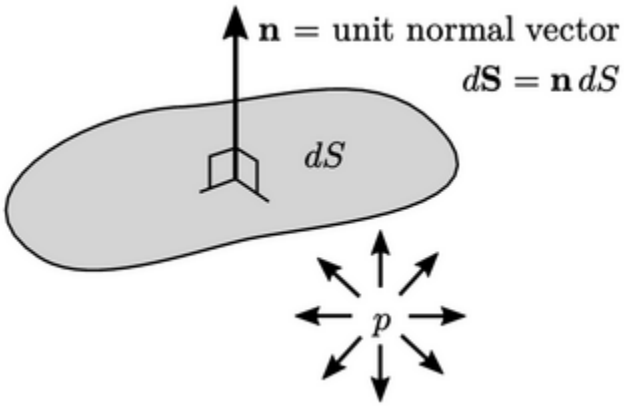
Pressure

The equations of fluid dynamics in CFD treat the fluid as a continuous medium, or *continuum*. It is continuous in the sense that we consider the fluid as having no “empty space” by ignoring its molecular nature. We assume it has properties that vary from point to point and are continuous throughout the solution domain, and whose derivatives are also continuous.

Pressure is an important property of a fluid, denoted by p. It describes the amount of force per unit surface area which *acts on* a surface, in the direction *perpendicular* to the surface.



Pressure p is a *scalar* that produces a force *vector* with direction normal to the surface. When pressure is applied to one side of a segment of surface with area dS, the force $d\mathbf{f}$ points *away* from that side by

$$d\mathbf{f} = (\mathbf{n}\, dS)\, p = d\mathbf{S}\, p, \tag{2.3}$$

where $\mathbf{n}$ is the vector of *unit length*, normal to the surface dS. The term *vector* denotes a geometric entity with magnitude and direction; a surface can be represented by a *surface area vector* $d\mathbf{S} = \mathbf{n}\, dS$ of magnitude dS and direction $\mathbf{n}$.

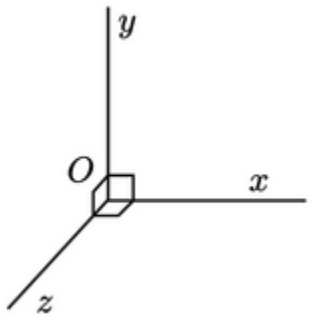
While pressure exerts a force *on a surface*, the fluid experiences a force which is compressive in nature, assuming p is positive.

Pressure is measured in SI units of pascal $\mathrm{Pa} \equiv \mathrm{kgm^{-1}s^{-2}}$. From Sec. 2.13 onwards, however, we generally use p to represent the *kinematic* pressure, in units $\mathrm{m^2s^{-2}}$, obtained by dividing the true pressure by a constant density ρ.

Scalar fields

The majority of properties, *e.g.* pressure, temperature, energy, density, volume, *etc.*, can be represented by a single number, or *scalar*. A *scalar field* describes a scalar property, *e.g.* pressure, which varies from point to point across some spatial domain.

Point locations can be defined in any coordinate system of axes, *e.g.* Cartesian (x, y, z), and in any orientation. A scalar field is *invariant*, meaning the scalar values are the same irrespective of the coordinate system used.

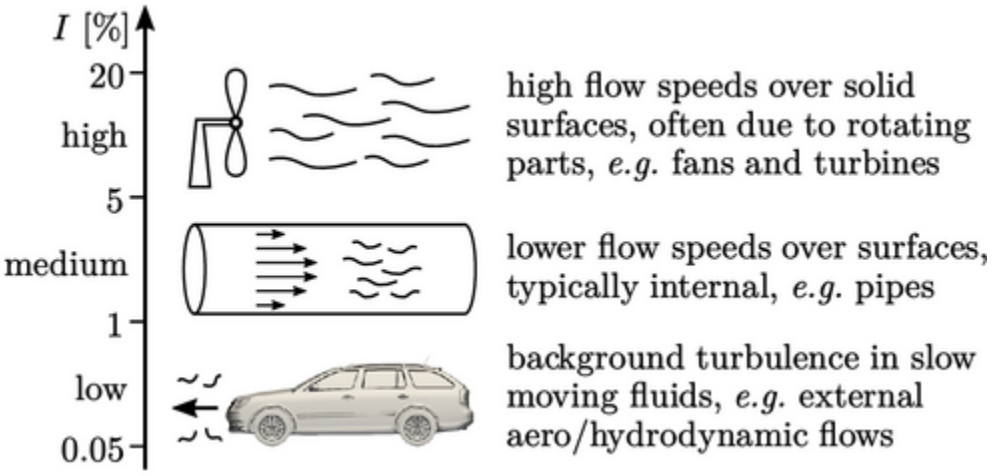


In this book, space and fields will be described in a coordinate system with right-handed rectangular Cartesian axes. The axes are constructed by defining an origin O from which three lines are drawn at right angles to each other, termed the Ox, Oy, Oz axes. A right-handed set of axes requires that looking down the Oz axis with O nearest, an arc from the Ox axis to the Oy the axis is in a clockwise sense.

Inlet turbulence

Expressions are presented in Sec. 7.2 to estimate inlet and initial values of k_{in} and $\varepsilon_{\mathrm{in}}$. They include parameters I and l_{m} which must themselves be estimated sufficiently accurately to calculate k_{in} and $\varepsilon_{\mathrm{in}}$ reliably.

The values of I and l_m at domain inlets depend on the flow conditions upstream of the inlet. The figure below shows typical ranges of intensity I for different upstream flow conditions.



A medium intensity $1\% < I < 5\%$ is most commonly specified in CFD problems, in particular for internal flows. For these flows, I can be calculated from a power-law function of Re, fitted to measurements at the central axis in fully developed flow along a smooth-wall pipe, according to[4]

$$I = 0.055\mathrm{Re}^{-0.041}. \tag{7.7}$$

An estimate of $l_\mathrm{m} = 0.07D$ at the centre axis of a pipe, see Sec. 6.12, can be used in conjunction with I from Eq. (7.7). For ducts and channels of non-circular cross-section, D can be calculated by $D = 4 \times A/P$, where A is the cross-sectional area and P is the perimeter length. For a partially filled pipe or duct, P corresponds to the wetted region where the fluid is in contact with the boundary.

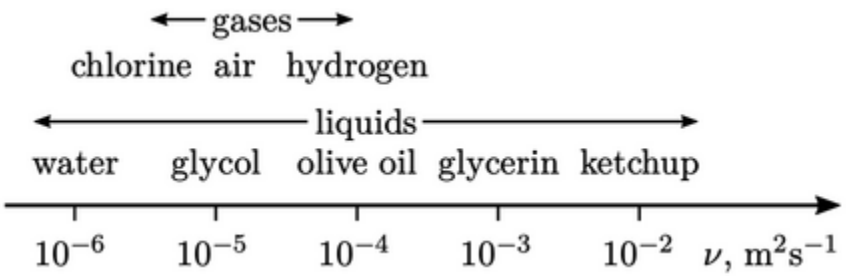
For wall-bounded flows with a boundary layer of thickness δ, an estimate of $l_\mathrm{m} = \kappa\delta \approx 0.4\delta$ is often used. This relation (see also Sec. 6.12) requires δ to be estimated, *e.g.* from the δ/x expression for a turbulent layer at the end of Sec. 6.4.

Verifying turbulent viscosity

Combining Eq. (7.4), Eq. (7.6) and Eq. (6.31) gives the following expression for ν_t in terms of length l_m and velocity $|\mathbf{u}|I$ scales:

$$\nu_\mathrm{t} = c_\mu \frac{k_\mathrm{in}^2}{\varepsilon_\mathrm{in}} = \left(\frac{3}{2}\right)^{\frac{1}{2}} c_\mu^{1/4} \, |\mathbf{u}|I \, l_\mathrm{m} \approx 0.67 \, |\mathbf{u}|I \, l_\mathrm{m}. \tag{7.8}$$

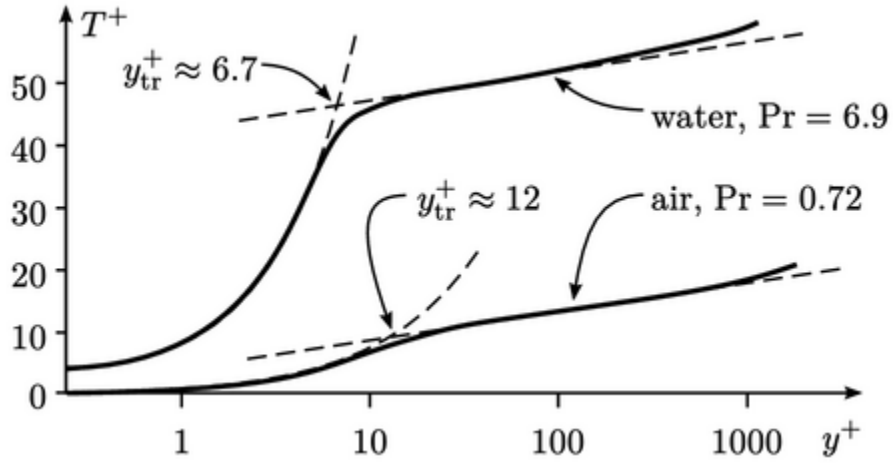
Values of ν_t need to be realistic. Realistic values usually fall within the range of molecular viscosities ν for common fluids at standard temperature shown below.



The range is presented in terms of *kinematic* viscosity ν which governs the *rate* of momentum *diffusion*, *e.g.* the rate of growth of boundary layers. By contrast, *forces* are governed by dynamic viscosity $\mu = \rho\nu$, which make liquids “feel” more viscous.

Thermal wall functions

Wall functions were introduced in Sec. 7.5 in order to improve the calculation of wall shear stress τ_w when cells are too large near a wall to resolve $\partial u_x/\partial y$ accurately. The same problem exists with heat flux q_w and an under-predicted $\partial T/\partial y$. As before, the universal character of the boundary layer can be exploited, this time to improve the calculation of q_w.



The temperature distribution is characterised by Eq. (7.51) for the viscous sub-layer, and the log law Eq. (7.52) for the inertial sub-layer. The transition y_{tr}^+ for T occurs at the intersection of the two equations, *i.e.* when

$$y_{\mathrm{tr}}^+ = \frac{\mathrm{Pr_t}}{\mathrm{Pr}} \frac{1}{\kappa} \ln y_{\mathrm{tr}}^+ + \frac{1}{\mathrm{Pr}} B_T \tag{7.57}$$

While the transition between these regimes is fixed at $y_{\mathrm{tr}}^+ \approx 11$ by Eq. (7.18) for the u_x profile, the (iterative) solution of Eq. (7.57) is dependent on Pr and $\mathrm{Pr_t}$.

Using $\mathrm{Pr_t} = 0.85$ and Eq. (7.55) for B_T, $y^+_{\mathrm{tr}} \approx 12$ for air at $20\,^{\circ}\mathrm{C}$ with $\mathrm{Pr} = 0.72$. For water under the same conditions, $\mathrm{Pr} = 6.9$ and the corresponding $y^+_{\mathrm{tr}} \approx 6.7$.

A wall function can be derived which adjusts the turbulent conductivity κ_{t}, in a similar manner to ν_{t} in the standard wall function in Sec. 7.5. The model calculates κ_{t} for each patch face based on the near-wall cell y^+_{P}.

No adjustment is made to κ_{t} when y^+_{P} corresponds to the viscous sub-layer. When y^+_{P} corresponds to the inertial sub-layer, κ_{t} is calculated as

$$\kappa_{\mathrm{t}} = \kappa_{\ell}\left(\frac{\mathrm{Pr}\, y^+_{\mathrm{P}}}{\mathrm{Pr_t} \ln y^+_{\mathrm{P}}/\kappa + B_T} - 1\right) \quad \text{for} \quad y^+_{\mathrm{P}} > y^+_{\mathrm{tr}}, \tag{7.58}$$

where κ_{ℓ} denotes the laminar thermal conductivity, *i.e.* κ from Eq. (2.54), to distinguish it from Kármán's constant κ. Eq. (7.58) uses y^+_{P} from Eq. (7.19), as in the standard ν_{t} wall function.

The wall function is derived based on adjusting κ_{t} to improve the numerical calculation of q_{w} by

$$q_{\mathrm{w}} = -\kappa_{\mathrm{eff}} \frac{\partial T}{\partial y} = -(\kappa_{\mathrm{t}} + \kappa_{\ell}) \frac{T - T_{\mathrm{w}}}{y}, \tag{7.59}$$

where T represents a value close to the wall, *e.g.* in a near-wall cell. By comparison, Eq. (7.49) and Eq. (7.50) combine to give

$$q_{\mathrm{w}} = -\frac{y^+}{T^+}\frac{T - T_{\mathrm{w}}}{y}\,\mathrm{Pr}\,\kappa_\ell. \tag{7.60}$$

The heat flux q_{w} is consistent between Eq. (7.59) and Eq. (7.60) when

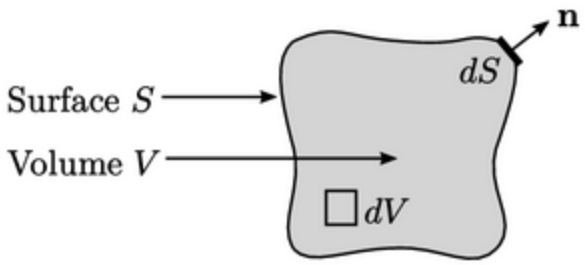
$$T^+ = \frac{\kappa_\ell}{\kappa_{\mathrm{t}} + \kappa_\ell}\,\mathrm{Pr}\,y^+. \tag{7.61}$$

When the log law Eq. (7.52) is then substituted in Eq. (7.61), it provides the thermal wall function model which adjusts κ_{t} according to Eq. (7.58).

This chapter introduces fluid dynamics for CFD. It describes: governing equations, *i.e.* conservation of mass, momentum and energy; and, associated physical models, *e.g.* for viscosity, heat conduction and thermodynamics.

The equations describe fluid motion, forces and heat in time and three-dimensional (3D) space. Vector notation provides a mathematical framework to present the equations in a compact form. It enables the equations to be presented independently of any co-ordinate system, *e.g.* Cartesian ($x/y/z$) or spherical ($r/\theta/\varphi$). It includes a standard set of algebraic operations, *e.g.* the inner (dot) and outer products.

The notation helps to ensure that the terms in equations are unchanged, or *invariant*, under a co-ordinate system transformation. Without invariance, a flow solution, *e.g.* along a pipe, would be dependent on the orientation of the pipe with respect to the co-ordinate system. Logically this dependence cannot exist; the laws of motion are the same in all "inertial frames".1



The derivation of the governing equations uses a control volume V bounded by a surface S, presented using the two-dimensional (2D) illustration above. We use dV and dS to describe an infinitesimally small volume and surface, respectively, and $\mathbf{n}$ is the unit normal vector for each increment of surface dS, discussed in Sec. 2.1. It is important to note in any derivation whether the volume is defined as fixed in space or moving with the fluid.

Each derivation generally begins with a definite integral of some quantity, *e.g.* $\mathbf{\Psi}$, over the volume V denoted by

$$\int_V \mathbf{\Psi}\, dV. \tag{2.1}$$

If this notation is unfamiliar, understand it to mean a summation for all increments of volume dV that make up the total volume V. The summed values are $\mathbf{\Psi} \times dV$, where $\mathbf{\Psi}$ is the value in the respective dV.

The derivations also use integrals over the surface S, *e.g.*

$$\int_S \mathbf{n}\mathbf{\Psi}\, dS \quad \text{or} \quad \int_S (d\mathbf{S}\, \mathbf{\Psi}),$$