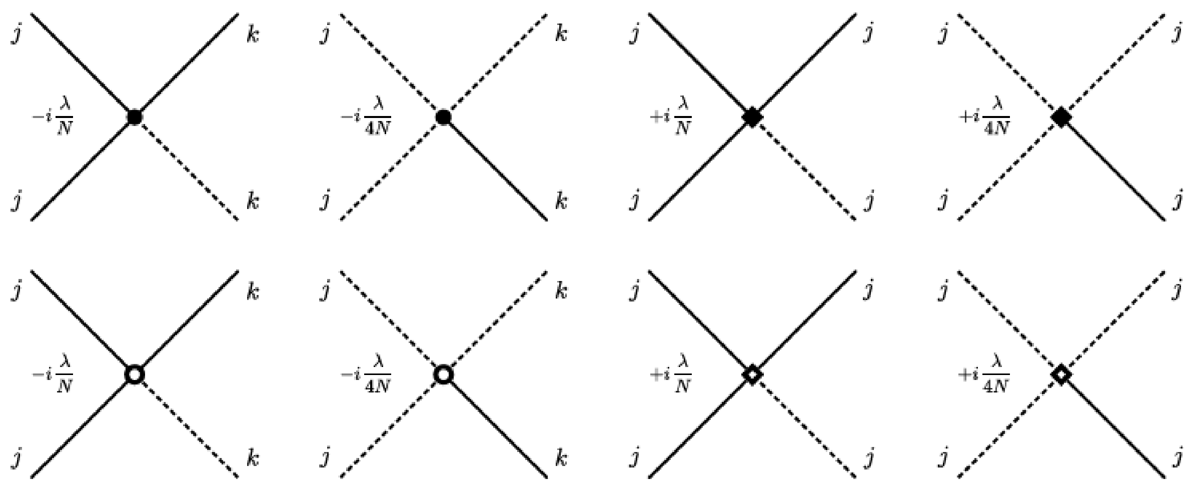
Introduction

In classical chaotic systems, even a minor disturbance in initial conditions leads to an exponential divergence of trajectories, represented as z(t)≈eλtz(0)z(t) \approx e^{\lambda t} z(0)z(t)≈eλtz(0), where zzz denotes the phase space distance between two trajectories, and λ\lambdaλ is the Lyapunov exponent. This sensitivity to initial conditions underpins the classical "butterfly effect," foundational to fields like thermodynamics and hydrodynamics.

Quantum systems, however, don’t exhibit this exponential trajectory divergence directly due to the uncertainty principle, which restricts the infinitesimal shifts that define classical Lyapunov exponents. Instead, alternative metrics—such as Out-of-Time-Ordered Correlators (OTOCs), spectral statistics, and entanglement entropy—are used to capture quantum chaos, revealing how quantum correlations dissipate over time. This loss of correlation is particularly intriguing in the context of quantum gravity, where spacetime itself may behave chaotically at small scales.



Quantum Chaos and the Quantum Butterfly Effect

In quantum systems, the butterfly effect is represented by the OTOC, a diagnostic that approximates classical Lyapunov behavior in a quantum context. The OTOC function Cij(t)=⟨[qi(t),pj(0)]2⟩C_{ij}(t) = \langle [q_i(t), p_j(0)]^2 \rangleCij(t)=⟨[qi(t),pj(0)]2⟩ reflects how two initially commuting operators (like position and momentum) can develop correlations over time, effectively "scrambling" information across the system.

This quantum butterfly effect, when viewed through the lens of quantum gravity, suggests a fundamental instability or sensitivity in spacetime geometry itself. In quantum gravity theories, particularly those considering black holes or wormholes, OTOCs have been connected to the idea of "scrambling" within a black hole’s event horizon, where information is rapidly lost, pointing to the chaotic nature of spacetime at quantum scales.

Summary: Quantum Chaos, Lyapunov Exponent, and Spatial Quantum Waves in Non-Symmetric Systems

The butterfly effect in quantum systems manifests through the Out-of-Time-Ordered Correlator (OTOC), a tool that parallels classical chaos diagnostics by revealing how quantum correlations dissipate. In quantum gravity, this dissipation hints at the underlying chaotic nature of spacetime. In a specialized model, we analyze quantum chaos by intentionally breaking O(N)O(N)O(N) symmetry. This breakdown, achieved by excluding self-interaction terms, disrupts the integrable structure of the model, introducing richer dynamics that include non-zero quantum Lyapunov exponents under certain conditions.

When the symmetry is preserved, there are NNN conserved quantities, making the system classically integrable and reducing it to a manageable form with predictable dynamics. However, in the deformed, non-symmetric version, the dynamics become more complex and can be characterized by a small but positive quantum Lyapunov exponent at high temperatures, indicating sensitivity to initial conditions even in the quantum regime.

Integrating this with recent observations, a type of spatial quantum wave has been detected, presenting characteristics of non-linear oscillations that propagate through high-dimensional quantum fields. These waves resonate with chaotic dynamics, and their behavior could provide insights into the unpredictable nature of quantum spacetime, potentially observable in Qiskit Metal circuits. Together, these concepts form a framework for exploring how quantum systems, spacetime dynamics, and chaotic signals interact at the smallest scales.

In the study of quantum field theories, specifically within the model described in equation (1.3), propagators and vertices undergo loop corrections that are summed to the leading order, denoted as O(1)O(1)O(1). These corrections, which occur within a two-fold Keldysh contour framework, contribute to the refinement of the system's dynamics.

At the leading order O(1/N)O(1/N)O(1/N), the main contribution to the corrections is from diagrams known as tadpoles or cactus diagrams. These diagrams represent shifts in the tree-level mass of the system and are critical for understanding how the propagators evolve. The tadpole diagrams effectively modify the propagator masses by adding corrections to the bare (tree-level) values. These corrections can be calculated using Dyson-Schwinger equations, which describe the self-consistent propagation of particles in the system.

In the context of the O(N)O(N)O(N)-symmetric and full models, the loop corrections to propagators are essentially the same in leading order, as the nonsymmetric vertices become negligible. This means that, for the purposes of calculating the propagator, the systems behave similarly under both approximations.

The propagators themselves are modified through these corrections. The Dyson-Schwinger equations for the Green's function (denoted GRGRGR, GKGKGK, and GWGWGW) provide a recursive relationship, where the corrected propagators are related to the initial propagators and the loop-corrected terms. These terms involve integrals over time, indicating the way particles propagate through the system under quantum corrections.

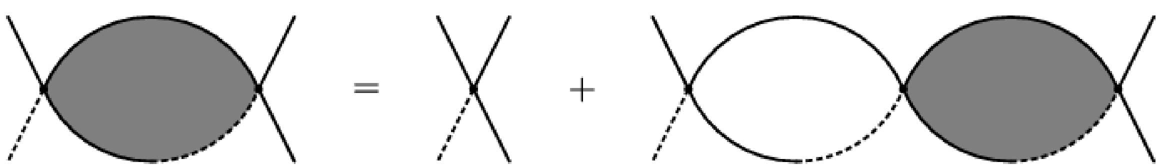
To summarize:

1. **Loop corrections**: The primary loop corrections come from tadpole diagrams, which shift the mass of the system.
2. **Dyson-Schwinger equations**: These equations are used to describe how the propagators evolve due to quantum corrections.
3. **Symmetry considerations**: In the O(N)O(N)O(N)-symmetric model, the loop corrections are the same as those in the full model at leading order, due to the suppression of nonsymmetric interactions.

Quantum Gravity and Higher-Dimensional Chaos

In quantum gravity, the chaotic behavior of spacetime is a topic of considerable interest. One approach is through models that propose higher-dimensional spaces, where quantum fluctuations in spacetime might follow chaotic dynamics similar to those seen in the classical butterfly effect. This chaotic behavior in higher dimensions implies that perturbations in quantum fields could propagate non-linearly, affecting structures such as black holes or quantum circuits in unpredictable ways.

Theories of quantum gravity that incorporate higher dimensions, like string theory, propose that quantum fluctuations are amplified in compactified dimensions. Here, Lyapunov exponents in higher-dimensional spaces can describe how quantum fields diverge or "scramble" over time, paralleling chaotic dynamics in classical systems. This perspective can lead to a broader understanding of spacetime instability and may offer insights into gravity's quantum nature.



Implementing Quantum Chaos in a Qiskit Metal Circuit

In Qiskit Metal, a superconducting quantum circuit design framework, it’s possible to simulate aspects of quantum chaos. One could model an array of coupled Josephson junctions that exhibit quantum chaotic behavior under certain conditions. By adjusting parameters, it’s feasible to emulate a system where OTOCs or spectral statistics can be measured, observing how these diagnostics evolve over time.

For example, a network of quantum resonators or qubits in a Qiskit Metal simulation could act as a “quantum chaotic circuit” by introducing controlled perturbations to initial states and tracking how quantum entanglement, OTOCs, or spectral form factors behave. This approach could mirror quantum gravity’s chaotic dynamics, providing a testing ground for theories where spacetime behaves like a quantum chaotic system.

Leading Order Corrections to Vertices and Their Application in Quantum Channels or Circuits

The Dyson-Schwinger equation for summing leading order loop corrections to vertices in a quantum model (equation 1.3) can be extended to quantum channels or circuits, especially in the context of quantum information processing. The corrections to the vertices are represented by bubble chain diagrams, which are a series of loop corrections that modify the interaction vertices in the system.

The general form of the Dyson-Schwinger equation that sums the corrections to a vertex is:

B(t1,t2)=δ(t1,t2)+2i∫0∞dt3 GR(t1,t3) GK(t1,t3) B(t3,t2)B(t_1, t_2) = \delta(t_1, t_2) + 2i \int_0^\infty dt_3 \, GR(t_1, t_3) \, GK(t_1, t_3) \, B(t_3, t_2)B(t1,t2)=δ(t1,t2)+2i∫0∞dt3GR(t1,t3)GK(t1,t3)B(t3,t2)

Here, B(t1,t2)B(t_1, t_2)B(t1,t2) denotes the resummed chain of leading order corrections to the vertex, while GRGRGR and GKGKGK represent the resummed propagators. The equation sums up the contributions from multiple loops (bubbles) to the vertex, with the integral accounting for the time evolution in the system.

For quantum circuits, where time evolution is of critical importance, this equation can be applied to the interactions between qubits or other quantum systems. The key idea is that these loop corrections introduce modifications to the interaction vertices, leading to a more accurate description of the quantum dynamics.

Key Features:

1. **Bubble Chain Diagrams**: The loop corrections to the vertices are illustrated by bubble chain diagrams. These diagrams represent a series of quantum fluctuations that modify the interaction strength at each vertex in the quantum circuit.
2. **Dyson-Schwinger Equation**: This equation describes how the resummed propagators (Green's functions) evolve over time and modify the system's dynamics. For quantum circuits, the propagators could correspond to the evolution of quantum states, such as qubit interactions or entanglement dynamics.
3. **Vertex Equation**: The equation for the vertex B(t1,t2)B(t_1, t_2)B(t1,t2) shows how it is related to the propagators and how it evolves over time. It highlights the importance of time-ordering and loop corrections, which modify the strength and nature of interactions between quantum components (such as qubits in a quantum circuit).
4. **Ansatz for Solution**: The ansatz proposed to solve this equation suggests that the vertex correction behaves similarly to a single bubble, incorporating oscillatory functions. This ansatz can be applied to quantum channels or circuits, where the corrections to interactions manifest as oscillations or phase shifts in quantum states.
   The ansatz is:

are parameters to be determined. This form reflects how the corrections oscillate over time, which could be related to phenomena such as coherence or entanglement dynamics in quantum channels.

5. **New Vertices and Causality**: New types of vertices appear after summing the corrections. However, due to causality reasons, these new vertices do not contribute to the perturbative expansion of higher-point correlators in the system.

Application to Quantum Circuits:

In the context of quantum channels or quantum circuits, such as those simulated with tools like Qiskit, this formalism can be useful for understanding the impact of quantum noise, decoherence, and interaction corrections in a system of qubits. These quantum circuit models can be viewed as systems with vertices corresponding to interactions between quantum gates (e.g., CNOT gates, Hadamard gates), and the Dyson-Schwinger equation helps refine the modeling of how errors or perturbations propagate through the circuit.

For example:

- **Quantum Noise**: The loop corrections could model how quantum noise affects the fidelity of a quantum circuit over time.
- **Entanglement Dynamics**: The ansatz for the corrected vertex could describe how entanglement evolves in a quantum system, introducing oscillatory patterns that reflect the quantum correlations.

Thus, by applying the concepts of Dyson-Schwinger equations and resummed propagators, we can improve the simulation of complex quantum circuits and better understand the subtleties of quantum chaos and information processing in quantum channels.

To implement the given equation in python

```
import numpy as np

# Define the function B(t1, t2)

def B(t1, t2, m):

    t12 = np.abs(t1 - t2)  # Calculate t12 = |t1 - t2|

    if t12 == 0:

        return 0  # Avoid division by zero

    else:

        return m * (np.sin(m * t12) / t12)  # Apply the formula
```

```
# Example usage

t1 = 2.0  # Time t1

t2 = 1.0  # Time t2

m = 1.5   # Value of m


# Calculate B(t1, t2)

result = B(t1, t2, m)

print(f'B({t1}, {t2}) = {result}')
```

Explanation/variables in functions:

1. `B(t1, t2, m)` **function**: This function accepts two time values t1t_1t1 and t2t_2t2, and a constant mmm. It calculates t12t_{12}t12 as the absolute difference between t1t_1t1 and t2t_2t2, then evaluates the expression m⋅sin(mt12)t12m \cdot \frac{\sin(m t_{12})}{t_{12}}m⋅t12sin(mt12), as required by the equation.
2. **Avoiding division by zero**: In cases where t1=t2t_1 = t_2t1=t2 (leading to t12=0t_{12} = 0t12=0), the function returns 0 to prevent indeterminate values.
3. **Using NumPy**: The code uses `numpy` for efficient computation of absolute values (`np.abs`) and the sine function (`np.sin`).

This code allows you to compute B(t1,t2)B(t_1, t_2)B(t1,t2) for any pair of time values t1t_1t1 and t2t_2t2, along with the constant mmm.

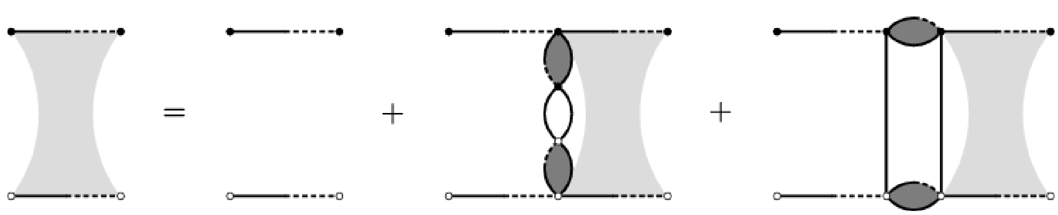


Figure 4: A diagrammatic representation of the Bethe-Salpeter equation that sums the leading order corrections to the averaged correlator (2.2). The lines denote the resummed tadpole diagrams from Fig. 2. The dark gray loops denote the resummed bubble chain diagrams from Fig. 3. The light gray block denotes the resummed four-point correlator (2.2).