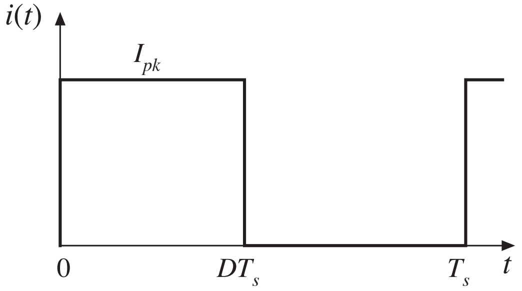
10.4.7 PWM waveform harmonics

Fourier series:

$$i(t) = I_0 + \sum_{j=1}^{\infty} \sqrt{2}\, I_j \cos(j\omega t)$$

with

$$I_j = \frac{\sqrt{2}\, I_{pk}}{j\pi} \sin(j\pi D) \qquad I_0 = DI_{pk}$$



Copper loss:

Dc $\quad P_{dc} = I_0^2 R_{dc}$

Ac $\quad P_j = I_j^2 R_{dc} \sqrt{j}\, \varphi_1 \left[G_1(\sqrt{j}\, \varphi_1) + \frac{2}{3}\left(M^2 - 1\right)\left(G_1(\sqrt{j}\, \varphi_1) - 2G_2(\sqrt{j}\, \varphi_1)\right) \right]$

Total, relative to value predicted by low-frequency analysis:

$$\frac{P_{cu}}{DI_{pk}^2 R_{dc}} = D + \frac{2\varphi_1}{D\pi^2} \sum_{j=1}^{\infty} \frac{\sin^2(j\pi D)}{j\sqrt{j}} \left[G_1(\sqrt{j}\, \varphi_1) + \frac{2}{3}\left(M^2 - 1\right)\left(G_1(\sqrt{j}\, \varphi_1) - 2G_2(\sqrt{j}\, \varphi_1)\right) \right]$$

Harmonic loss factor F_H

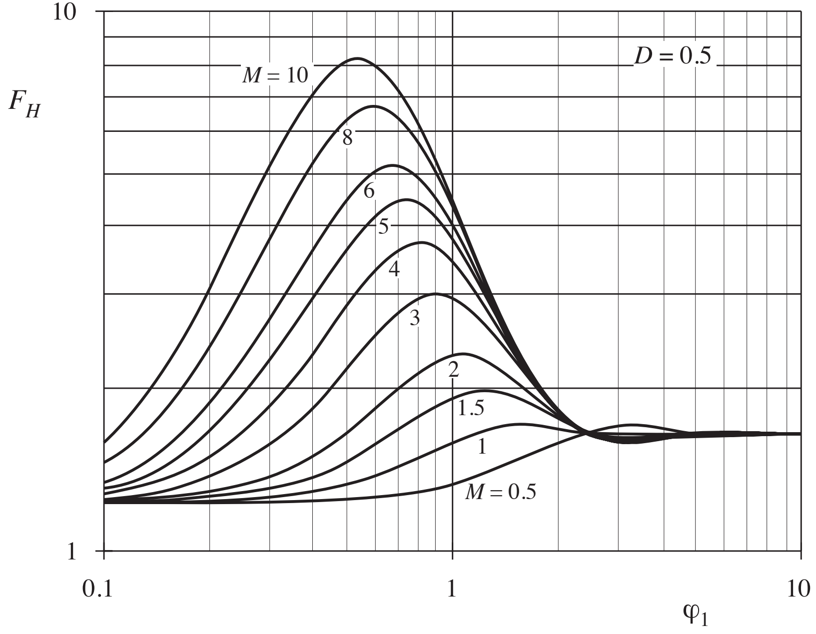
Effect of harmonics: F_H = ratio of total ac copper loss to fundamental copper loss

$$F_H = \frac{\sum_{j=1}^{\infty} P_j}{P_1}$$

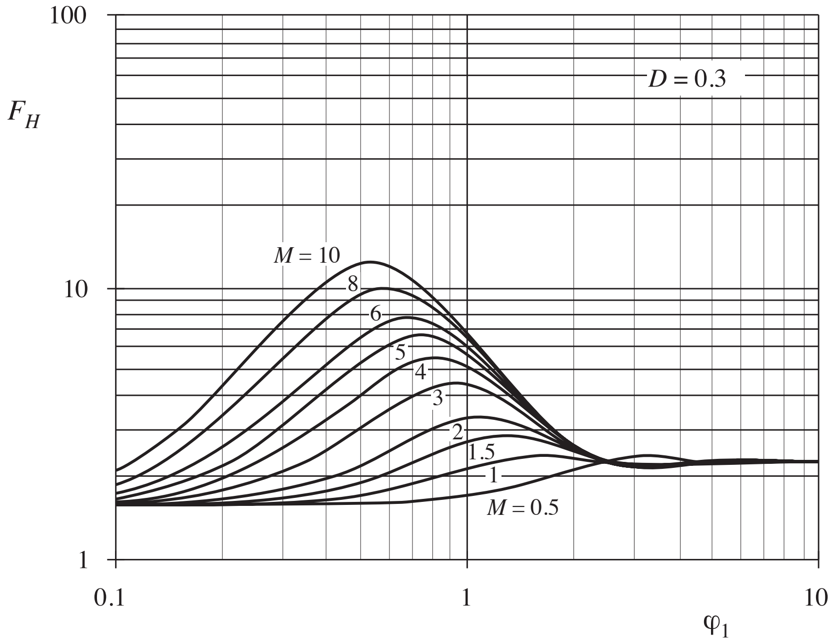
The total winding copper loss can then be written

$$P_{cu} = I_0^2 R_{dc} + F_H F_R I_1^2 R_{dc}$$

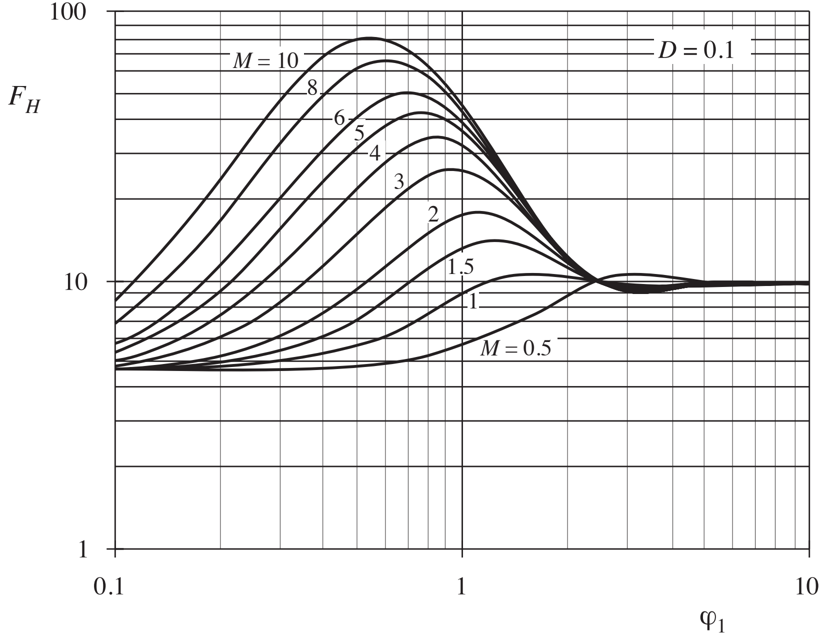
Increased proximity losses induced by PWM waveform harmonics: D = 0.5



Increased proximity losses induced by PWM waveform harmonics: D = 0.3



Increased proximity losses induced by PWM waveform harmonics: $D = 0.1$



Discussion: waveform harmonics

- Harmonic factor F_H accounts for effects of harmonics
- Harmonics are most significant for φ_1 in the vicinity of 1
- Harmonics can radically alter the conclusion regarding optimal wire gauge
- A substantial dc component can drive the design towards larger wire gauge
- Harmonics can increase proximity losses by orders of magnitude, when there are many layers and when φ_1 lies in the vicinity of 1
- For sufficiently small φ_1, F_H tends to the value $1 + (\mathrm{THD})^2$, where the total harmonic distortion of the current is

$$\mathrm{THD} = \frac{\sqrt{\sum_{j=2}^{\infty} I_j^2}}{I_1}$$