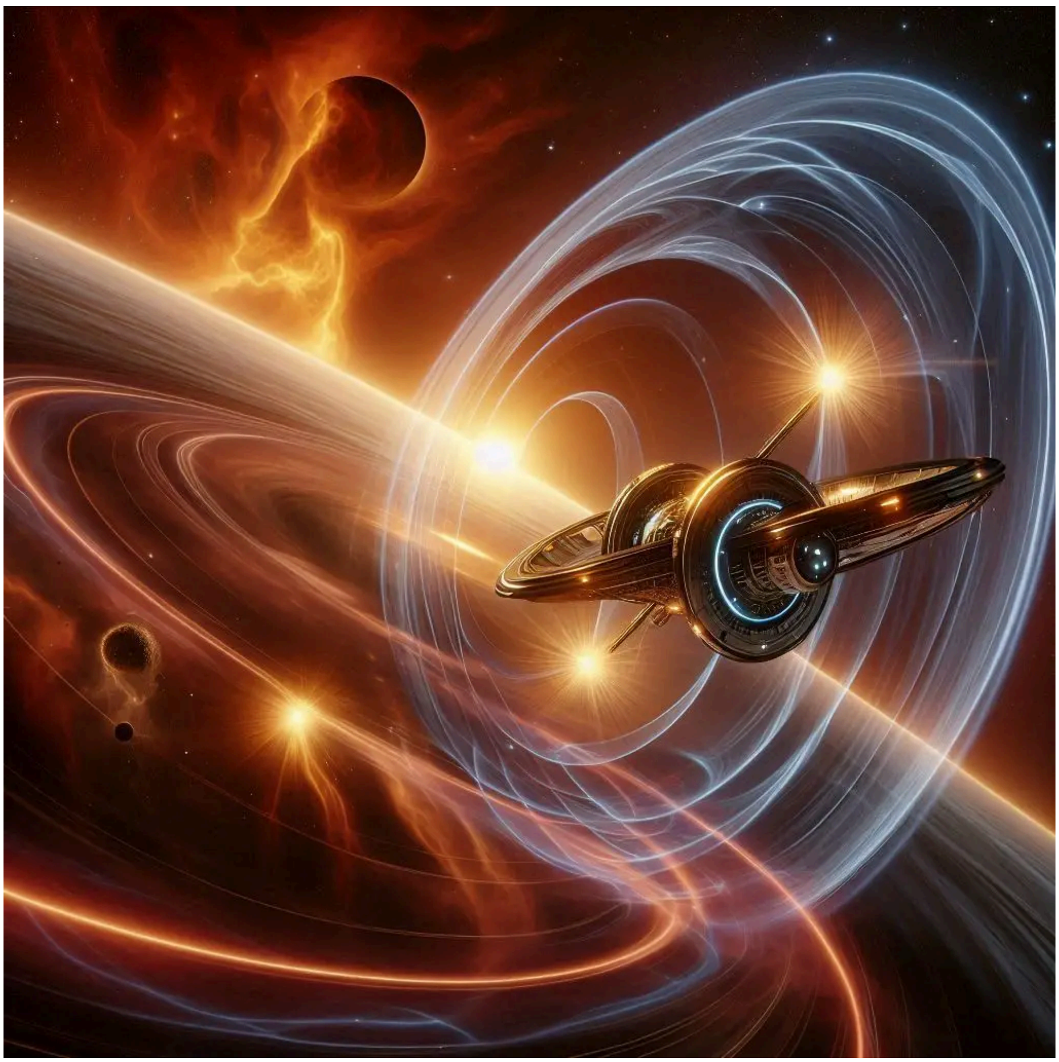
Black holes & Plasma Physics(Python)

In Cheng's book Intro to Plasma Physics, I thought it could ovalize or reshape the structure inside a soft, cold black hole coming from a small star. If you fall in, gravity stretches you and breaks you into molecules that gravity itself stretches. If this is correct, then the solution would be to stretch or reshape the plasma of a propulsion system in order to manipulate the structures or metrics of an object that could propel itself and survive a phenomenon similar to a black hole. It is possible that there are local themes or smaller, colder local black holes whose gravity cannot kill you or stretch or destroy matter. The question is where to find them. There is talk of white holes or stars that I thought that if they collapse when they die and are smaller, they may not finish collapsing, forming magnetic fields and even deuterium-type ice, and then these less serious or less risky scenarios could change the future of gravity or series of humans being able to handle these structures and become another type of civilization. I want to put Riemann metrics inside the plasma structures so that it is the black hole piece itself that combines with the plasma.



There exists a duality relation between a closed conformal Killing–Yano tensor and a Killing Yano tensor. To obtain this relation we use the following properties of the totally skew symmetric tensor eμ1μ2μ3μ4 eν1ν2ν3ν4 =−24δ[ν1 μ1 δν2 μ2 δν3 eμ1μ2μ3λeν1ν2ν3λ =−6δ[ν1 μ1 δν2 μ3 δν4] μ4 , μ2 δν3] μ3 , eμ1μ2 λeν1ν2 λ =−4δ[ν1 μ1 δν2] μ2 , eμ1π λeν1π λ =−6δν1 μ1 , εσπ λεσπ λ=−24.

Idea: introducing the waves & perturbations into the pg μν metric to see if certain plasma shapes adapt to the environment and generate less resistance to local curvature.

1) ABSTRACT

Black hole: first hidden symmetries in the 4D spacetime. Brief remarks on the hidden symmetries in the higher dimensions are given in Sections D.9 and D.10. Additional material can be found in the review article (Frolov and Kubizˇn´ak 2008). By definition, a spacetime has a hidden symmetry if the geodesic equations possess conserved quantities higher than the first order in momentum. The geometric structure responsible for such a conservation law is the Killing tensor. We define first a conformal Killing tensor. This is a symmetric tensor Kμ1...μp of rank p that obeys the equation K(μ1...μp;ν) = gν(μ1 ˜ Kμ2...μp)

˜ Kμ2...μp is a symmetric tensor of rank p − 1. For null geodesics xμ(λ), where λ is the affine parameter, in a spacetime with the conformal Killing tensor the following quantity is conserved Kμ1...μp uμ1 ...uμp , uμ = dxμ dλ .

A **Killing tensor** is a natural symmetric generalization of the Killing vector. There exists also an antisymmetric generalization, known as a Killing–Yano tensor. We introduce first a conformal Killing–Yano tensor. This is an anti–symmetric tensor hμ1μ2...μp of the rank p (p-form) that obeys the equation ∇μhμ1μ2...μp =∇[μhμ1μ2...μp] + pgμ[μ1 ˜ hμ2...μp]

• kμ1μ2...μp uμp is parallelly propagated along a geodesic with a tangent vector uμp. • Kμν =(k•k)μν ≡ kμμ2...p k μ2...μp ν is a Killing tensor:
Malleable plasma influenced by the Riemann metric and its shape could modify how it interacts with gravity.

"Soft" or local black holes, less extreme than the known ones and the possibility of passing through them without destroying matter.

A ship/structure that self-adjusts to the curvature environment (like an "animal" that adapts to the local gravitational field)

For a simple initial model, we use a modified Schwarzschild metric:
*ds*2=−(1−2*GMc*2*r*)*c*2*dt*2+(1−2*GMc*2*r*)−1*dr*2+*r*2*d*Ω2ds2 =−(1−c2 r2GM)c2 dt2 +(1−c2

r2GM)−1 dr2 +r2dΩ2

But if you suggest that the black hole didn't completely collapse, we could assume a modified metric with a curvature-smoothing term at a certain scale:

$ds2=-(1-2GMc2(r+\epsilon))c2dt2+$...ds2=−(1−c2(r+ϵ)2GM)c2dt2+...
(This simulates less extreme curvature at small scales)

I propose a new fusion model between the plasma equations (Cheng) and Kruskal-Szekeres (Frolov) space-time metrics to create a “propulsive vacuum,” a kind of environment where the plasma not only behaves as a charged fluid, but as an adaptive geometry that interacts with the curvature of space-time.

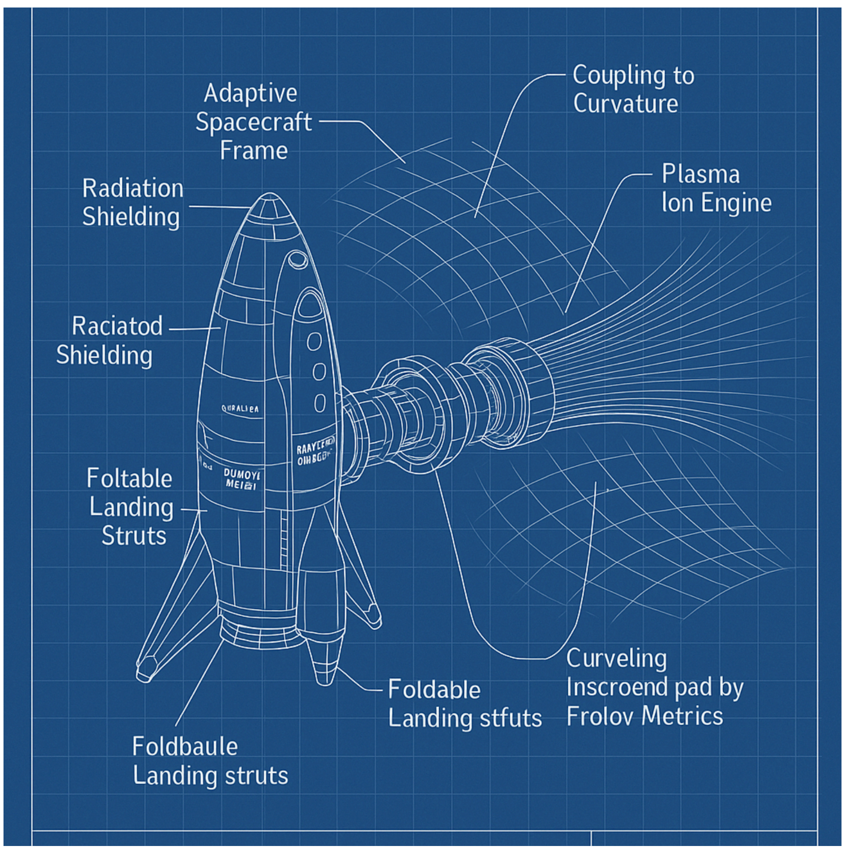
The variables combine some parameters called: ϱ ϱ: plasma energy density $u \mu$ and μ : 4-vector velocity $F \mu \nu$ F μn : electromagnetic field tensor:

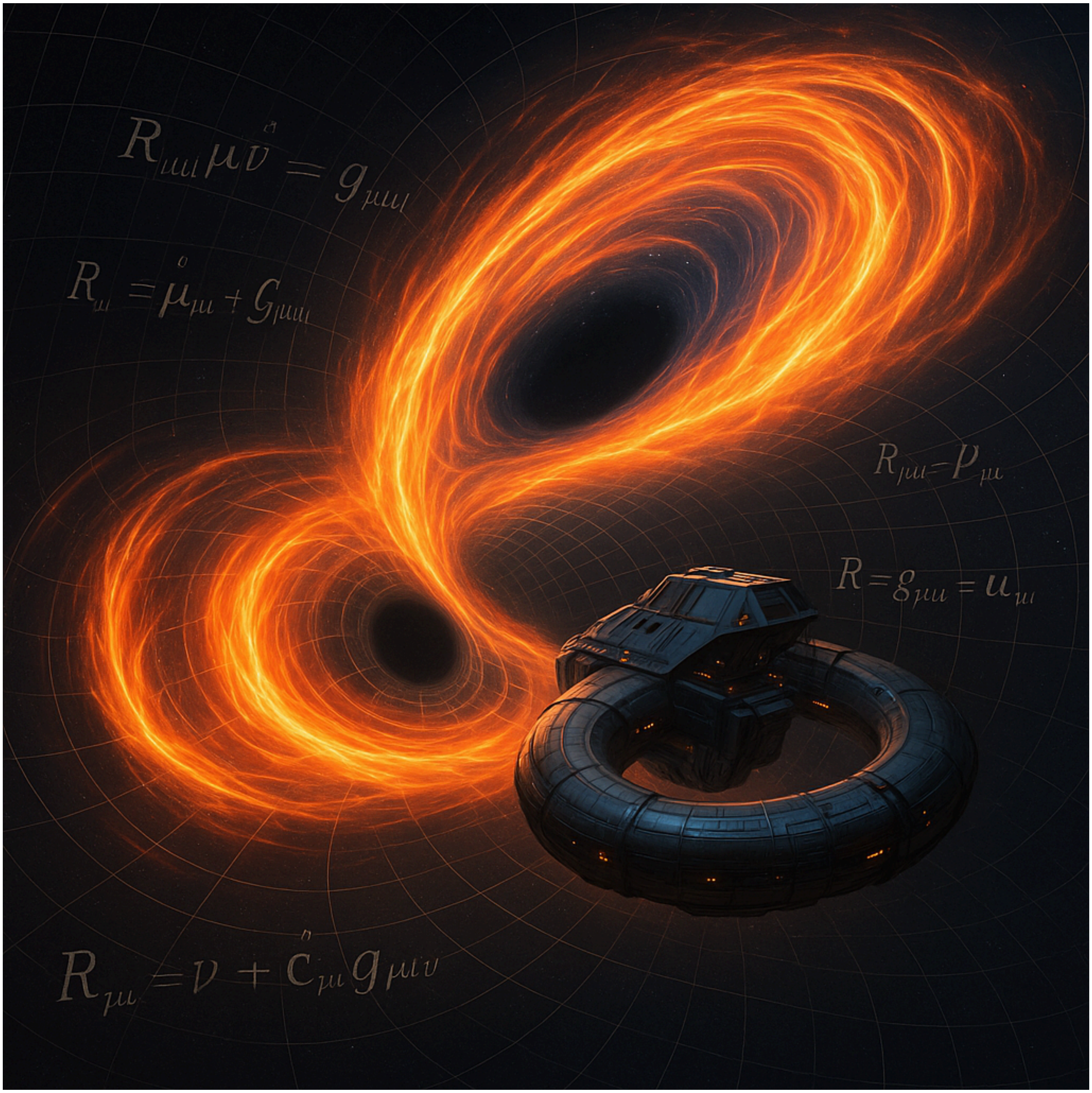
A spacecraft thruster that:
Injects plasma into a region with a variable magnetic field and modulates the magnetic field and the shape of the confinement to adjust the local geometry of the plasma. The spacecraft exploits local curvature to turn the plasma into a kind of "gravitational tuner" that adjusts the object's effective mass in space-time.

1. Primordial black holes (PBHs) from the Big Bang: These could have a small mass, a lower temperature, and would not immediately destroy matter.

2. Quark stars or cold Q-balls: Dense structures that have not completely collapsed, with strong fields but no event horizon.

3. Cold boson clusters or self-contained dark matter: These are still hypothetical, but could have a dense, localizable gravitational field.
4. Example





The inner Variables and metrics in black holes are:

∇[μhμ1μ2...μp] = 0, (D.2.4) the conformal Killing–Yano tensor is closed. Such a p-form h can be written (at least locally) as h =db. (D.2.5) The (p −1)-form b is called a potential-generating closed conformal Killing–Yano tensor. Consider a rank-2 skew-symmetric tensor hμν in a four-dimensional spacetime obeying the equation hμν;λ = gλνξμ −μην. The contraction of this relation gives This tensor obeys the relations ξμ = 1 3 h λ μ ;λ. ∇(λhμ)ν = gν(λξμ) − λμξ, ∇[λhμν] = 0, (D.2.6) (D.2.7) (D.2.8)

We define the dielectric tensor of a plasma as follows.

References: The fourth Maxwell equation is ∇ B¼μ0 jþE0 _ E ðB:1Þ where j is the plasma current due to the motion of the various charged particle species s, with density ns, charge qs, and velocity vs: X j ¼ nsqsvs s ðB:2Þ Considering the plasma to be a dielectric with internal currents j, we may write Eq. (B.1)as ∇ B¼μ0 _ D where D¼E0Eþ i ω j ðB:3Þ

Magnetohydrodynamics (MHD) equations

They are most useful if the plasma has a macroscopic structure and there is a B field:

1. *∂v∂t*+(*v*·∇)*v*=−1*ϱ*∇*p*+1*ϱ*(*J*×*B*)+*g*∂t∂v+(v·∇)v=−ϱ1∇p+ϱ1(J×B)+g
2. *∂B∂t*=∇×(*v*×*B*) ∂t ∂b =∇×(v ×b) ∇ · *B* =0, *J*=1*μ*0∇×*B*∇·b =0,J. =μ0 1∇×b

- V/J : plasma velocity*J*
- J: current *B*

- B: magnetic field*g*
- →g: external gravitational field

p

- p: pressure, combine *g* g or ∇*p*
- ∇p with spatial curvature.

Kruskal-Szekeres (Frolov) metric: This metric extends the Schwarzschild metric to eliminate degenerate coordinates at the horizon. It is practical in computing & physics for modeling flows of matter or energy crossing inward or outward from the black hole.

Computational-equation: ds^2 = -\frac{32G^3M^3}{c^6r} e^{-r/2GM} (-dT^2 + dX^2) + r^2 d\Omega^2

Plasma in a curved geometry
The trick is to couple the electric, magnetic, and pressure fields of the plasma to spacetime. This is done with the energy-momentum tensor

Variables:
T
μ
ν
T
μn,

which enters directly into Einstein's equation and thus the effective dielectric constant of the plasma is the tensor e¼E0 Iþiσ=E0ω ð Þ ð:7Þ where the unit tensor. Toevaluateσ,we use the linearized fluidequationofmotionfor species s, neglecting the collision and pressure terms: ms ∂vs ∂t ¼qsEþvs B0 ð Þ ð:8Þ

(ϱ + p) u^μ u^ν + p g^μν : Perfect fluid component (internal energy and pressure) + F^να − (1/4) g^μν F^αβ F_αβ : Electromagnetic energy-momentum tensor (electric and magnetic fields) + T^μν_cin and 'titic : Kinetic contributions of individual particles

Plasma bubble within an adaptive metric
This bubble would have the following characteristics:

Inside: the plasma follows relativistic MHD equations.the plasma tensor

T
μ
ν
T

Mn modifies the local metric, creating a directed or curved "vacuum pocket." The curvature is calculated using Einstein's equations and is influenced by the dynamic shapes of the plasma, as if it were an intelligent fluid. The question is how do we configure plasma propulsion and metrics of a black hole and what type of object could perform manipulations or something locally like a ship or something similar? How do I filter that energy or those curves in a ship and not destroy it in the case that that gravity is not as strong as that of the typical black hole?

You generate a magnetically stabilized plasma bubble (torus, spherical shell, or toroidal shell) (confined MHD). The plasma's electromagnetic energy locally modifies the curvature of space-time. According to Einstein: G^μν = (8πG / c^4) T^μν ...and if

T
μ
ν
plasma
T
μn
Plasma, that has certain symmetries or anisotropies, it can create regions with less curvature "ahead" than behind thus generating metric gradients that "push."

1) The ship moves like a bubble within its own manipulated geometry, without being destroyed.

WHAT IF GRAVITY ISN'T SO INTENSE?

In these types of environments, we can try to better understand the conditions for critical space: De Sitter or gentle Reissner–Nordström curvatures (no immediate horizon).

1. Non-extreme electromagnetic fields and energy-momentum tensors.
2. Plasma can adapt and not collapse due to tidal forces.

1. Curved Space-Time] Warp
2. Controlled External Plasma as a Propulsion Engine
3. Electromagnetic Cortex + Shielding
4. [Stable Hollow Interior + Ship/Cargo]

It is theoretically conceivable to design an object capable of adapting to the plasma configurations and local space-time structures around black holes using them not merely as abstract cosmological features, but as viable components in future energy extraction systems. This object may require dynamic or elastic qualities, enabling it to stretch or conform to magnetically confined plasma geometries such as toroidal bubbles or shells.

To extract energy from such systems, we would need a coupling between gravitational softness and plasma responsiveness, an interplay allowing viable propulsion mechanisms and scalable energy conversion. This would likely involve advanced magnetohydrodynamic control, superconducting containment structures, and dynamic adjustment to relativistic electromagnetic feedback. An intermediate gravity, strong enough to warp space-time and generate extractable energy, but not so strong as to destroy:

1. Less than that of a stellar black hole.

2. Greater than that of a planet.

3. Comparable to that of a white star core or dense exoplanet.

Conditions for how gravity stretches objects in local regions of outer space: | Requirement | Explanation Gravity less than a classical black hole
To avoid spaghettification Stable region with exploitable curved field To be able to extract energy and not collapse Exploitable curvature area Space must allow local manipulation of metrics and plasma.

Variables and section references(Frolov metrics)

$h\mu ;\lambda;\nu - h\mu ;\nu;\lambda = h\alpha R\alpha\mu\lambda\nu + h\mu\alpha R\alpha\ \lambda\nu$. By multiplying this relation by $g\ \lambda$ we get $h\mu\lambda ;\lambda ;\nu - h\mu\lambda;\nu ;\lambda = hr\ \alpha\mu\lambda\nu - hR$. Consider a vector $\xi\mu$ defined by Eq. (D.2.7). For this vector $\xi\mu;\nu = 1\ 3$ and Eq. (D.3.4) implies $h\mu\lambda ;\lambda ;\nu, 2\xi\mu;\nu + g\mu\nu\ \xi\lambda ;\lambda = hr\ \alpha\mu\lambda\nu - hR$. Contraction of this equation gives $\xi ;\lambda\ \lambda =0$. After symmetrization of the relation Eq. (D.3.6) we get $\xi(\mu;\nu) =-1\ 2R\alpha\ (\nu h\mu)\alpha$. (D.3.2) (D.3.3) (D.3.4) (D.3.5) (D.3.6) (D.3.7) (D.3.8)

Einstein Field Equation (curvature):

G_{μν} = (8πG / c^4) · T_{μν}

Where:
- G_{μν} is spacetime curvature
- T_{μν} is plasma stress-energy tensor

Stress-energy of relativistic plasma:

T^{μν} = (ϱ + p)u^μu^ν + p g^{μν} + (F^{μλ}F^ν_λ - ¼g^{μν}F^2)

Plasma propulsion via magnetic field gradient:

$\partial B/\partial t = \nabla \times (v \times B)$

Combined with curvature resistance tensor modulation.

Some advanced materials are already in use or being explored for technologies involving plasma control and gravitational dynamics. Graphene aerogel is notable for being extremely light, heat-resistant, and a great conductor. YBCO superconductors are able to produce intense magnetic fields and are used in plasma confinement systems like tokamaks. Researchers have also created diamond-sapphire composites that combine strength and electromagnetic transparency, although they're still experimental. For protecting against heat and impact, carbon-carbon nanofoam has proven effective, especially in space capsules. Mercury-based liquid metals are being tested to allow structural flexibility and adaptive shaping. As for fusion fuel, tritium combined with deuterium remains a key ingredient in nuclear energy experiments.

Toroidal hull structure:
  Carbon-carbon nanofoam (lightweight & thermal shielding)
  Liquid metal lattice (mercury-alloy for dynamic shape tuning)

Containment and control:
  YBCO superconductors (magnetic confinement)
  Graphene aerogel (for EM transparency and lightweight rigidity)

Plasma interaction:
  Deuterium-Tritium mix (initial plasma fuel)
  Boron-carbon shell (to isolate fusion side-reactions)
  Sapphire-diamond composite viewports (transparent to EM, durable)

Orbital Assembly Base
  Constructed in L2 or deep lunar orbit
  Protected against gravitational lensing and radiation

Drone deployment
  Self-adaptive plasma pods to test local curvature
  Adjust navigation algorithms

GAPV-1 deployment
  Full-scale jump-capable vehicle

```
BEGIN MODULE GAPV_MAIN

  // --------------------------
  // [1] INITIALIZE SYSTEM DATA
  // --------------------------
  DEFINE PlasmaShellState AS STRUCT:
      float temperature
      float pressure
      Vector3 velocity
      Tensor EM_Field_Tensor
      Tensor Stress_Energy_Tensor
  END STRUCT

  DEFINE MetricField AS STRUCT:
      Tensor g_mu_nu         // Local spacetime metric
      Tensor RiemannTensor   // Curvature tensor
      float gravityGradient  // dΦ/dr
  END STRUCT
```

```
DEFINE Spacecraft AS STRUCT:
    PlasmaShellState plasma
    MetricField localMetric
    float structuralIntegrity
    Function AdaptiveControl()
    Function CurvatureNavigation()
END STRUCT

// -------------------------
// [2] PLASMA-METRIC COUPLING
// -------------------------
FUNCTION AdaptiveControl(plasma, metric):
    UPDATE plasma.Stress_Energy_Tensor USING:
        T^{μν} = (ρ + p)u^μu^ν + p g^{μν} + F^{μλ}F^ν_λ -
(1/4)g^{μν}F²

    IF metric.gravityGradient > threshold THEN
        ALTER plasma.geometry TO elongate along minimal
curvature axis
        ENHANCE EM_Field_Tensor FOR ADDITIONAL CONFINEMENT
    END IF

    RETURN plasma
END FUNCTION

// -------------------------
// [3] NAVIGATION VIA CURVATURE
// -------------------------
FUNCTION CurvatureNavigation(plasma, metric):
    COMPUTE ∇_μ T^{μν} = 0   // Energy-momentum conservation
    IF plasma deformation produces directional anisotropy
THEN
        COMPUTE net_force_direction FROM gradient(T^{μν})
        SET trajectory_vector TO follow
geodesic(metric.g_mu_nu, net_force_direction)
    END IF

    RETURN trajectory_vector
END FUNCTION

// -------------------------
// [4] SIMULATION LOOP
// -------------------------
FUNCTION MainSimulationLoop():
    INITIALIZE GAPV AS Spacecraft
```

```
      LOOP WHILE GAPV.structuralIntegrity > safe_limit:
          GAPV.plasma = AdaptiveControl(GAPV.plasma,
GAPV.localMetric)
          trajectory = CurvatureNavigation(GAPV.plasma,
GAPV.localMetric)

          UPDATE GAPV.position ALONG trajectory
          MONITOR plasma.temperature AND curvature feedback

          IF plasma unstable THEN
              EMIT WARNING "PLASMA DECOUPLING DETECTED"
              ACTIVATE CONTAINMENT FIELD BOOST
          END IF
      END LOOP

      OUTPUT "SIMULATION COMPLETE: Final Position Reached"
  END FUNCTION

  CALL MainSimulationLoop()

END MODULE
```

```
NOTES:
- This pseudocode models the logic of a spacecraft using
plasma-metric
  feedback to navigate gravitational curvature safely.
- Real implementation would require tensor calculus and general
relativity
  solvers in symbolic or numerical frameworks (e.g., SymPy,
MATLAB, C++)
```

$\xi^\alpha h_{\mu\nu;\alpha} = 0$. Proof: Using Eq. (D.2.17) we get $\xi^\alpha h_{\mu\nu;\alpha} = -2\xi_{[\mu}\xi_{\nu]} = 0$. Wedenote $\zeta_\alpha = -h_{\alpha\beta}\xi^\beta$. This vector obeys the property $\zeta_\mu\xi^\mu = 0$. Lemma7: Thefollowing relation is valid: $\zeta_\mu =$ 1 4 Proof: One has $(h_{\alpha\beta}h^{\alpha\beta})_{;\mu}$. $(h_{\alpha\beta}h^{\alpha\beta})_{;\mu} = 2h^{\alpha\beta}h_{\alpha\beta;\mu} = 2h^{\alpha\beta}(g_{\mu\beta}\xi_\alpha - g_{\mu\alpha}\xi_\beta) = 4\zeta_\mu$. (D.4.1) (D.4.2) (D.4.3) (D.4.4) Lemma8: Let ξ beaprimaryKilling vector for the closed conformal Killing–Yano tensor h. Then, the following relation is valid: $L_\xi h_{\mu\nu} = 0$. Proof: Using the definition of the Lie derivative we have $L_\xi h_{\mu\nu} = \xi^\alpha h_{\mu\nu;\alpha} + \xi$;α μ hαν +ξ ;α Wealso have $\xi_{\mu;\alpha}h^\alpha{}_\nu = -\xi_{\alpha;\mu}h^\alpha{}_\nu = -(\xi_\alpha h^\alpha{}_\nu)_{;\mu} + \xi_\alpha h^\alpha{}_{\nu;\mu} = g_{\mu\nu}\xi^2 - \zeta_{\nu;\mu} - \xi_\mu\xi_\nu$. Using Eq. (D.4.1) we obtain $L_\xi h_{\mu\nu} = 2\zeta_{[\mu;\nu]}$. Finally, we use the relation Eq. (D.4.3) to prove Eq. (D.4.5). D.5 Secondary Killing Vector (D.4.5) ν hμα. (D.4.6) (D.4.7) (D.4.8) Let $h_{\mu\nu}$ be a closed conformal Killing–Yano tensor and $k_{\mu\nu}$ is its Hodge dual Killing–Yano tensor. Then, the Killing tensor associated with μν Mν = μ λλ ν (D.5.1)

References-Plasma: The dielectric tensor of a plasma as follows. The fourth Maxwell equation is ∇ B¼μ0 jþE0 _ E ðB:1Þ where j is the plasma current due to the motion of the various charged particle species s, with density ns, charge qs, and velocity vs: X j ¼ nsqsvs s ðB:2Þ Considering the plasma to be a dielectric with internal currents j, we may write Eq. (B.1)as ∇ B¼μ0 _ D where D¼E0Eþ i ω j ðB:3Þ ðB:4Þ Here we have assumed an exp (iωt) dependence for all plasma motions. Let the current j be proportional to E but not necessarily in the same direction (because of the magnetic field B0zˆ); we may then define a conductivity tensor σ by the relation ©Springer International Publishing Switzerland 2016 F.F. Chen, Introduction to Plasma Physics and Controlled Fusion, DOI 10.1007/978-3-319-22309-4 417 j¼σ E ðB:5Þ Equation(B.4)becomes D¼E0 Iþ i E0ωσ E¼e E ðB:6Þ Thustheeffectivedielectricconstantoftheplasmaisthetensor e¼E0 Iþiσ=E0ω ð Þ ðB:7Þ whereIistheunit tensor. Toevaluateσ,we use the linearized fluidequationofmotionfor species s, neglecting the collision and pressure terms: ms ∂vs ∂t ¼qsEþvs B0 ð Þ ð:8Þ Defining the cyclotron and plasma frequency foreach species as ωcs qsB0 ms ω2 ps n0q2 s E0ms , ðB:9Þ

Space time correlation Photo/curved gravity

Plasma fusion for computational python models:

Collisions dominate, and the simple equations of ordinary fluid dynamics suffice. At the other extreme in very low-density devices like the alternating-gradient synchrotron, only single-particle trajectories need be considered; collective effects are often unimportant. Plasmas behave sometimes like fluids, and sometimes like a collection of individual particles. The first step in learning how to deal with this schizophrenic personality is to understand how single particles behave in electric and magnetic fields. This chapter differs from succeeding ones in that the E and B fields are assumed to be prescribed and not affected by the charged particles. 2.2 Uniform E and B Fields 2.2.1 E¼0 In this case, a charged particle has a simple cyclotron gyration. The equation of motion is mdv dt ¼ qv B Taking ^ z to be the direction of B (B¼B ^ z),

Python variables reference:

We have Switzerland 2016 F.F. Chen, Introduction to Plasma Physics and Controlled Fusion, DOI 10.1007/978-3-319-22309-4_2 ð2:1Þ 19 20 2 Single-Particle Motions m_ vx ¼ qBvy m_ vy ¼ qBvx m_ vz ¼ 0 2 € vx ¼qB m _ vy ¼ € vy ¼ qB m _ vx ¼ qB m qB m vx 2 vy ð2:2Þ This describes a simple harmonic oscillator at the cyclotron frequency, which we define to be ωc q j jB m ð2:3Þ By the convention we have chosen, ωc is always nonnegative. B is measured in tesla, or webers/m2, a unit equal to 104 G. The solution of Eq. (2.2) is then vx, y ¼ v⊥exp iωctþiδx,y the denoting the sign of q. We may choose the phase δ so that vx ¼ v⊥eiωct ¼ _ x ð2:4aÞ where v⊥ is a positive constant denoting the speed in the plane perpendicular to B. Then vy ¼m qB _ vx ¼ Integrating once again, we have x x0¼ i v⊥ 1 ωc _ vx ¼ iv⊥eiωct ¼ _ y ωc eiωct We define the Larmor radius to be y y0¼ v⊥ ωc eiωct rL v⊥ ωc ¼ mv⊥ q j jB Taking the real part of Eq. (2.5), we have x x0¼rLsinωct y y0 ¼ rLcosωct ð2:4bÞ ð2:5Þ ð2:6Þ ð2:7Þ This describes a circular orbit around a guiding center (x0, y0) which is fixed (Fig. 2.1). The direction of the gyration is always such that the magnetic field

Parker Solar Probe hydrazine thruster

Python variables-metrics(examples)

$\zeta_\mu = -h_{\mu\nu}\xi^\nu$, then direct calculations give $H_{\mu\nu;\lambda} = 2\, h_{\lambda(\mu}\xi_{\nu)} - g_{\lambda(\mu}\zeta_{\nu)}$, $K_{\mu\nu;\lambda} = 2\, h_{\lambda(\mu}\xi_{\nu)} - g_{\lambda(\mu}\zeta_{\nu)} + g_{\mu\nu}\, \zeta_\lambda$, $K_{\mu\lambda}{}^{;\lambda} = -2\, \zeta_\mu$.

(D.5.3) Lemma9: If$K_{\mu\nu}$ is a Killing tensor associated with the closed conformal Killing–Yano tensor $h_{\mu\nu}$ and ξ^μ is a primaryKilling vector then $\eta_\mu = K_\Lambda \xi^\lambda$ is a Killing vector. We call this Killing vector a secondary Killing vector. Proof: One has Thus, $\eta_{\mu;\nu} = K_{\mu\lambda;\nu}\xi^\lambda + K_{\mu\lambda}\xi^\lambda{}_{;\nu}$. $\eta_{(\mu;\nu)} = K_{\lambda(\mu;\nu)}\xi^\lambda + 1\ 2\ [K_{\mu\lambda}\xi^\lambda{}_{;\nu} + K_{\nu\lambda}\xi^\lambda{}_{;\mu}]$. Using the definition of the Killing tensor we get $K_{\lambda(\mu;\nu)} = -1$ (D.5.4) (D.5.5) (D.5.6) $2K_{\mu\nu;\lambda}$. The expression in the squared brackets in Eq.

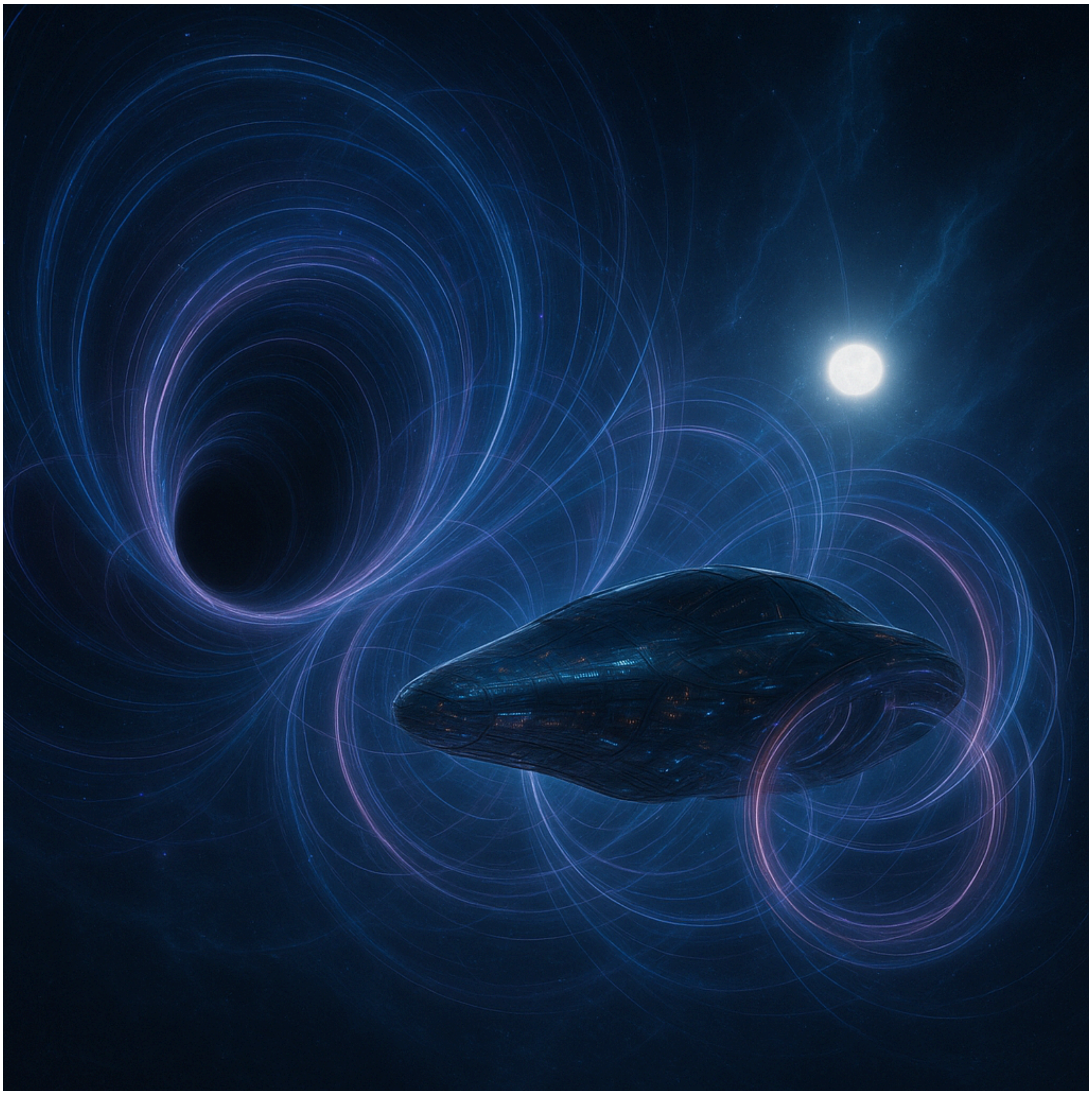
(D.5.6) can be rewritten as $K_{\mu\lambda}\xi^\lambda{}_{;\nu} + N\xi^\lambda{}_{;\mu} = \xi K_{\mu\nu} - \xi\kappa_{\mu\nu;\lambda}$. Combining these results we obtain $\eta_{(\mu;\nu)} = 1\ 2 L_\xi K_{\mu\nu} - \xi^\lambda K_{\mu\nu;\lambda}$. (D.5.7) (D.5.8) The first term vanishes because $K_{\mu\nu}$ is constructed in terms of $h_{\mu\nu}$ and $g_{\mu\nu}$, forwhich $L_\xi h_{\mu\nu} = 0$ and $L_\xi g_{\mu\nu} = 0$. Using Eq. (D.5.3) one can check that the second term also vanishes. Thus, we proved that $\eta_{(\mu;\nu)} = 0$,

6.1.1 Spherically symmetric spacetime Aspacetime is called spherically symmetric if there exist coordinates in which its metric takes the form $ds^2 = \gamma_{AB}dx^A dx^B + r^2 d\omega^2$, (6.1.1) where $\gamma_{AB} = \gamma_{AB}(x)$, $r = r(x)$ $(A,B = 0,1)$, and $d\omega^2$ is a metric on a unit 2-dimensional sphere S^2 $d\omega^2 = \omega_{XY}\, d\zeta^X d\zeta^Y$. $= d\theta^2 + \sin^2\theta\, d\varphi^2$. This metric admits 3 Killing vectors $\xi_1 = -\cos\varphi\partial_\theta + \cot\theta\, \sin\varphi\partial_\varphi$, $\xi_2 = \sin\varphi\partial_\theta + \cot\theta\, \cos\varphi\partial_\varphi$, $\xi_3 = \partial_\varphi$. Problem 6.1: Prove that the commutators of these vector fields are of the form where ijk is a 3D Levi-Civita symbol. $[\xi_i, \xi_j]= {}_{ijk}\, \xi_k$, (6.1.2) (6.1.3) (6.1.4) In the flat spacetime the vectors ξ_i are the generators of the rotation group, that is usual operators LX, LY and LZ of the angular momentum. Under the action of the symmetry transformations a point remains on a surface of constant radius r. Any two points on this 2D surface can be obtained from one another by the action of the symmetry transformation. Such a surface is called a transitivity surface of the symmetry group. Besides the continuous isometries generated by the Killing vectors, the metric Eq. (6.1.1) possesses discrete symmetries: $\varphi \rightarrow -\varphi$, $\theta \rightarrow \pi-\theta$. (6.1.5) Spherically Symmetric Gravitational Field 163 Problem 6.2: Let a, aX, and aX Y be scalar, vector, and tensor functions on S2 that are invariant under all the isometries of a two-sphere.Prove that a =const, aX =0, aX Y = const δX Y. (6.1.6) In a spherically symmetric spacetime a tensor $Q^\mu{}_\nu$ that obeys this symmetry satisfies the conditions

$L_i Q^\mu{}_\nu = 0$, $i = 1,2,3$. In the coordinates (x^A,ζ^X) it has the following form $Q^\mu{}_\nu =$ Q0 0 Q0 1 0 0 Q0 1 Q1 1 0 0 0 0 ˆ Q 0 0 0 0 ˆ Q , $\partial Q^\mu{}_\nu/\partial\zeta^X = 0$. (6.1.7) (6.1.8) In particular, this is valid for the Einstein tensor $G^\mu{}_\nu$. The conservation law $G^\mu{}_{\nu;\mu} = 0$, implies GA B:A + 2r:A r GA B − 2r:B r ˆ G=0. Here, (...):A is the covariant derivative in the 2D metric γ_{AB}. (6.1.9) 6.1.2 Dimensional reduction of the action In order to study the spherically symmetric spacetimes it is sufficient to substitute the metric ansatz Eq. (6.1.1) into the Einstein equations. But there exists an alternative way. Namely, one can substitute the ansatz Eq. (6.1.1) into the Einstein–Hilbert action Eq. (5.1.6). Since neither R nor depends on the angular variables, one can integrate over them. As a result, the Einstein–Hilbert action reduces to the action Ssph, which is a function of the two dimensional metric γ_{AB} and the scalar field r, Ssph = Ssph[γ,r]. Both of these field variables depend only on the

x^A coordinates, so that the original 4D problem is reduced to the 2D one. Let us note that in the general case, substitution of an ansatz chosen for the metric into the action does not guarantee that after the variation of the reduced action the obtained equations reproduce correctly the reduced Einstein equations. In a spherically symmetric case the variation of S_{sph} with respect to γ_{AB} and r gives a set of equations that is equivalent to the Einstein equations. Indeed, the variation of the Einstein Hilbert action Eq. (5.1.6) is1 $\delta S[g]=-\frac{1}{16\pi} V d^4x\sqrt{-g}E_{\mu\nu}\,\delta g_{\mu\nu}$, $E_{\mu\nu} = G_{\mu\nu} + g_{\mu\nu}$. (6. 1. 10

Image example: Important, see the image below to understand how the system works:



Physicists such as Frolov and Carter have explored how hidden symmetries inside black holes allow us to define conformal Killing-Yano tensors—geometric objects linked to conserved quantities in curved spacetime. These structures underlie some of the internal mechanisms of black hole geometry and help describe how matter behaves in such intense gravitational fields.

Now, if we shift the focus to plasma physics, especially within a gravitational background, things get truly interesting. Plasma—electrically charged and responsive to magnetic fields—can be described using magnetohydrodynamic (MHD) equations. These equations combine fluid dynamics and electromagnetism, forming the backbone of fusion research, space propulsion, and even astrophysical jet modeling.

When placed near or inside curved spacetime (like near a black hole), plasma contributes its own energy and momentum to the curvature itself through the Einstein Field Equation:

$$G^{\mu\nu} = (8\pi G / c^4) \cdot T^{\mu\nu}$$

This means the plasma isn't just influenced by gravity—it shapes it. That opens the door to ideas where plasma is intentionally manipulated to bend spacetime locally. Using magnetic confinement (such as toroidal shells), one could theoretically generate regions of weaker or asymmetric curvature and potentially useful for propulsion or energy extraction.

The vision sketched here goes beyond academic theory. Imagine a spacecraft enveloped in a magnetically stabilized plasma bubble that carves out its own geometry—moving not by brute force, but by following gradients in the local metric. If the gravitational field is gentle (similar to a neutron star or dense exoplanet, but not a classic black hole), such a bubble might allow for propulsion without destruction.

Materials like graphene aerogel, YBCO superconductors, diamond-sapphire composites, and carbon-carbon nanofoams, as well as liquid metals and nuclear fuels like tritium–deuterium, all point toward realistic pathways for building such systems. These aren't speculative inventions—they're in labs now, or already flying on spacecraft.

The question isn't whether this is possible in some distant future. The question is: how close are we to building the scaffolding needed to test these principles? As Hermann Minkowski suggested, space and time are not separate—they form a unity. We may be on the verge of bending that unity to serve propulsion.