p1), on(c3,c1)

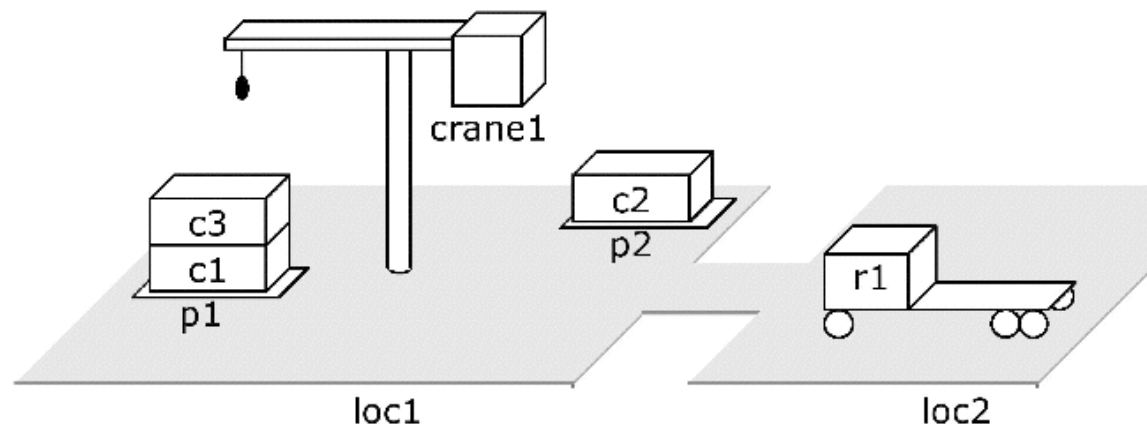
effects: holding(crane1,c3), ¬empty(crane1), ¬in(c3,p1),  
¬top(c3,p1), ¬on(c3,c1), top(c1,p1)

# Applicability and Result of Actions

- Let  $S$  be a set of literals. Then
$$S^+ = \{\text{atoms that appear positively in } S\}$$
$$S^- = \{\text{atoms that appear negatively in } S\}$$
- Let  $a$  be an operator or action. Then
$$\text{precond}^+(a) = \{\text{atoms that appear positively in } a\text{'s preconditions}\}$$
$$\text{precond}^-(a) = \{\text{atoms that appear negatively in } a\text{'s preconditions}\}$$
$$\text{effects}^+(a) = \{\text{atoms that appear positively in } a\text{'s effects}\}$$
$$\text{effects}^-(a) = \{\text{atoms that appear negatively in } a\text{'s effects}\}$$

- Action  $a$  is **applicable** to (or **executable** in)  $S$  if
  - $\text{precond}^+(a) \subseteq s$
  - $\text{precond}^-(a) \cap s = \emptyset$
- The **result** of applying action  $a$  to state  $S$  is
  - $\gamma(s, a) = (s \setminus \text{effects}^-(a)) \cup \text{effects}^+(a)$

# Example: Applicability



- An action:

`take(crane1,loc1,c3,c1,p1)`

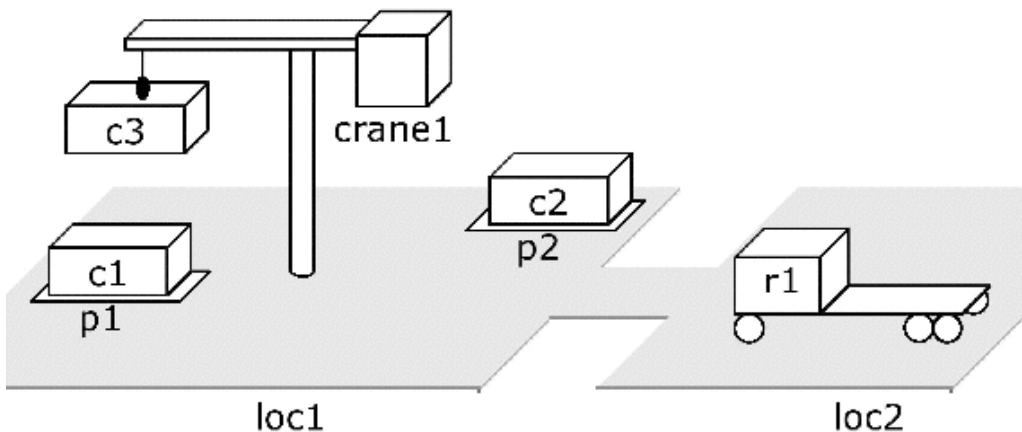
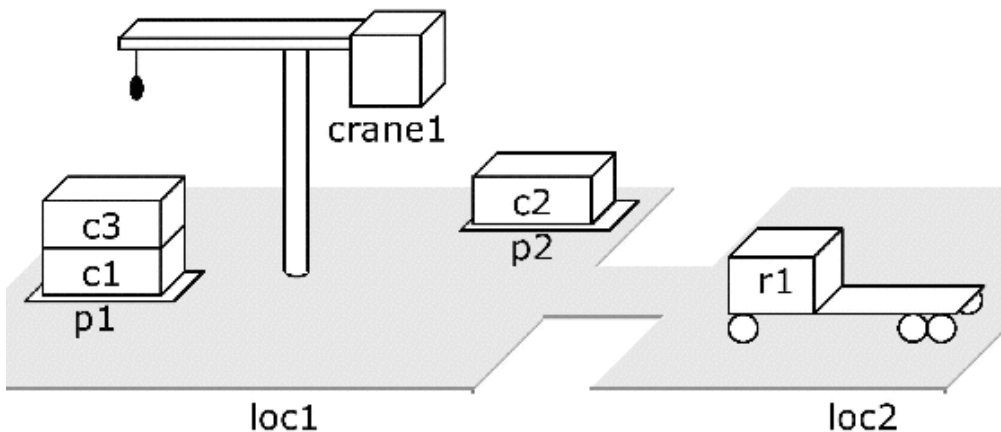
precond: `belong(crane,loc1),`  
`attached(p1,loc1),`  
`empty(crane1), top(c3,p1),`  
`on(c3,c1)`

effects: `holding(crane1,c3),`  
`¬empty(crane1),`  
`¬in(c3,p1), ¬top(c3,p1),`  
`¬on(c3,c1), top(c1,p1)`

- A state it's applicable to

$s_1 = \{\text{attached}(\text{p1}, \text{loc1}), \text{in}(\text{c1}, \text{p1}), \text{in}(\text{c3}, \text{p1}),$   
 $\text{top}(\text{c3}, \text{p1}), \text{on}(\text{c3}, \text{c1}), \text{on}(\text{c1}, \text{pallet}),$   
 $\text{attached}(\text{p2}, \text{loc1}), \text{in}(\text{c2}, \text{p2}),$   
 $\text{top}(\text{c2}, \text{p2}), \text{on}(\text{c2}, \text{pallet}),$   
 $\text{belong}(\text{crane1}, \text{loc1}), \text{empty}(\text{crane1}),$   
 $\text{adjacent}(\text{loc1}, \text{loc2}),$   
 $\text{adjacent}(\text{loc2}, \text{loc1}), \text{at}(\text{r1}, \text{loc2}),$   
 $\text{occupied}(\text{loc2}, \text{unloaded}(\text{r1}))\}$

# Example: Result



`take(crane1, loc1, c3, c1, p1)`

precond: `belong(crane, loc1),`  
`attached(p1, loc1),`  
`empty(crane1), top(c3, p1),`  
`on(c3, c1)`

effects: `holding(crane1, c3),`  
 $\neg$ `empty(crane1),`  
 $\neg$ `in(c3, p1),`  $\neg$ `top(c3, p1),`  
 $\neg$ `on(c3, c1), top(c1, p1)`

$s_2 = \{ \text{attached}(p1, loc1), \text{in}(c1, p1), \text{in}(c3, p1),$   
~~`top(c3, p1), on(c3, c1),`~~ `on(c1, pallet),`  
`attached(p2, loc1), in(c2, p2),`  
`top(c2, p2), on(c2, pallet),`  
`belong(crane1, loc1),` ~~`empty(crane1),`~~  
`adjacent(loc1, loc2),`  
`adjacent(loc2, loc1), at(r1, loc2),`  
`occupied(loc2, unloaded(r1),`  
`holding(crane1, c3), top(c1, p1)`  $\}$

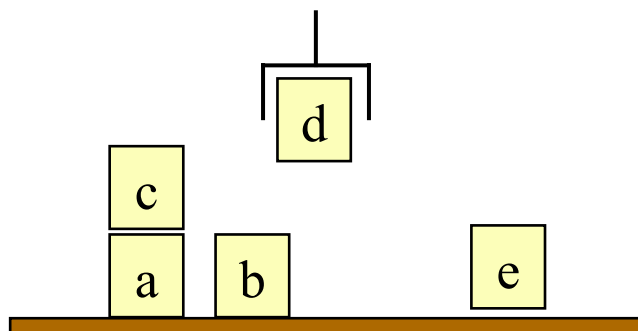
# Exercise

# Exercise: The Blocks World

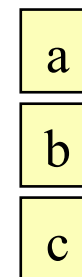
- Infinitely wide table, finite number of children's blocks
- Ignore where a block is located on the table
- A block can sit on the table or on another block
- There's a robot gripper that can hold at most one block
- Want to move blocks from one configuration to another

- e.g.,

initial state



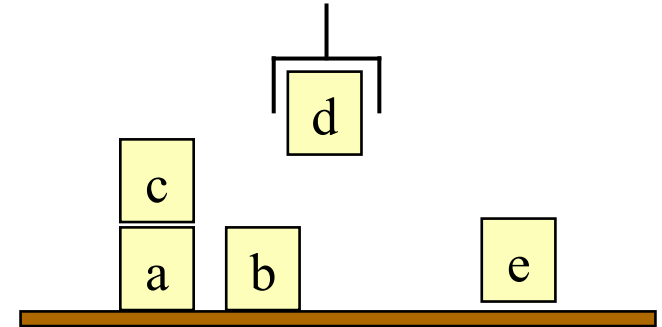
goal





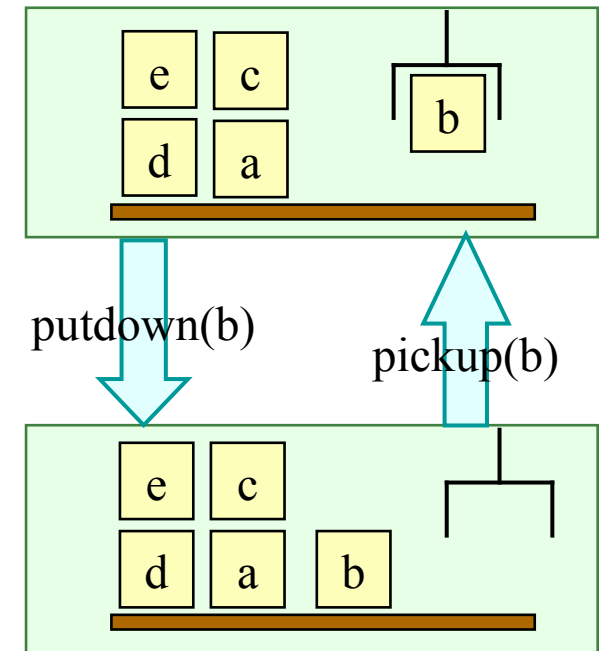
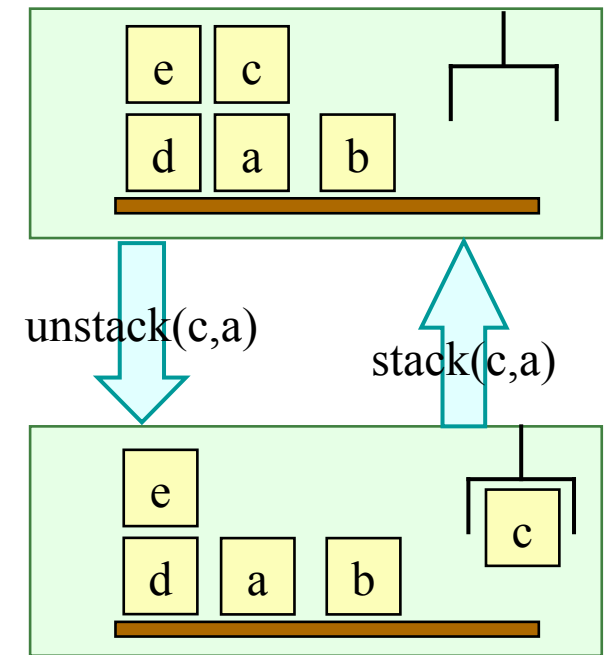
# Exercise: Classical Representation – Symbols

- Constant symbols:
  - The blocks: a, b, c, d, e
- Dynamic relations?



# Exercise: Classical Operators

- Preconditions and effects?



# Summary: Planning Problems

Given a planning domain (language  $L$ , operators  $O$ )

- **Representation** of a planning problem: a triple  $P = (O, s_0, g)$ 
  - $O$  is the collection of operators
  - $s_0$  is a state (the initial state)
  - $g$  is a set of literals (the goal formula)

# Plans and Solutions

Let  $P = (O, s_0, g)$  be a planning problem

- **Plan:** any sequence of actions  $\pi = \langle a_1, a_2, \dots, a_n \rangle$  such that each  $a_i$  is an instance of an operator in  $O$
- Plan  $\pi$  is a **solution** for  $P = (O, s_0, g)$  if it is executable and achieves  $g$ 
  - i.e., if there are states  $s_0, s_1, \dots, s_n$  such that
$$\gamma(s_0, a_1) = s_1$$
$$\gamma(s_1, a_2) = s_2$$
$$\vdots$$
$$\gamma(s_{n-1}, a_n) = s_n$$
$$s_n \text{ satisfies } g$$

# Example: The 5 DWR Operators

$\text{move}(r, l, m)$

;; robot  $r$  moves from location  $l$  to location  $m$

precond:  $\text{adjacent}(l, m), \text{at}(r, l), \neg \text{occupied}(m)$

effects:  $\text{at}(r, m), \text{occupied}(m), \neg \text{occupied}(l), \neg \text{at}(r, l)$

$\text{load}(k, l, c, r)$

;; crane  $k$  at location  $l$  loads container  $c$  onto robot  $r$

precond:  $\text{belong}(k, l), \text{holding}(k, c), \text{at}(r, l), \text{unloaded}(r)$

effects:  $\text{empty}(k), \neg \text{holding}(k, c), \text{loaded}(r, c), \neg \text{unloaded}(r)$

$\text{unload}(k, l, c, r)$

;; crane  $k$  at location  $l$  takes container  $c$  from robot  $r$

precond:  $\text{belong}(k, l), \text{at}(r, l), \text{loaded}(r, c), \text{empty}(k)$

effects:  $\neg \text{empty}(k), \text{holding}(k, c), \text{unloaded}(r), \neg \text{loaded}(r, c)$

$\text{put}(k, l, c, d, p)$

;; crane  $k$  at location  $l$  puts  $c$  onto  $d$  in pile  $p$

precond:  $\text{belong}(k, l), \text{attached}(p, l), \text{holding}(k, c), \text{top}(d, p)$

effects:  $\neg \text{holding}(k, c), \text{empty}(k), \text{in}(c, p), \text{top}(c, p), \text{on}(c, d), \neg \text{top}(d, p)$

$\text{take}(k, l, c, d, p)$

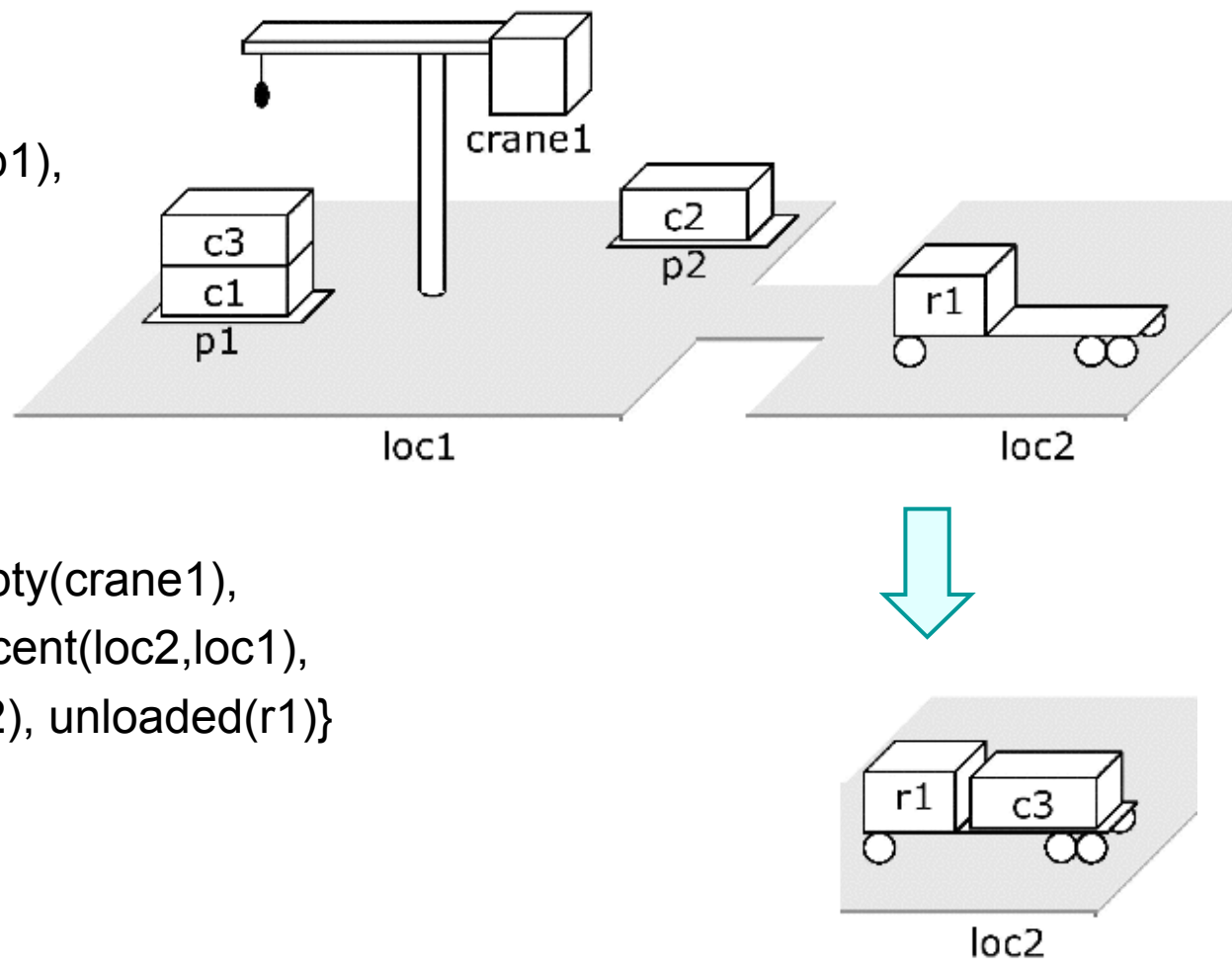
;; crane  $k$  at location  $l$  takes  $c$  off of  $d$  in pile  $p$

precond:  $\text{belong}(k, l), \text{attached}(p, l), \text{empty}(k), \text{top}(c, p), \text{on}(c, d)$

effects:  $\text{holding}(k, c), \neg \text{empty}(k), \neg \text{in}(c, p), \neg \text{top}(c, p), \neg \text{on}(c, d), \text{top}(d, p)$

# Example

- Let  $P = (O, s_0, g)$ , where
  - $O = \{\text{the 5 DWR operators}\}$
  - $s_0 = \{\text{attached}(p1, loc1), \text{in}(c1, p1), \text{in}(c3, p1), \text{top}(c3, p1), \text{on}(c3, c1), \text{on}(c1, \text{pallet}), \text{attached}(p2, loc1), \text{in}(c2, p2), \text{top}(c2, p2), \text{on}(c2, \text{pallet}), \text{belong}(\text{crane1}, loc1), \text{empty}(\text{crane1}), \text{adjacent}(loc1, loc2), \text{adjacent}(loc2, loc1), \text{at}(r1, loc2), \text{occupied}(loc2), \text{unloaded}(r1)\}$
  - $g = \{\text{loaded}(r1, c3), \text{at}(r1, loc2)\}$



- Two *redundant* solutions (can remove actions and still have a solution):

```

⟨move(r1,loc2,loc1),
 take(crane1,loc1,c3,c1,p1),
 move(r1,loc1,loc2),
 move(r1,loc2,loc1),
 load(crane1,loc1,c3,r1),
 move(r1,loc1,loc2)⟩

```

```

⟨take(crane1,loc1,c3,c1,p1),
 put(crane1,loc1,c3,c2,p2),
 move(r1,loc2,loc1),
 take(crane1,loc1,c3,c2,p2),
 load(crane1,loc1,c3,r1),
 move(r1,loc1,loc2)⟩

```

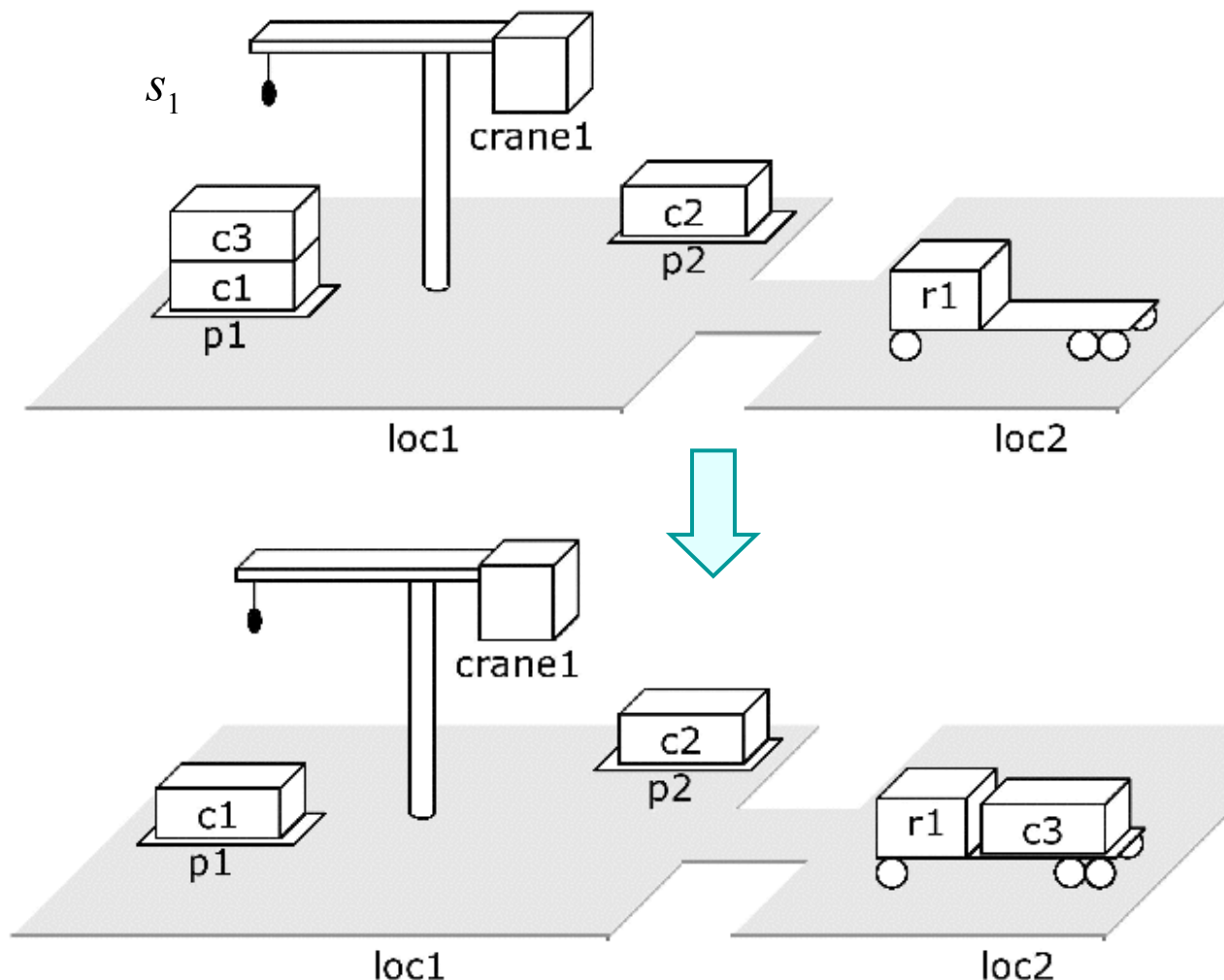
- A solution that is both *irredundant* and *shortest*:

```

⟨move(r1,loc2,loc1), take(crane1,loc1,c3,c1,p1),
 load(crane1,loc1,c3,r1), move(r1,loc1,loc2)⟩

```

- Are there any other shortest solutions? Are irredundant solutions always shortest?

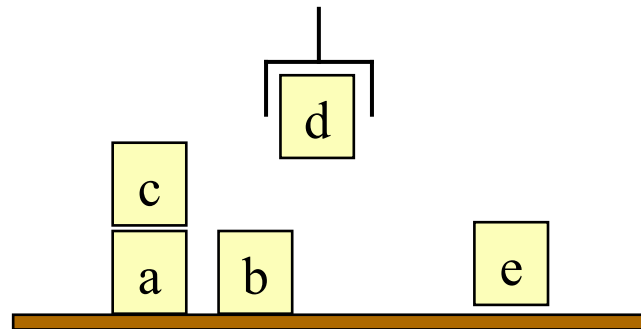


# Exercise

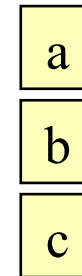


# Exercise: Plans

initial state



goal



- Solution?

# State-Variable Representation

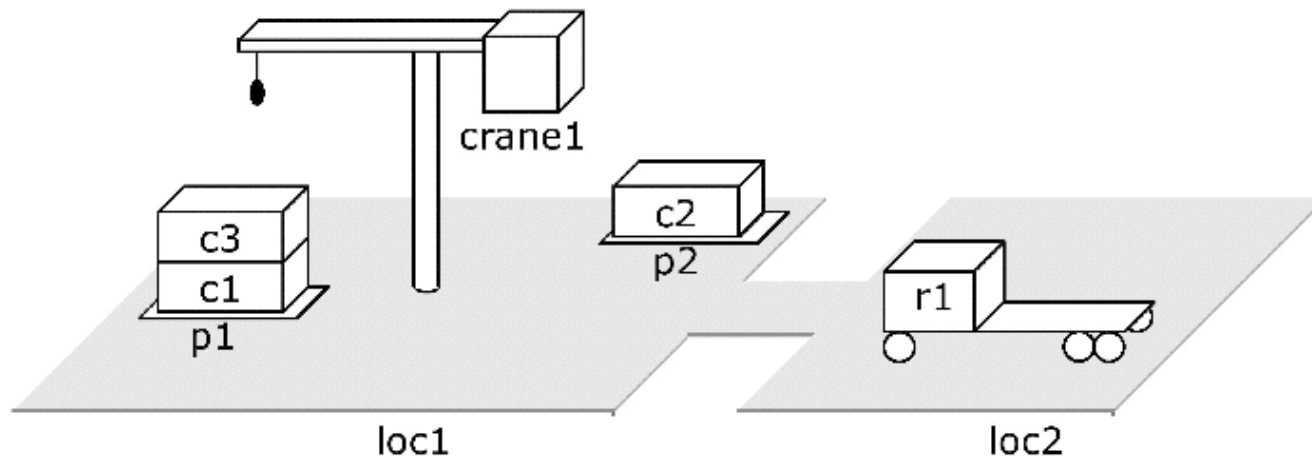
- Use ground atoms for properties that do not change, e.g., `adjacent(loc1,loc2)`
- For properties that can change, assign values to **state variables**
  - Like fields in a record structure
- Classical and state-variable representations take similar amounts of space
  - Each can be translated into the other in low-order polynomial time

`move(r, l, m)`

*;;* robot *r* at location *l* moves to an adjacent location *m*

precond: `rloc(r) = l, adjacent(l, m)`

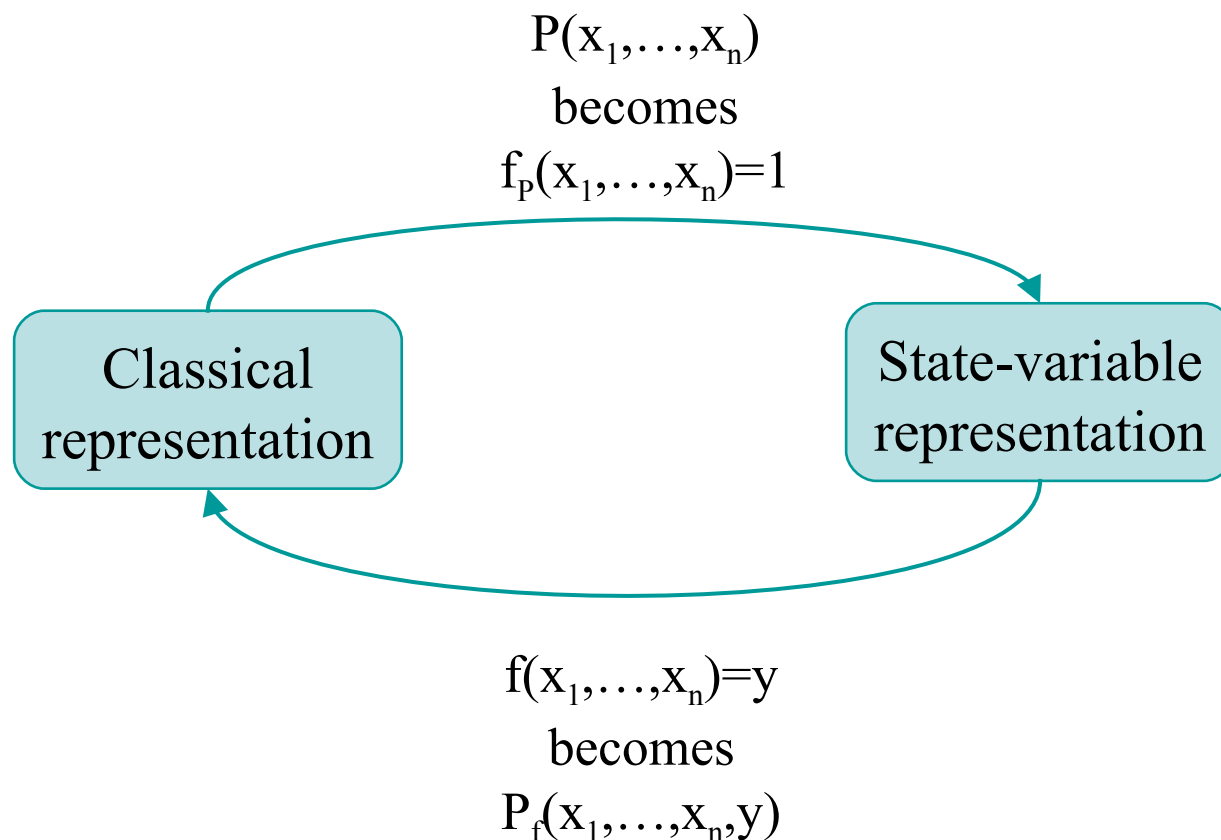
effects: `rloc(r)  $\leftarrow$  m`



$s_1 = \{\text{top}(p1)=c3,$   
 $\text{cpos}(c3)=c1,$   
 $\text{cpo}(c1)=\text{pallet},$   
 $\text{holding}(\text{crane1})=\text{nil},$   
 $\text{rloc}(r1)=\text{loc2},$   
 $\text{loaded}(r1)=\text{nil}, \dots\}$

# Expressive Power

- Any problem that can be represented in one representation can also be represented in the other
- Can convert in linear time and space



# Comparison

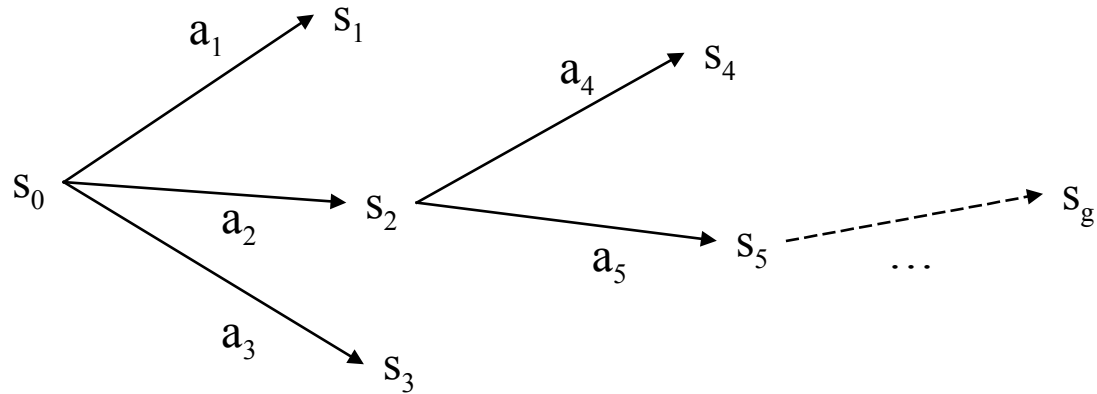
- Classical representation
  - The most popular for classical planning, partly for historical reasons
- State-variable representation
  - Equivalent to classical representation in expressive power
  - Less natural for logicians, more natural for engineers and most computer scientists
  - Useful in non-classical planning problems as a way to handle numbers, functions, time

# State-Space Planning

# Search Algorithms

## Search tree

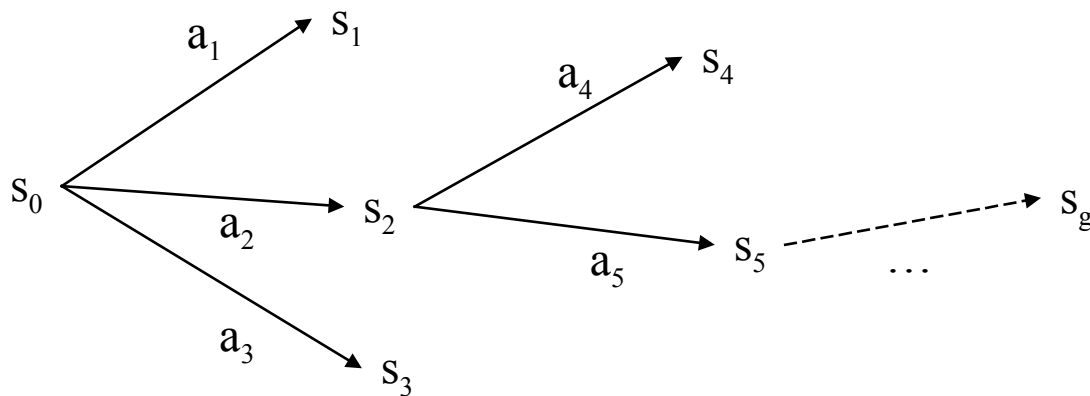
- nodes = states
- edges = actions



# Search Algorithms

## Search tree

- nodes = states
- edges = actions



- **Breadth-first** search is sound and complete
    - But usually not practical because it requires too much memory (exponential in the length of the solution)
  - In practice, more likely to use **depth-first** search
    - Worst-case memory requirement is linear in the length of the solution
    - In general, sound but not complete
      - But classical planning has only finitely many states
- ➡ can make depth-first search complete by doing loop-checking

# Exercise



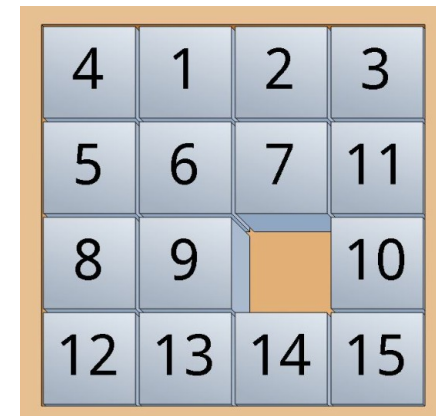
# Exercise: Interchange Values of Variables

- Operator `assign(v,w,x,y)`  
precond: `value(v,x), value(w,y)`  
effects: `¬value(v,x), value(v,y)`
- Initial state  $s_0 = \{ \text{value}(a,3), \text{value}(b,5), \text{value}(c,0) \}$
- Goal  $g = \{ \text{value}(a,5), \text{value}(b,3) \}$
- In the search tree for this planning problem,
  - what is the length of the shortest path to a solution?
  - what is the length of the longest path in the tree?

# Planning with Heuristic Search

- Explicitly search with heuristic  $h(s)$  that estimates cost from  $s$  to goal
- General idea:  
heuristic function = length of optimal plan for a **relaxed problem**

- Examples:
  - Manhattan distance in 15-puzzle
  - Euclidean distance in route finding



- How to get and solve suitable relaxations?
- How to get heuristics automatically?

# Heuristics for Classical Planning

- Automatic extraction of informative heuristic function **from the problem P itself**
- Most common relaxation in planning: **ignore all negative effects** of the operators.

Let  $P^+$  be obtained from planning problem  $P$  by dropping the negative effects.  
If  $c^*(P^+, s)$  is optimal cost of  $P^+$  with initial state  $s$ , then the heuristic is set to

$$h(s) = c^*(P^+, s)$$

- This heuristic is intractable in general, but easy to approximate

## Example.

- Operator `assign(v, w, x, y)`  

```
precond:  value(v, x), value(w, y)
effects:  ¬value(v, x), value(v, y)
```
- $s_0 = \{ \text{value}(a, 3), \text{value}(b, 5), \text{value}(c, 0) \}$ ,  $g = \{ \text{value}(a, 5), \text{value}(b, 3) \}$
- Optimal relaxed plan: `assign(a, b, 3, 5)`, `assign(b, a, 5, 3)`, hence  $h(s_0) = 2$

# Example

- Operator `assign(v,w,x,y)`  
precond: `value(v,x), value(w,y)`  
effects: `¬value(v,x), value(v,y)`
- $g = \{ \text{value}(a,5), \text{value}(b,3) \}$
- $s_0 = \{ \text{value}(a,3), \text{value}(b,5), \text{value}(c,0) \}$

Consider all possible successor states after one action:

$s_1 = \{ \text{value}(a,5), \text{value}(b,5), \text{value}(c,0) \}$

$h(s_1) = \infty$

$s_2 = \{ \text{value}(a,3), \text{value}(b,3), \text{value}(c,0) \}$

$h(s_2) = \infty$

$s_3 = \{ \text{value}(a,0), \text{value}(b,5), \text{value}(c,0) \}$

$h(s_3) = \infty$

$s_4 = \{ \text{value}(a,3), \text{value}(b,5), \text{value}(c,3) \}$

$h(s_4) = 2$

$s_5 = \{ \text{value}(a,3), \text{value}(b,0), \text{value}(c,0) \}$

$h(s_5) = \infty$

$s_6 = \{ \text{value}(a,3), \text{value}(b,5), \text{value}(c,5) \}$

$h(s_6) = 2$

No relaxed plan exists

# Example

- Operator `assign(v,w,x,y)`  
precond: `value(v,x), value(w,y)`  
effects: `¬value(v,x), value(v,y)`
- $g = \{ \text{value}(a,5), \text{value}(b,3) \}$
- $s_4 = \{ \text{value}(a,3), \text{value}(b,5), \text{value}(c,3) \}$

Consider all possible successor states after next action:

$s_7 = \{ \text{value}(a,5), \text{value}(b,5), \text{value}(c,3) \}$        $h(s_1) = 1$

$s_8 = \{ \text{value}(a,3), \text{value}(b,3), \text{value}(c,3) \}$        $h(s_8) = \infty$

$s_9 = \{ \text{value}(a,3), \text{value}(b,5), \text{value}(c,5) \}$        $h(s_9) = 2$

One of the successor states of  $s_7$  is a goal state:

$s_{10} = \{ \text{value}(a,5), \text{value}(b,3), \text{value}(c,3) \}$

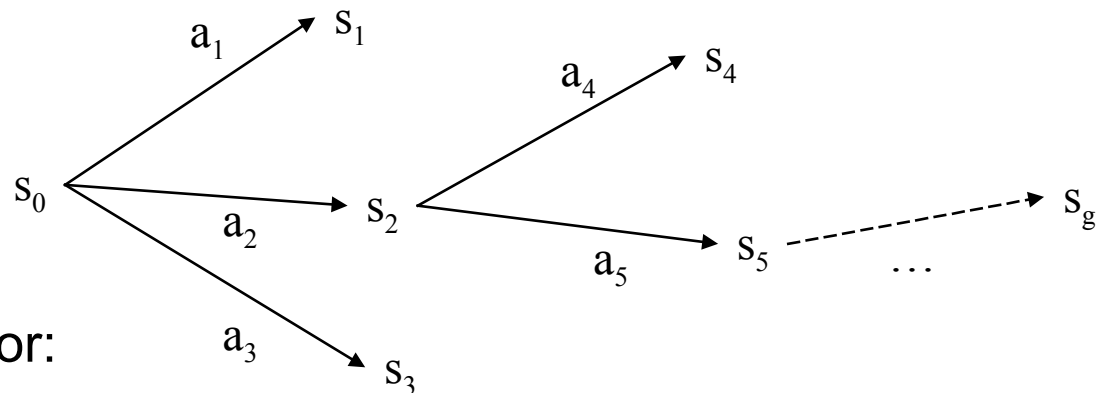
# Planning-Graph Techniques

# History

- Before Graphplan came out, most planning researchers were working on Plan Space Search-like planners
- **Graphplan** caused a sensation because it was so much faster
- Many subsequent planning systems have used ideas from it
  - IPP, STAN, GraphHTN, SGP, Blackbox, Medic, TGP, LPG
  - Many of them even much faster than the original Graphplan

# Motivation

- A big source of inefficiency in search algorithms is the **branching factor** (= the number of children of each node)
- A standard tree search may try lots of actions that are unrelated to the goal



- One way to reduce branching factor:
- First create a **relaxed problem**
  - Remove some restrictions of the original problem  
⇒ Want the relaxed problem to be easy to solve (polynomial time)
  - The solutions to the relaxed problem will include all solutions to the original problem
- Then do a modified version of the original search
  - Restrict its search space to include only those actions that occur in solutions to the relaxed problem



# Graphplan

procedure Graphplan:

- for  $k = 0, 1, 2, \dots$ 
  - *Graph expansion:*
    - ➡ create a “planning graph” that contains  $k$  “levels”
  - Check whether the planning graph satisfies a necessary (but insufficient) condition for plan existence
- If it does, then
  - ➡ do **solution extraction**:
    - backward search, modified to consider only the actions in the planning graph
    - if we find a solution, then return it

relaxed  
problem

# Example: Have the Cake and Eat it Too

- | Operator Name | Preconditions  | Effects                  |
|---------------|----------------|--------------------------|
| eat(c)        | have(c)        | $\neg$ have(c), eaten(c) |
| bake(c)       | $\neg$ have(c) | have(c)                  |
- Also have the maintenance actions: one for each literal
  - $s0 = \{ \text{have}(\text{cake}) \}$
  - $g = \{ \text{have}(\text{cake}), \text{eaten}(\text{cake}) \}$

state-level 0

have(cake)

$\neg$ eaten(cake)

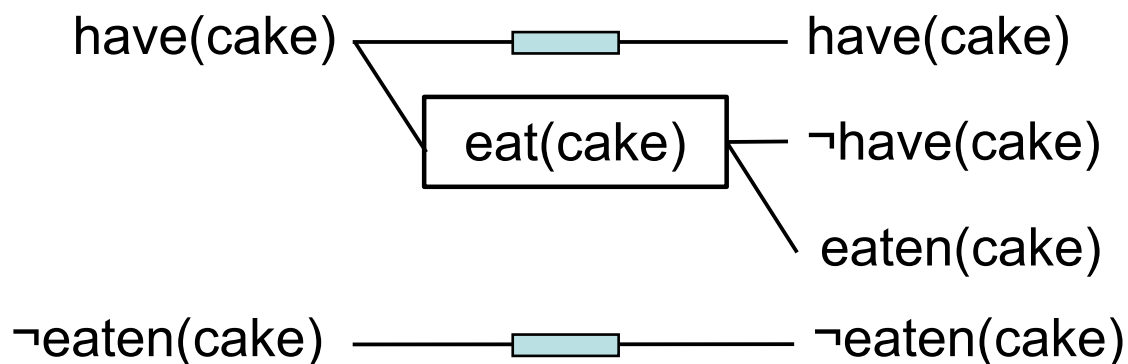
# Example: Have the Cake and Eat it Too

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- $s_0 = \{ \text{have}(\text{cake}) \}$
- $g = \{ \text{have}(\text{cake}), \text{eaten}(\text{cake}) \}$

state-level 0

action-level 1

state-level 1



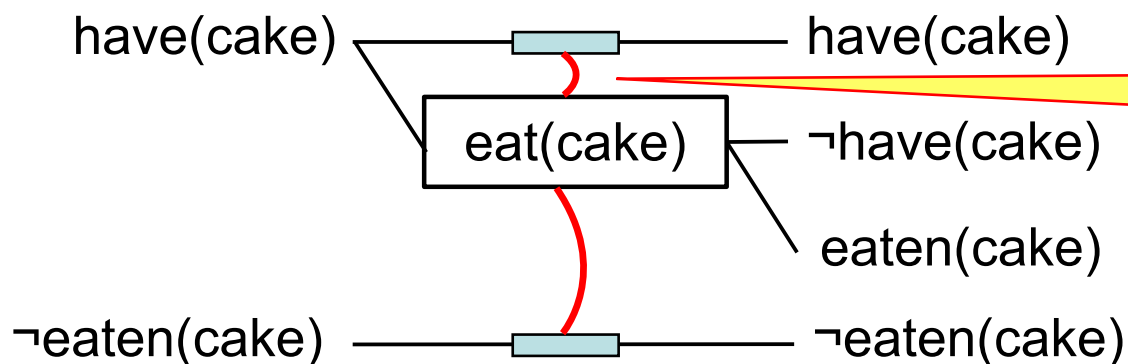
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state-level 0

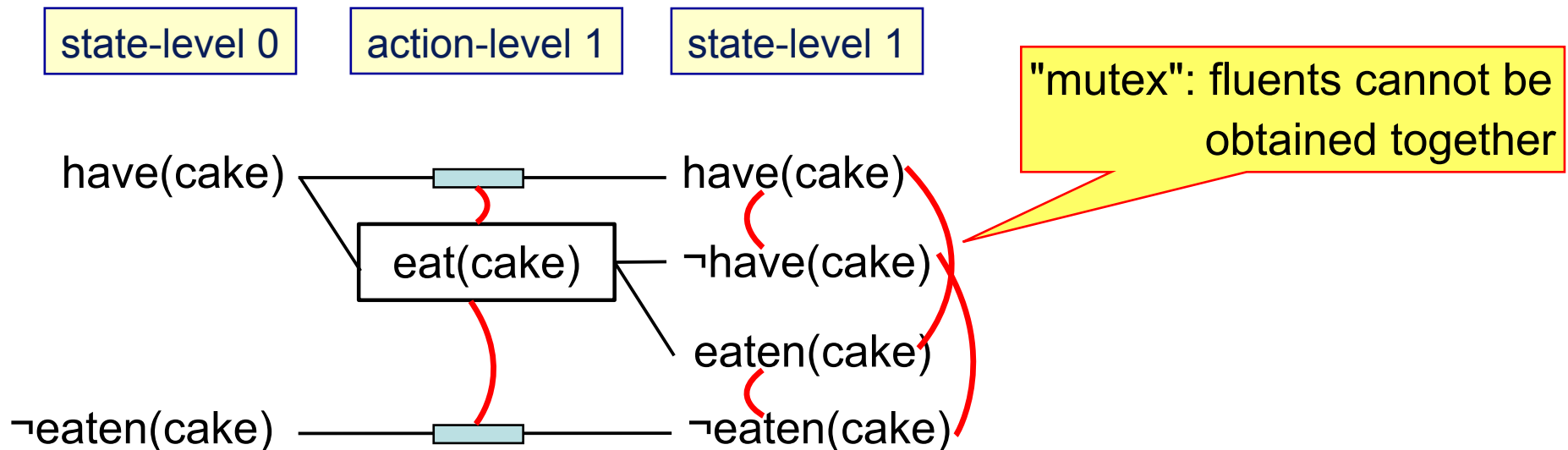
action-level 1

state-level 1



# Example: Have the Cake and Eat it Too

- | Operator Name | Preconditions  | Effects                  |
|---------------|----------------|--------------------------|
| eat(c)        | have(c)        | $\neg$ have(c), eaten(c) |
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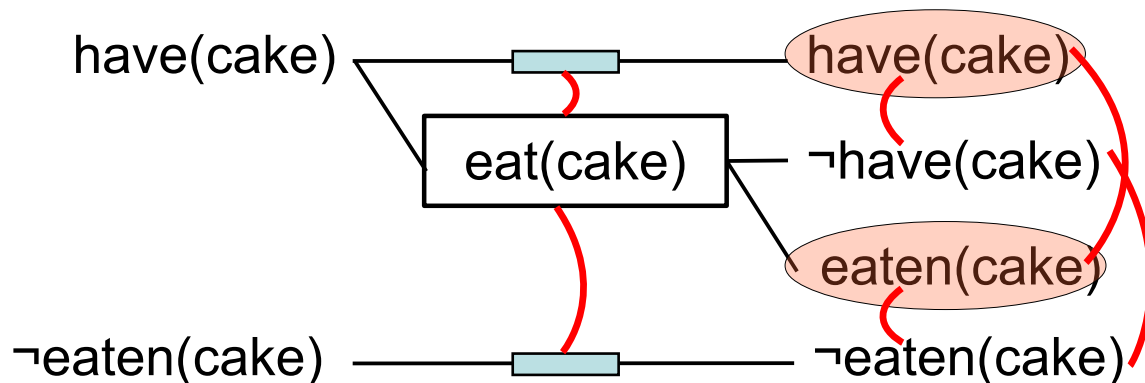
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- Also have the maintenance actions: one for each literal
- $s_0 = \{ \text{have}(\text{cake}) \}$
- $g = \{ \text{have}(\text{cake}), \text{eaten}(\text{cake}) \}$

state-level 0

action-level 1

state-level 1



Solution extraction **not** called since goals are mutex

# Example: Have the Cake and Eat it Too

- | Operator Name | Preconditions  | Effects                  |
|---------------|----------------|--------------------------|
| eat(c)        | have(c)        | $\neg$ have(c), eaten(c) |
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- $g = \{ \text{have}(\text{cake}), \text{eaten}(\text{cake}) \}$

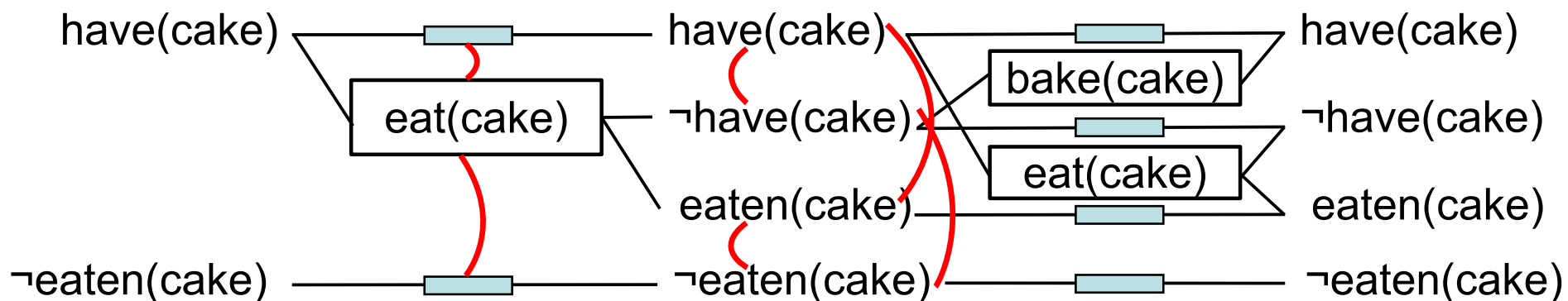
state-level 0

action-level 1

state-level 1

action-level 2

state-level 2



# Example: Have the Cake and Eat it Too

- | Operator Name | Preconditions  | Effects                  |
|---------------|----------------|--------------------------|
| eat(c)        | have(c)        | $\neg$ have(c), eaten(c) |
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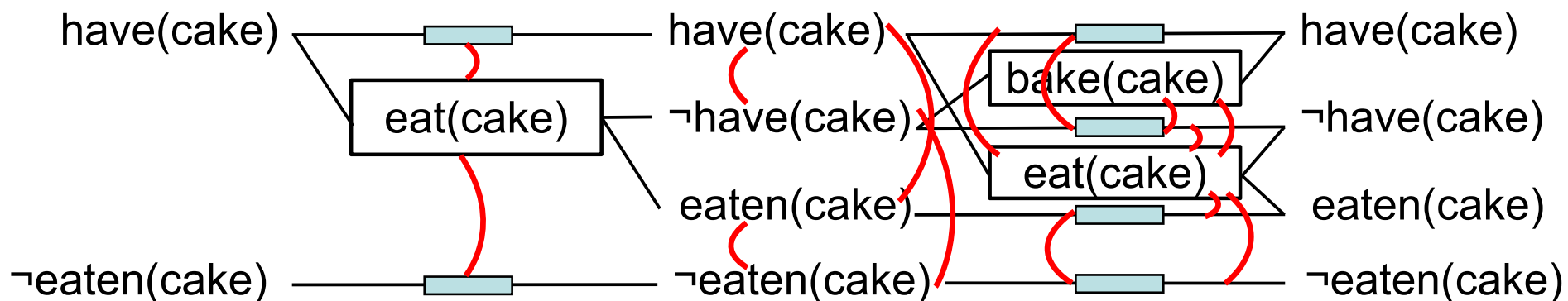
state-level 0

action-level 1

state-level 1

action-level 2

state-level 2





# Example: Have the Cake and Eat it Too

- | Operator Name | Preconditions  | Effects                  |
|---------------|----------------|--------------------------|
| eat(c)        | have(c)        | $\neg$ have(c), eaten(c) |
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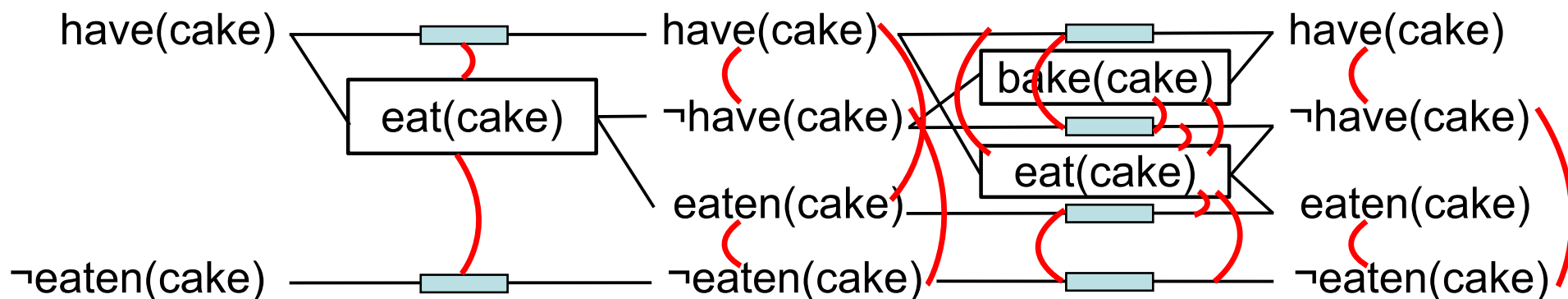
state-level 0

action-level 1

state-level 1

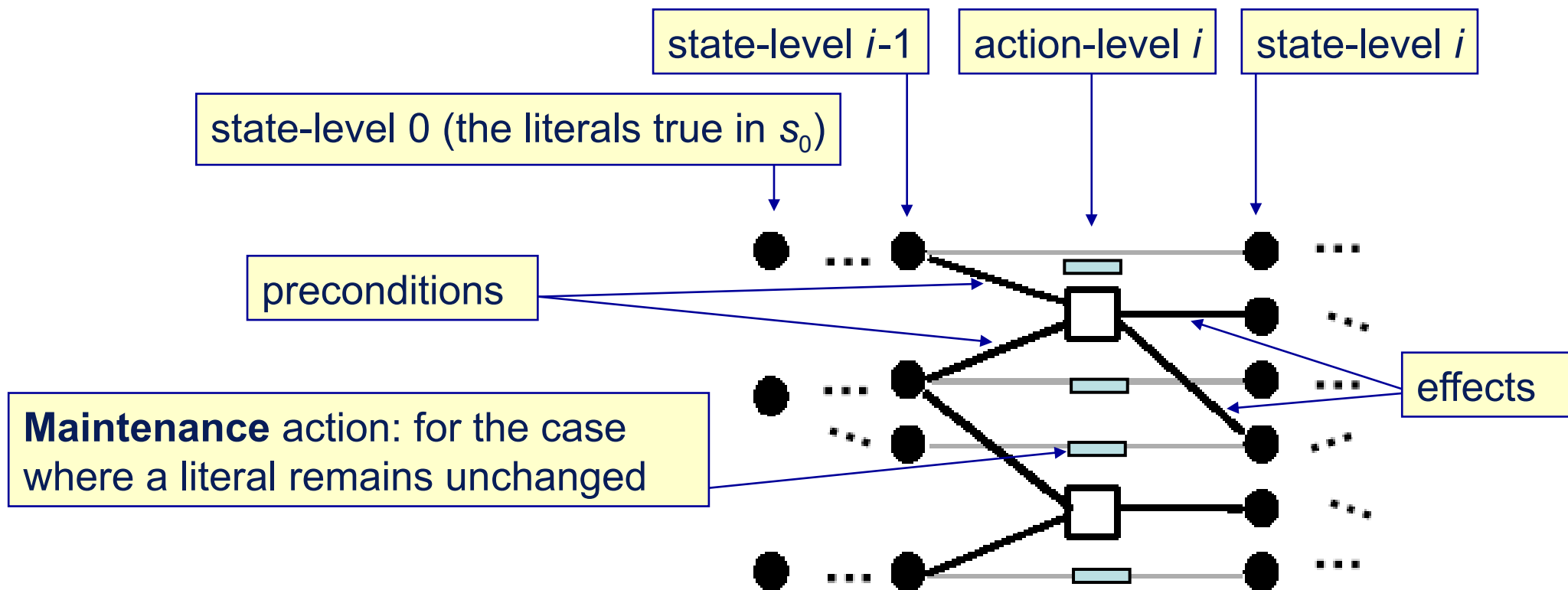
action-level 2

state-level 2

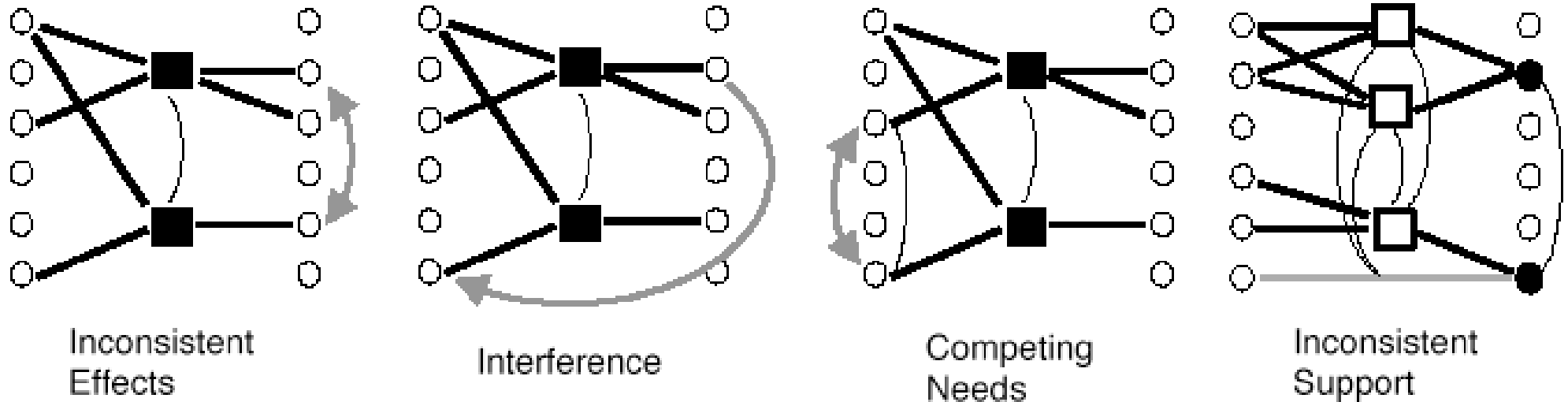


# The Planning Graph

- Search space for a relaxed version of the planning problem
- Alternating layers of ground literals and actions
  - Nodes at action-level  $i$ : actions that might be possible to execute at time  $i$
  - Nodes at state-level  $i$ : literals that might possibly be true at time  $i$
  - Edges: preconditions and effects



# Mutual Exclusion



- Two actions at the same action-level are mutex if
  1. **Inconsistent effects:** an effect of one negates an effect of the other
  2. **Interference:** one deletes a precondition of the other
  3. **Competing needs:** **they have mutually exclusive preconditions**
- Otherwise they don't interfere with each other
  - Both may appear in a solution plan
- Two literals at the same state-level are mutex if
  4. **Inconsistent support:** one is the negation of the other, **or all ways of achieving them are pairwise mutex**

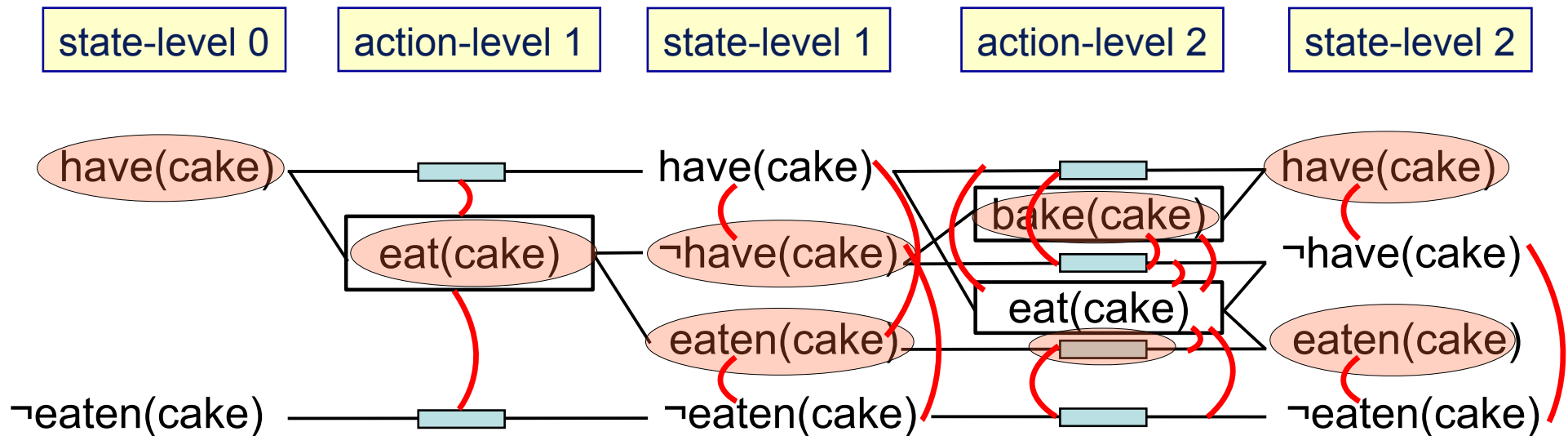
Recursive  
propagation  
of mutexes

# Mutexes in the Cake-Example

Level	Mutexes		Rule
A1	<i>eat(cake)</i>	$m_{have(cake)}$	1 (also 2)
A1	<i>eat(cake)</i>	$m_{\neg eaten(cake)}$	1, 2
S1	<i>have(cake)</i>	$\neg have(cake)$	4
S1	<i>eaten(cake)</i>	$\neg eaten(cake)$	4
S1	<i>have(cake)</i>	<i>eaten(cake)</i>	4
S1	$\neg have(cake)$	$\neg eaten(cake)$	4
A2	<i>bake(cake)</i>	<i>eat(cake)</i>	1, 3
A2	<i>bake(cake)</i>	$m_{\neg have(cake)}$	1, 2
A2	<i>eat(cake)</i>	$m_{have(cake)}$	1, 2
A2	<i>eat(cake)</i>	$m_{\neg have(cake)}$	3
A2	<i>eat(cake)</i>	$m_{eaten(cake)}$	<b>3</b>
A2	<i>eat(cake)</i>	$m_{\neg eaten(cake)}$	1, 2
A2	$m_{have(cake)}$	$m_{\neg have(cake)}$	1, 2, 3
A2	$m_{eaten(cake)}$	$m_{\neg eaten(cake)}$	1, 2, 3
S2	<i>have(cake)</i>	$\neg have(cake)$	4
S2	<i>eaten(cake)</i>	$\neg eaten(cake)$	4
S2	$\neg have(cake)$	$\neg eaten(cake)$	4

# Example: Have the Cake and Eat it Too

Solution extraction **succeeds**  
( = plan without mutexes)



# Solution Extraction

The set of goals we are trying to achieve

The level of the state  $s_j$

procedure Solution-extraction( $g, j$ )

if  $j = 0$  then return the solution

for each literal  $l$  in  $g$

nondeterministically choose an action

to use in state  $s_{j-1}$  to achieve  $l$

if any pair of chosen actions are mutex

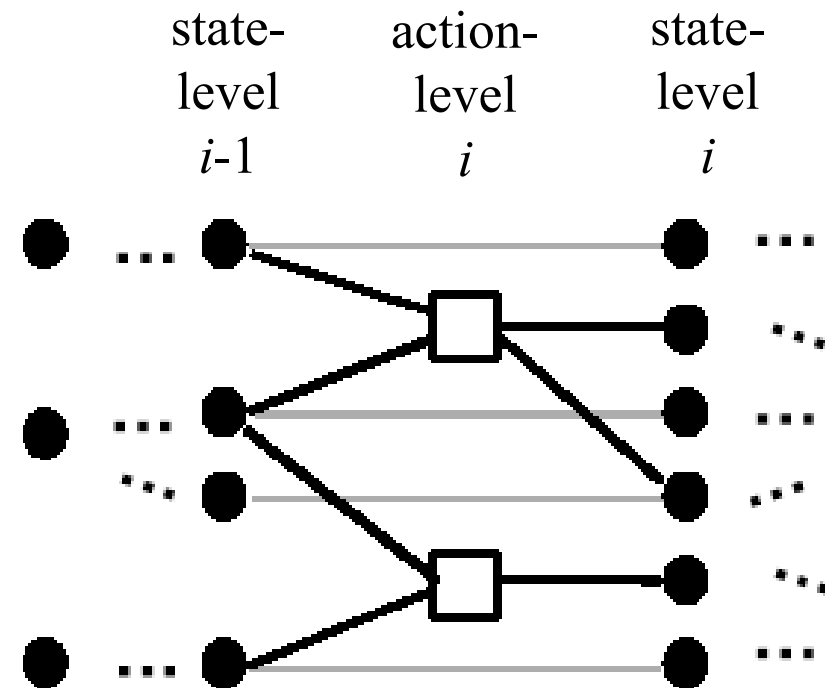
then backtrack

$g' := \{\text{the preconditions of the chosen actions}\}$

Solution-extraction( $g', j-1$ )

end Solution-extraction

A real action or a maintenance action



# Comparison with State-Space Planning

- Advantage:
  - The backward-search part (solution extraction) of Graphplan—which is the hard part—will only look at the actions in the planning graph
  - smaller search space than state-space planning; thus faster
- Disadvantage:
  - To generate the planning graph, Graphplan creates a huge number of ground atoms
  - Many of them may be irrelevant
- For classical planning, the advantage outweighs the disadvantage
  - GraphPlan solves classical planning problems much faster than SSP without heuristics

# Summary

- Representations for classical planning
  - Classical representation
  - State-variable representation
- State-space planning
  - with heuristics
- Planning graphs
  - Creating the graph
  - Adding mutexes
  - Searching the graph