Black Holes and Limiting Curvature Gravity (LCG) Theory

Main Concepts

Black Holes:

- **Formation of New Universes:** Inside a black hole.
- **Bouncing Cosmological Solutions:** Describing oscillating universes.
- **Mass Inflation Problem:** Addressing growth of mass in certain scenarios.

Limiting Curvature Gravity (LCG) Theory:

- Covariant action imposing constraints on curvature invariants.
- **Subcritical Regime:**
  - $Q < 0$ →\rightarrow $\text{symbol}^2 = -Q$, $X = 0$.
- **Supercritical Regime:**
  - $Q = 0$ →\rightarrow $\text{symbol} = 0$, $X \neq 0$.

Mathematical Framework

- **Lagrangian:** L(q,q′)=L(q,q′)+Z[Q(q,q′)+symbol2].L(q, q') = L(q, q') + Z[Q(q, q') + \text{symbol}^2].
- **Constraints:** Q(q,q′)+symbol2=0,X×symbol=0.Q(q, q') + \text{symbol}^2 = 0, \quad X \times \text{symbol} = 0.

2D Limiting Curvature Gravity Model

- **Action:**
  ILCG=IDG+Ix,Ix=12∫d2x∣g∣X′(R−A+symbol2),I_{\text{LCG}} = I_{\text{DG}} + I_x, \quad I_x = \frac{1}{2}\int \mathrm{d}^2x\sqrt{|g|} X'(R - A + \text{symbol}^2),
  where $X' = X + \epsilon^2$.

- **Subcritical Regime:**
  X=0,C2=A−R.X = 0, \quad C^2 = A - R.
- **Equation for ϵ:**
  −ϵ(γ2+14R)ϵ=0.-\epsilon(\gamma^2 + \frac{1}{4}R)\epsilon = 0.
  Solution: $\epsilon = \exp(-\text{symbol}_0)$.

- **Metric for a Static 2D Black Hole:**
  ds2=−fdt2+f−1dr2,f=1−mγe−2γr.ds^2 = -f dt^2 + f^{-1} dr^2, \quad f = 1 - \frac{m}{\gamma}e^{-2\gamma r}.
- **Boundary Conditions:**
  P(±π2)=±π2,Q(±π2)=±π2.P(\pm\frac{\pi}{2}) = \pm\frac{\pi}{2}, \quad Q(\pm\frac{\pi}{2}) = \pm\frac{\pi}{2}.

Key Variables

- q, q': Coordinates and derivatives.
- X, symbol: Lagrange multipliers.
- A, R: Parameters in curvature equations.
- γ, β: Scaling constants.
- P, Q: Null coordinates.

Formulas for Curvature Metrics

- Boundary Value Condition: RA−RH=12γlnβ,β=4γ2A<1.R_A - R_H = 12\gamma \ln \beta, \quad \beta = \frac{4\gamma^2}{A} < 1.
- Temperature: T−,A=1γarccosβ.T_{-,A} = \frac{1}{\gamma} \arccos \sqrt{\beta}.

Python Script

```
import numpy as np
import matplotlib.pyplot as plt

def metric_function(r, m, gamma):
    return 1 - (m / gamma) * np.exp(-2 * gamma * r)

# Parameters
m = 1.0   # Mass parameter
gamma = 0.5  # Scaling constant
r = np.linspace(0, 10, 500)

# Metric computation
f = metric_function(r, m, gamma)

# Plotting the metric
plt.plot(r, f, label="Metric function f(r)")
plt.axhline(0, color='red', linestyle='--', label="Event Horizon")
plt.xlabel("r")
plt.ylabel("f(r)")
plt.title("Metric Function of a Static 2D Black Hole")
plt.legend()
```

```
plt.grid()
plt.show()
```

GNU Octave Script

```
% Parameters
m = 1.0; % Mass parameter
gamma = 0.5; % Scaling constant
r = linspace(0, 10, 500);

% Metric function
f = 1 - (m / gamma) * exp(-2 * gamma * r);

% Plot
figure;
plot(r, f, 'b', 'LineWidth', 1.5);
hold on;
line([0, 10], [0, 0], 'color', 'red', 'linestyle', '--', 'linewidth',
1.5);
xlabel('r');
ylabel('f(r)');
title('Metric Function of a Static 2D Black Hole');
grid on;
hold off;
```

Summary

This document outlines the theoretical framework of black holes in the context of Limiting Curvature Gravity (LCG) theory. Python and Octave scripts are provided for simulating the metric of a static 2D black hole, which serves as a foundation for understanding the behavior of curvature and associated metrics in this model.