Homework Assignment #1

Basic Magnetics and Loss Mechanisms

Chapter 10

Magnetics for Power Electronic Converters

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Analysis of an Inductor

Appendix B illustrates ferrite E cores, with the top and side views of an E core, reproduced below.

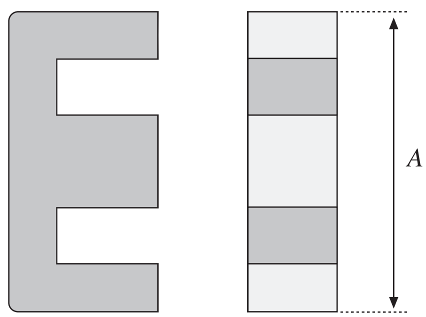

Figure 1 A ferrite E core.

Two of these E cores can be placed together to form a ferrite EE core set as illustrated in Fig. 2. The dimensions in mm of a specific EE 70 core set are included in this figure. The cores are symmetrical.

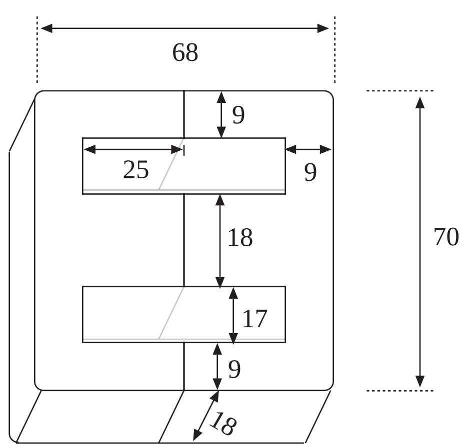

Figure 2 A ferrite EE core set. Dimensions in mm are labeled.

The center leg has square cross section, 18 mm $\times$ 18 mm. The outer legs have rectangular cross sections, 9 mm $\times$ 18 mm. The ferrite core material has relative permeability $\mu_r = 1000$. The mean magnetic path length ℓ_m for this core set is 170 mm; this is the distance traveled by magnetic flux flowing around the core through the center leg and one of the outside legs.

An inductor is constructed using this core set. An air gap of length ℓ_g is added to each of the three legs, and a winding of n turns is placed on the center leg. For this problem, $\ell_g = 0.1$ mm and $n = 20$. The wire gauge is # 16 AWG. The winding mean length per turn is $MLT = 14$ cm; this is the average length (circumference or perimeter) of one turn of the winding. The copper wire has resistivity $\rho = 1.724 \cdot 10^{-6}$ ohm cm at room temperature 25° C.

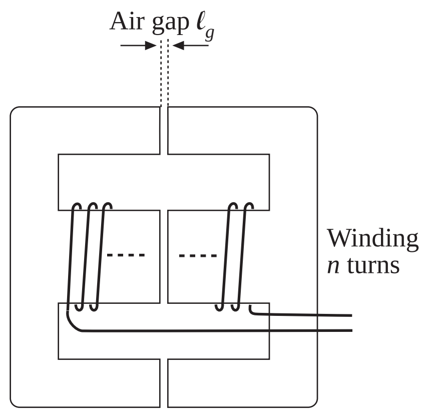

Figure 3 An inductor, consisting of the EE core of Fig. 2, air gaps placed in each core leg, and n turns of wire wound around the center leg.

1. What is the dc resistance of the winding at 25° C? Enter a numerical result in milliohms (mΩ), with an accuracy of ±0.5%.
2. What is the inductance of the winding? You should include the reluctances of the air gaps and the cores. Enter a numerical answer in microhenries (μH), with an accuracy of ±0.5%.
3. The saturation flux density of the ferrite core material is $B_{sat} = 0.4$ T. At what winding current does the inductor saturate? Again, account for the reluctances of the air gaps and the cores. Enter a numerical result in amperes (A), with an accuracy of ±0.5%.
4. When the winding current of Question 3 flows in the winding, what is the magnetomotive force (MMF) across the center leg air gap? Express your answer in amperes (A), with an accuracy of ±0.5%.
5. The manufacturer's data sheet includes the plot of Fig. 4 for the core loss of the ferrite material. This plot passes through the points
   - 0.2 W/cm^3 at $\Delta B = 0.1$T
   - 0.01 W/cm^3 at $\Delta B = 0.04$T

   Find the Steinmetz empirical equation for the core loss at 200 kHz, of the form

$$(power\ loss\ density) = K_{fe} B_{max}^{\beta}$$

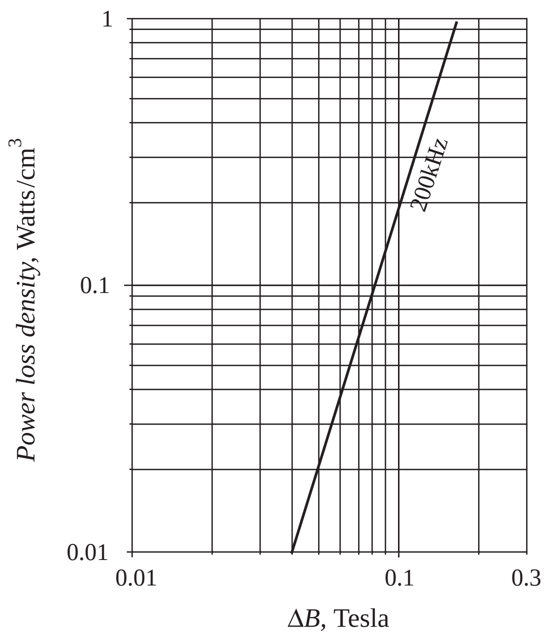

Figure 4 Core loss data for the ferrite core material.

where B_{max} is the peak sinusoidal flux density in Tesla (i.e., ΔB), and the power loss density is expressed in W/cm^3. Enter the numerical value of K_{fe}, accurate to $\pm 1\%$.

6. For the core loss data of Question 5, enter the numerical value of β, accurate to $\pm 1\%$
7. A sinusoidal current of magnitude 1.5 A rms and frequency 200 kHz flows through the inductor winding. Find the core loss, expressed in watts (W), with an accuracy of $\pm 1\%$.

Analysis of a Coupled Inductor Structure

The magnetic device illustrated in Fig. 5 contains two windings on two legs of an E-E core, with n_1 and n_2 turns respectively. For this problem, all three legs of the core have the same cross-sectional area A_c. Each leg has an air gap of length g. The core permeability μ_{core} is very large (so $\mu_{core} \to \infty$).

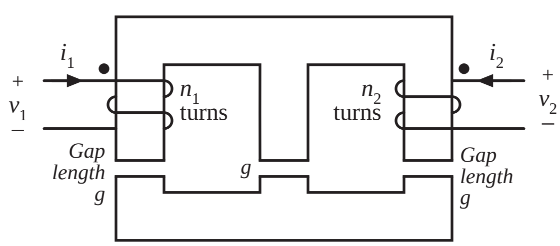

Figure 5 Magnetic device for Questions 8 to 12.

Derive a magnetic circuit model for this device, and use your model to answer Questions 8 to 12. Your answers should be analytical (math) expressions, written as functions of the following variable names:

- Winding 1 turns n_1: n1
- Winding 2 turns n_2: n2
- Air gap length g: g
- Cross-sectional area A_c: Ac
- Permeability of air (free space) μ_0: u

Neglect fringing, and assume that all core legs and air gaps have the same cross-sectional area A_c.

Write the electrical terminal equations of this device in the matrix form

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

8. Enter your analytical expression for L_{11} in the field below.
9. Enter your analytical expression for L_{12} below.
10. Enter your analytical expression for L_{22} below.
11. The two windings are connected in series as illustrated in Fig. 6

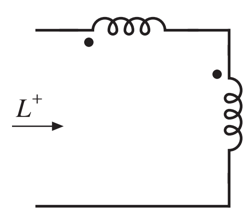

Figure 6 Series connection of the two windings, for Question 11

Note the winding polarities shown. Find an analytical expression for the inductance L^+, in terms of the variables defined in Question 8, and enter your expression below.

12. The two windings are connected in anti-series as illustrated in Fig. 7

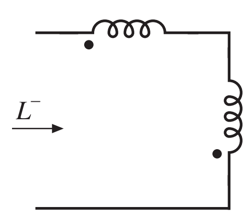

Figure 7 Anti-series connection of the two windings, for Question 12

Note the change in winding polarities. Find an analytical expression for the inductance L^-, in terms of the variables defined in Question 8, and enter your expression below.